

САМ СЕБЕ РЕПЕТИТОР®

ОТВЕТЫ и РЕШЕНИЯ

+ РЕШЕНИЯ ЗАДАЧ ПОВЫШЕННОЙ ТРУДНОСТИ
И НА ПОСТРОЕНИЕ

10



К задачкику

А.Г. Мордковича, Л.О. Денищевой и др.

АЛГЕБРА
и начала анализа

А.А. Белова

ПОДРОБНЫЙ РАЗБОР ЗАДАНИЙ ИЗ ЗАДАЧНИКА ПО АЛГЕБРЕ И НАЧАЛАМ АНАЛИЗА

10 класс

авторов: А.Г. Мордкович
Л.О. Денищева
Т.А. Корешкова и др.
(М.: Мнемозина, 2002-2004)

+ ЗАДАЧИ ПОВЫШЕННОЙ СЛОЖНОСТИ

+ ЗАДАЧИ НА ПОСТРОЕНИЕ

Москва «ВАКО» 2004

Белова А.А.

Б43 Алгебра и начала анализа. 10–11 кл. Подробный разбор заданий из учебника авторов: А.Г. Мордкович, Л.О. Денищева, Т.А. Корешкова и др. – М.: ВАКО, 2004. – 208 с. – (Сам себе репетитор).

ISBN 5-94665-171-4

Издание содержит алгоритмы решения типовых задач, подробный разбор абсолютно всех заданий, включая задачи на построение графиков и задачи повышенной сложности из учебника по алгебре для 10–11 класса авторов: А.Г. Мордкович, Л.О. Денищева, Т.А. Корешкова и др. (М.: Мнемозина, 2002–2004).

Пособие будет незаменимым помощником школьникам при приготовлении домашних работ, подготовке к экзамену, а также будет способствовать обретению прочных навыков самопроверки.

УДК 373.167:1:[512+517.1]
ББК 22.141я721+22.161я721

Глава 1. Тригонометрические функции

§1. Введение

В задачах 1 – 8 требуется найти длину дуги. Она находится по формуле: $l = \alpha \cdot r$, где α – радианная мера дуги, r – радиус окружности. Так как рассматривается единичная окружность, то $r = 1$, т.е. $l = \alpha$. Переход от градусной к радианной мере:

$$\alpha = \frac{\beta}{180} \cdot \pi, \text{ где } \beta - \text{мера угла в градусах.}$$

1. $AM = 90^\circ + 45^\circ = 135^\circ$, $BK = 90^\circ + 30^\circ = 120^\circ$.

$$MP = 45^\circ + 60^\circ = 105^\circ, DC = 270^\circ,$$

$$KA = 150^\circ, BP = 150^\circ,$$

$$CB = 270^\circ, BC = 90^\circ.$$

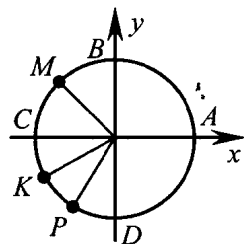
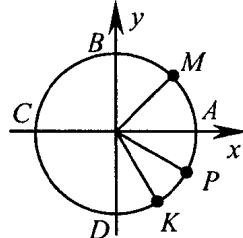
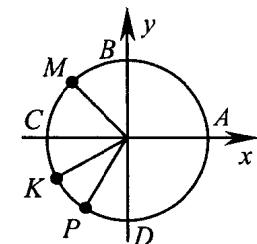
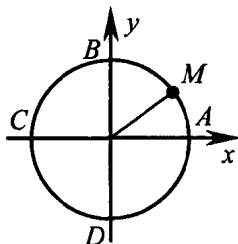
2. $AM = 45^\circ$, $BD = 180^\circ$,

$$CK = 120^\circ, MP = 285^\circ,$$

$$DM = 135^\circ, MK = 360^\circ - 105^\circ = 255^\circ,$$

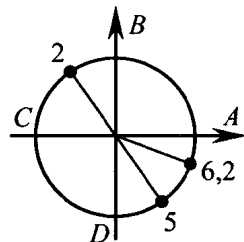
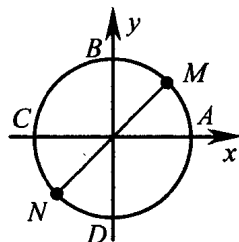
$$CP = 150^\circ, PC = 210^\circ.$$

3. $AM = 36^\circ$, $MB = 54^\circ$, $DM = 126^\circ$, $MC = 144^\circ$. 4. $CP = 15^\circ$, $PD = 75^\circ$, $AP = 105^\circ$.



5. $AN = 225^\circ$.

6. а) Да; б) да; в) да; г) нет, т.к. длина всей окружности $l = 2\pi r = 2 \cdot 3,1415... < 6,3$.



7. $\angle AOM = 60^\circ$; $MN = 2 \cdot \sin \angle AOM = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$;

$$AM = 60^\circ, MB = 30^\circ, AN = 300^\circ, NA = 60^\circ.$$

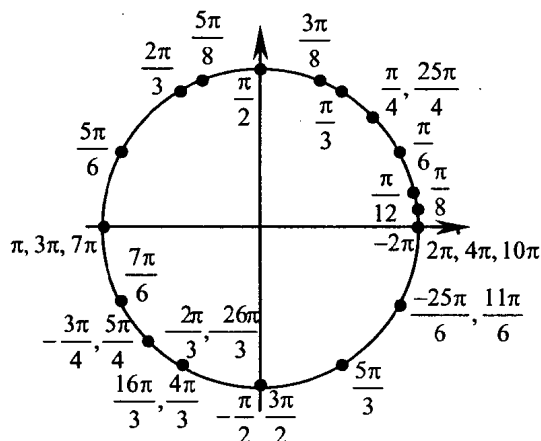
8. Воспользуемся результатами задачи № 7.

$\overset{\frown}{AM} = \overset{\frown}{NA} = 60^\circ$, $\overset{\frown}{MB} = \overset{\frown}{DN} = 30^\circ$. Аналогично $\overset{\frown}{BP} = \overset{\frown}{QD} = 30^\circ$, $\overset{\frown}{PC} = \overset{\frown}{CQ} = 60^\circ$.

Окончательно получаем: $\overset{\frown}{AM} = \overset{\frown}{MP} = \overset{\frown}{PC} = \overset{\frown}{CQ} = \overset{\frown}{QN} = \overset{\frown}{NA} = 60^\circ$, ч.т.д.

§2. Числовая окружность

С № 9 по № 16 см. рисунок.



9. а) $\frac{\pi}{2}; (0; 1)$; б) $\pi; (-1; 0)$; в) $\frac{3\pi}{2}; (0; -1)$; г) $2\pi; (1; 0)$.

10. а) $7\pi; (-1; 0)$; б) $4\pi; (1; 0)$; в) $10\pi; (1; 0)$; г) $3\pi; (-1; 0)$.

11. а) $\frac{\pi}{3}; \left(\frac{1}{2}; \frac{\sqrt{3}}{2}\right)$; б) $\frac{\pi}{4}; \left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$; в) $\frac{\pi}{6}; \left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$.

г) $\frac{\pi}{8}; \cos^2 \frac{\pi}{8} = \frac{1 + \cos \frac{\pi}{4}}{2}; \cos \frac{\pi}{8} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2};$

$\sin^2 \frac{\pi}{8} = \frac{1 - \sin^2 \frac{\pi}{4}}{2}; \sin \frac{\pi}{8} = \sqrt{\frac{2 - \sqrt{2}}{2}}; \left(\sqrt{\frac{2 + \sqrt{2}}{2}}; \sqrt{\frac{2 - \sqrt{2}}{2}}\right).$

12. а) $\frac{2\pi}{3}; \left(-\frac{1}{2}; \frac{\sqrt{3}}{2}\right)$; б) $\frac{3\pi}{4}; \left(-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$; в) $\frac{5\pi}{6}; \left(-\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$; г) $\frac{5\pi}{4}; \left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right).$

13. а) $\frac{4\pi}{3}; \left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right)$; б) $\frac{5\pi}{3}; \left(\frac{1}{2}; -\frac{\sqrt{3}}{2}\right)$; в) $\frac{7\pi}{6}; \left(-\frac{\sqrt{3}}{2}; -\frac{1}{2}\right)$; г) $\frac{11\pi}{6}; \left(\frac{\sqrt{3}}{2}; -\frac{1}{2}\right).$

14. а) $\frac{\pi}{12}; \cos^2 \frac{\pi}{12} = \frac{1 + \cos \frac{\pi}{6}}{2}; \cos \frac{\pi}{12} = \frac{\sqrt{2 + \sqrt{3}}}{2};$

$$\sin^2 \frac{\pi}{12} = \frac{1 - \sin^2 \frac{\pi}{6}}{2}; \quad \sin \frac{\pi}{12} = \sqrt{\frac{2 - \sqrt{3}}{2}}; \quad \left(\sqrt{\frac{2 + \sqrt{3}}{2}}; \sqrt{\frac{2 - \sqrt{3}}{2}} \right);$$

$$\text{б) } \frac{5\pi}{12}; \frac{5}{12}\pi = \frac{\pi}{4} + \frac{\pi}{6}; \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4};$$

$$\sin \frac{5\pi}{12} = \frac{\sqrt{2}(\sqrt{3} + 1)}{4}; \quad \left(\frac{\sqrt{2}(\sqrt{3} - 1)}{4}; \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \right);$$

$$\text{в) } \frac{3\pi}{8}; \cos \frac{3\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}; \sin \frac{3\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}; \quad \left(\frac{\sqrt{2 - \sqrt{2}}}{2}; \frac{\sqrt{2 + \sqrt{2}}}{2} \right);$$

$$\text{г) } \frac{5\pi}{8}; \cos \frac{5\pi}{8} = -\frac{\sqrt{2 - \sqrt{2}}}{2}; \sin \frac{5\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}; \quad \left(-\frac{\sqrt{2 - \sqrt{2}}}{2}; \frac{\sqrt{2 + \sqrt{2}}}{2} \right).$$

$$15. \text{ а) } -\frac{\pi}{2}; (0; -1); \quad \text{б) } \frac{2\pi}{3}; \left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right); \quad \text{в) } -2\pi; (1; 0); \quad \text{г) } -\frac{3\pi}{4}; \left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right).$$

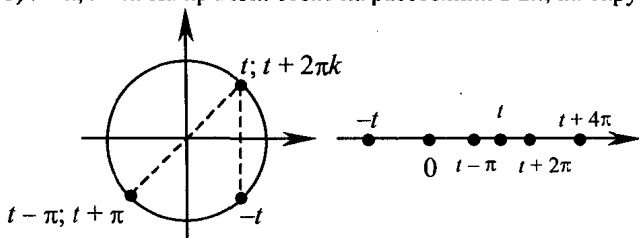
$$16. \text{ а) } \frac{25\pi}{4}; \left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right); \quad \text{б) } -\frac{25\pi}{3}; \left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right); \quad \text{в) } -\frac{25\pi}{6}; \left(\frac{\sqrt{3}}{2}; -\frac{1}{2}\right); \quad \text{г) } \frac{16\pi}{3}; \left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right).$$

17. а) $t; -t$. На прямой: симметрично относительно нуля; на окружности: симметрично относительно оси x .

б) $t; t + 2\pi k$. На прямой: стоят с периодом $2\pi k$; на окружности: совпадают.

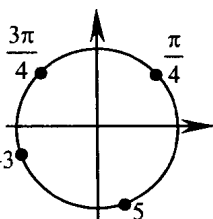
в) $t; t + \pi$. На прямой: стоят на расстоянии в π ; на окружности: диаметрально противоположны.

г) $t + \pi, t - \pi$. На прямой: стоят на расстоянии в 2π ; на окружности: совпадают.



$$18. \text{ а) } M\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right); \quad \text{б) } M(5) = (0,284; -0,959);$$

$$\text{в) } M\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right); \quad \text{г) } M(-3) = (-0,990; -0,141).$$



$$19. \text{ а) } A: 2\pi k; \quad \text{б) } C: \pi + 2\pi k; \quad \text{в) } A \text{ и } C: \pi n.$$

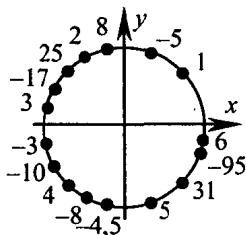
20. См. рисунок к № 19.

$$\text{а) } B: \frac{\pi}{2} + 2\pi k; \quad \text{б) } D: -\frac{\pi}{2} + 2\pi k; \quad \text{в) } B \text{ и } D: \frac{\pi}{2} + \pi n.$$

21. См. рисунок к № 19.

- а) A : \min положительное $= 2\pi$; \max отрицательное $= -2\pi$;
 б) B : \min положительное $= \frac{\pi}{2}$; \max отрицательное $= -\frac{3\pi}{2}$;
 в) C : \min положительное $= \pi$; \max отрицательное $= -\pi$;
 г) D : \min положительное $= \frac{3\pi}{2}$; \max отрицательное $= -\frac{\pi}{2}$.

С № 22 по № 25 см. рисунок.



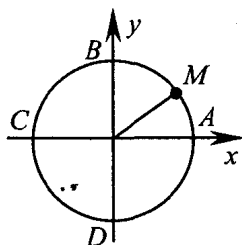
22. а) 1: (0,540; 0,841); б) -5: (0,284; 0,950);
 в) 4,5: (-0,211; -0,978); г) (-3): (-0,990; -0,141).
 23. а) 6: IV; б) 2: II; в) 3: II; г) 4: III.
 24. а) 5: IV; б) -5: I; в) 8: II; г) -8: III.
 25. а) 10: III; б) -17: II; в) 31: IV; г) -95: IV.

26. а) $AM: t \in \left(2\pi k; \frac{\pi}{4} + 2\pi k\right)$;

б) $CM: t \in \left(-\pi + 2\pi k; \frac{\pi}{4} + 2\pi k\right)$;

в) $MA: t \in \left(\frac{\pi}{4} + 2\pi k; 2\pi + 2\pi k\right)$;

г) $MC: t \in \left(\frac{\pi}{4} + 2\pi k; \pi + 2\pi k\right)$.



27. См. рисунок к № 26

а) $DM: t \in \left(-\frac{\pi}{2} + 2\pi k; \frac{3\pi}{4} + 2\pi k\right)$;

б) $BD: t \in \left(\frac{\pi}{2} + 2\pi k; \frac{3\pi}{2} + 2\pi k\right)$;

в) $MD: t \in \left(\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{2} + 2\pi k\right)$;

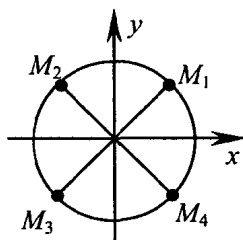
г) $DB: t \in \left(-\frac{\pi}{2} + 2\pi k; \frac{\pi}{2} + 2\pi k\right)$.

28. а) $M_1M_2: t \in \left(\frac{\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k\right)$;

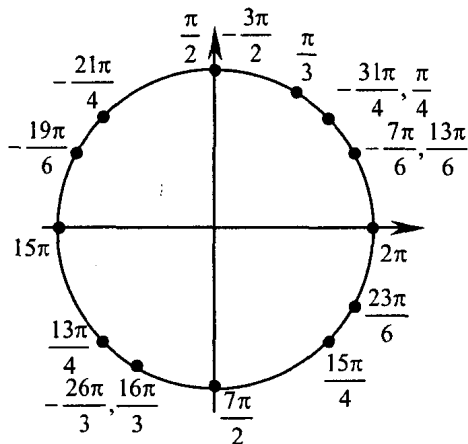
б) $M_4M_1: t \in \left(-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k\right)$;

в) $M_3M_2: t \in \left(-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k\right)$;

г) $M_1M_3: t \in \left(\frac{\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k\right)$.



С № 29 по № 32 см. рисунок.



$$\text{б) } M = \left(-\frac{\sqrt{3}}{2}; \frac{1}{2} \right); \text{ min положит.} = \frac{5\pi}{6}; \text{ max отрицат.} = -\frac{7\pi}{6};$$

$$\text{в) } M = \left(\frac{\sqrt{3}}{2}; -\frac{1}{2} \right); \min \text{ положит.} = \frac{11\pi}{6}; \max \text{ отрицат.} = -\frac{\pi}{6};$$

$$\text{г) } M = \left(-\frac{\sqrt{3}}{2}; -\frac{1}{2} \right); \min \text{ положит.} = \frac{7\pi}{6}; \max \text{ отрицат.} = -\frac{5\pi}{6}.$$

$$34. \text{ а) } M = \left(\frac{1}{2}; \frac{\sqrt{3}}{2} \right); \min \text{ положит.} = \frac{\pi}{3}; \max \text{ отрицат.} = -\frac{5\pi}{3};$$

$$\text{б) } M = \left(-\frac{1}{2}; \frac{\sqrt{3}}{2} \right); \min \text{ положит.} = \frac{2\pi}{3}; \max \text{ отрицат.} = -\frac{4\pi}{3};$$

$$\text{в) } M = \left(-\frac{1}{2}; -\frac{\sqrt{3}}{2} \right); \min \text{ положит.} = \frac{4\pi}{3}; \max \text{ отрицат.} = -\frac{2\pi}{3};$$

$$\text{г) } M = \left(\frac{1}{2}; -\frac{\sqrt{3}}{2} \right); \min \text{ положит.} = \frac{5\pi}{3}; \max \text{ отрицат.} = -\frac{\pi}{3}.$$

$$35. \text{ а) } y = \frac{\sqrt{2}}{2}; t = \frac{\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n;$$

$$\text{б) } y = \frac{1}{2}; t = \frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n;$$

$$\text{в) } y = 0; t = \pi n;$$

$$\text{г) } y = \frac{\sqrt{3}}{2}; t = \frac{\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n.$$

$$36. \text{ а) } y = -\frac{\sqrt{3}}{2}; x = (-1)^{k+1} \frac{\pi}{3} + \pi k;$$

$$\text{б) } y = 1; x = \frac{\pi}{2} + 2\pi k;$$

$$\text{в) } y = -\frac{\sqrt{2}}{2}; x = (-1)^{k+1} \frac{\pi}{4} + \pi k;$$

$$\text{г) } y = -1; x = -\frac{\pi}{2} + 2\pi k.$$

$$37. \text{ а) } x = \frac{\sqrt{3}}{2}; y = \pm \frac{\pi}{6} + 2\pi k;$$

$$\text{б) } x = \frac{1}{2}; y = \pm \frac{\pi}{3} + 2\pi k;$$

$$\text{в) } x = 1; y = 2\pi k;$$

$$\text{г) } x = \frac{\sqrt{2}}{2}; y = \pm \frac{\pi}{4} + 2\pi k.$$

$$38. \text{ а) } x = 0; y = \frac{\pi}{2} + \pi k;$$

$$\text{б) } x = -\frac{1}{2}; y = \pm \frac{2\pi}{3} + 2\pi k;$$

$$\text{в) } x = -\frac{\sqrt{3}}{2}; y = \pm \frac{5\pi}{6} + 2\pi k;$$

$$\text{г) } x = -1; y = \pi + 2\pi k.$$

$$39. \text{ а) да;}$$

$$\text{б) нет;}$$

$$\text{в) да;}$$

$$\text{г) нет.}$$

$$40. \text{ а) } E(2) - +;$$

$$\text{б) } K(4) - -;$$

$$\text{в) } F(1) + +;$$

$$\text{г) } L(6) + -.$$

$$41. \text{ а) } M(12) + -; \text{ б) } N(15) - +; \text{ в) } P(49) + -; \text{ г) } Q(100) + -.$$

$$42. \text{ а) } y = -\frac{\sqrt{3}}{2}, \quad x < 0 \quad \frac{4\pi}{3} + 2\pi n; \quad M\left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right);$$

$$\text{б) } y = \frac{1}{2}, \quad x < 0 \quad \frac{5\pi}{6} + 2\pi n; \quad M\left(-\frac{\sqrt{3}}{2}; \frac{1}{2}\right);$$

$$\text{в) } y = \frac{1}{2}, \quad x > 0 \quad \frac{\pi}{6} + 2\pi n; \quad M\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right);$$

$$\text{г) } y = -\frac{\sqrt{3}}{2}, \quad x > 0 \quad -\frac{\pi}{3} + 2\pi n; \quad M\left(\frac{1}{2}; -\frac{\sqrt{3}}{2}\right).$$

$$43. \text{ а) } x = \frac{\sqrt{3}}{2}, \quad y > 0 \quad \frac{\pi}{6} + 2\pi n; \quad M\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right);$$

$$\text{б) } x = -\frac{1}{2}, \quad y < 0 \quad -\frac{2\pi}{3} + 2\pi n; \quad M\left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right);$$

$$\text{в) } x = \frac{\sqrt{3}}{2}, \quad y < 0 \quad -\frac{\pi}{6} + 2\pi n; \quad M\left(\frac{\sqrt{3}}{2}; -\frac{1}{2}\right);$$

$$\text{г) } x = -\frac{1}{2}, \quad y > 0 \quad \frac{2\pi}{3} + 2\pi n; \quad M\left(-\frac{1}{2}; \frac{\sqrt{3}}{2}\right).$$

$$44. \text{ а) } x > 0, \quad t \in \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right); \quad \text{б) } x < \frac{1}{2}, \quad t \in \left(\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right);$$

$$\text{в) } x > \frac{1}{2}, \quad t \in \left(-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right); \quad \text{г) } x < 0, \quad t \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right).$$

$$45. \text{ а) } x < \frac{\sqrt{2}}{2}, \quad t \in \left(\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k\right); \quad \text{б) } x > -\frac{\sqrt{2}}{2}, \quad t \in \left(-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k\right);$$

$$\text{в) } x > \frac{\sqrt{2}}{2}, \quad t \in \left(-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k\right); \quad \text{г) } x < -\frac{\sqrt{2}}{2}, \quad t \in \left(\frac{3\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k\right).$$

$$46. \text{ а) } x \leq \frac{\sqrt{3}}{2}, \quad t \in \left(\frac{\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k\right); \quad \text{б) } x \leq -\frac{\sqrt{3}}{2}, \quad t \in \left(\frac{5\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k\right);$$

$$\text{в) } x \geq \frac{\sqrt{3}}{2}, \quad t \in \left(-\frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k\right); \quad \text{г) } x \geq -\frac{\sqrt{3}}{2}, \quad t \in \left(-\frac{5\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k\right).$$

$$47. \text{ а) } y > 0, \quad t \in (2\pi k; \pi + 2\pi k); \quad \text{б) } y < \frac{1}{2}, \quad t \in \left(\frac{7\pi}{6} + 2\pi k; 6\frac{\pi}{3} + 2\pi k\right);$$

$$\text{в) } y > \frac{1}{2}, \quad t \in \left(\frac{\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k\right); \quad \text{г) } y < 0, \quad t \in (-\pi + 2\pi k; 2\pi k).$$

$$48. \text{ а) } y < \frac{\sqrt{2}}{2}, \quad -\frac{5\pi}{4} + 2\pi k < t < \frac{\pi}{4} + 2\pi k; \quad \text{б) } y > -\frac{\sqrt{2}}{2}, \quad -\frac{\pi}{4} + 2\pi k < t < \frac{5\pi}{4} + 2\pi k;$$

- в) $y > \frac{\sqrt{2}}{2}, \frac{\pi}{4} + 2\pi k < t < \frac{3\pi}{4} + 2\pi k$; г) $y < -\frac{\sqrt{2}}{2}, \frac{5\pi}{4} + 2\pi k < t < \frac{7\pi}{4} + 2\pi k$.
49. а) $y \leq \frac{\sqrt{3}}{2}, -\frac{4\pi}{3} + 2\pi k < t < \frac{\pi}{3} + 2\pi k$; б) $y \leq -\frac{\sqrt{3}}{2}, \frac{4\pi}{3} + 2\pi k < t < \frac{5\pi}{3} + 2\pi k$;
- в) $y \geq \frac{\sqrt{3}}{2}, \frac{\pi}{3} + 2\pi k < t < \frac{2\pi}{3} + 2\pi k$; г) $y \geq -\frac{\sqrt{3}}{2}, -\frac{\pi}{3} + 2\pi k < t < \frac{4\pi}{3} + 2\pi k$.

§4. Синус и косинус

50. а) $\sin 0 = 0, \cos 0 = 1$; б) $\sin \frac{\pi}{2} = 1, \cos \frac{\pi}{2} = 0$;
- в) $\sin \frac{3\pi}{2} = -1, \cos \frac{3\pi}{2} = 0$; г) $\sin \pi = 0, \cos \pi = -1$.
51. а) $\sin(-2\pi) = 0, \cos(-2\pi) = 1$; б) $\sin\left(-\frac{\pi}{2}\right) = -1, \cos\left(-\frac{\pi}{2}\right) = 0$;
- в) $\sin\left(-\frac{3\pi}{2}\right) = 1, \cos\left(-\frac{3\pi}{2}\right) = 0$; г) $\sin(-\pi) = 0, \cos(-\pi) = -1$.
52. а) $\sin \frac{5\pi}{6} = \frac{1}{2}, \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; б) $\sin \frac{5\pi}{4} = \frac{\sqrt{2}}{2}, \cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$;
- в) $\sin \frac{7\pi}{6} = -\frac{1}{2}, \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$; г) $\sin \frac{9\pi}{4} = -\frac{\sqrt{2}}{2}, \cos \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$.
53. а) $\sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$; б) $\sin\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(-\frac{4\pi}{3}\right) = -\frac{1}{2}$;
- в) $\sin\left(-\frac{13\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(-\frac{13\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; г) $\sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}$.
54. а) $\sin \frac{13\pi}{6} = \frac{1}{2}, \cos \frac{13\pi}{6} = -\frac{\sqrt{3}}{2}$; б) $\sin\left(-\frac{8\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \cos\left(-\frac{8\pi}{3}\right) = -\frac{1}{2}$;
- в) $\sin \frac{23\pi}{6} = -\frac{1}{2}, \cos \frac{23\pi}{6} = \frac{\sqrt{3}}{2}$; г) $\sin\left(-\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \cos\left(-\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.
55. а) $\sin\left(-\frac{\pi}{4}\right) + \cos \frac{\pi}{3} + \cos\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} + 1 - \sqrt{2}}{2}$;
- б) $\cos \frac{\pi}{6} \cos \frac{\pi}{4} \cos \frac{\pi}{3} \cos \frac{\pi}{2} = 0$; в) $\sin\left(-\frac{\pi}{2}\right) \cos(-\pi) + \sin\left(-\frac{3\pi}{2}\right) = -1 + 1 + 1 = 1$;
- г) $\sin \frac{\pi}{6} \sin \frac{\pi}{4} \sin \frac{\pi}{3} \sin \frac{\pi}{2} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \cdot 1 = \frac{\sqrt{6}}{6}$.

$$56. \text{ а) } \sin\left(-\frac{3\pi}{4}\right) + \cos\left(-\frac{\pi}{4}\right) + \sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos 0 \sin\frac{\pi}{2} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 = 1;$$

$$\text{б) } \cos\frac{5\pi}{3} + \cos\frac{4\pi}{3} + \sin\frac{3\pi}{2}\sin\frac{5\pi}{8}\cos\frac{3\pi}{2} = \frac{1}{2} - \frac{1}{2} = 0.$$

$$57. \text{ а) } \cos 2t, t = \frac{\pi}{2}, \cos \pi = -1;$$

$$\text{б) } \sin\frac{t}{2}, t = -\frac{\pi}{3}, \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2};$$

$$\text{в) } \sin 2t, t = -\frac{\pi}{6}, \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2};$$

$$\text{г) } \cos\frac{t}{2}, t = -\frac{\pi}{3}, \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

$$58. \text{ а) } \sin^2 t - \cos^2 t, t = \frac{\pi}{3}, \sin^2\frac{\pi}{3} - \cos^2\frac{\pi}{3} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2};$$

$$\text{б) } \sin^2 t + \cos^2 t, t = \frac{\pi}{4}, \sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = \frac{1}{2} + \frac{1}{2} = 1;$$

$$\text{в) } \sin^2 t - \cos^2 t, t = \frac{\pi}{4}, \sin^2\frac{\pi}{4} - \cos^2\frac{\pi}{4} = 0;$$

$$\text{г) } \sin^2 t + \cos^2 t, t = \frac{\pi}{6}, \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} = \frac{1}{4} + \frac{3}{4} = 1.$$

$$59. \text{ а) } f(t) = 2\sin t \quad f_{\max} = 2 \quad f_{\min} = -2. \quad \text{б) } f(t) = 3 + 4\cos t \quad f_{\max} = 7 \quad f_{\min} = -1.$$

$$\text{в) } f(t) = -3\cos t \quad f_{\max} = 3 \quad f_{\min} = -3. \quad \text{г) } f(t) = 3 - 5\sin t \quad f_{\max} = 8 \quad f_{\min} = -2.$$

$$60. \text{ а) } 1 - \sin^2 t = \cos^2 t; \quad \text{б) } 1 - \cos^2 t = \sin^2 t;$$

$$\text{в) } 1 + \sin^2 t + \cos^2 t = 2; \quad \text{г) } \sin t - \sin t \cos^2 t = \sin t(1 - \cos^2 t) = \sin^3 t.$$

$$61. \text{ а) } (\sin t - \cos t)^2 + 2 \sin t \cos t = 1;$$

$$\text{б) } (\sin t + \cos t)^2 - 2 \sin t \cos t = \sin^2 t + \cos^2 t + 2 \sin t \cos t - 2 \sin t \cos t = 1.$$

$$62. \text{ а) } \sin^2 t(1,5 + 2\pi k) + \cos^2 1,5 + \cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{6}\right) = 1 + \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2} + 1}{2};$$

$$\text{б) } \cos^2\left(\frac{\pi}{8} + 4\pi\right) + \sin^2\left(\frac{\pi}{8} - 44\pi\right) = 1.$$

$$63. \text{ а) } \cos t = \frac{\sqrt{2}}{2}, t = \pm\frac{\pi}{4} + 2\pi n;$$

$$\text{б) } \sin t = -\frac{1}{2}, t = (-1)^{n+1}\frac{\pi}{6} + \pi n;$$

$$\text{в) } \cos t = -\frac{1}{2}, t = \pm\frac{2\pi}{3} + 2\pi n;$$

$$\text{г) } \sin t = \frac{\sqrt{2}}{2}, t = (-1)^{n+1}\frac{\pi}{4} + \pi n.$$

$$64. \text{ а) } \sin t = \frac{\sqrt{3}}{2}, -t = (-1)^{k+1}\frac{\pi}{3} + \pi k;$$

$$\text{б) } \cos = \sqrt{3}, \text{ решения нет } |\cos t| \leq 1;$$

$$\text{в) } \cos t = -\frac{\sqrt{3}}{2}, t = \pm\frac{5\pi}{6} + 2\pi n;$$

$$\text{г) } \cos = -\frac{\pi}{3}, \text{ решения нет } |\cos t| \leq 1.$$

$$65. \text{ а) } 0;$$

$$\text{б) } \frac{\pi}{2};$$

$$\text{в) } -\frac{\pi}{6};$$

$$\text{г) } \frac{\pi}{3}.$$

66. а) $\frac{\pi}{2}$; б) 0; в) $\frac{2\pi}{3}$; г) $\frac{5\pi}{6}$.

67. а) $\sin t + 1 = 0$, $\sin t = -1$, $t = -\frac{\pi}{2} + 2\pi n$;

б) $\cos t - 1 = 0$, $\cos t = 1$, $t = 2\pi n$;

в) $1 - 2 \sin t = 0$, $\sin t = \frac{1}{2}$, $t = (-1)^n \frac{\pi}{6} + \pi n$;

г) $2 \cos t + 1 = 0$, $\cos t = -\frac{1}{2}$, $t = \pm \frac{2\pi}{3} + 2\pi n$.

68. а) $\frac{\sin t - 1}{\cos t}$, $\cos t \neq 0$, $t \neq -\frac{\pi}{2} + \pi n$; б) $\frac{\cos t + 5}{2 \sin t - 1}$, $\sin t \neq \frac{1}{2}$, $t \neq (-1)^n \frac{\pi}{6} + \pi n$;

в) $\frac{\cos t}{1 - \sin t}$, $\sin t \neq 1$, $t \neq \frac{\pi}{2} + 2\pi n$; г) $\frac{\sin t}{1 + \cos t}$, $\cos t \neq -1$, $t \neq \pi + 2\pi n$.

69. а) $\sin \frac{4\pi}{7} > 0$; б) $\cos \left(-\frac{5\pi}{7}\right) < 0$; в) $\sin \frac{9\pi}{8} < 0$; г) $\sin \left(-\frac{3\pi}{8}\right) < 0$.

70. а) $\sin(-2) < 0$; б) $\cos 3 < 0$; в) $\sin 5 < 0$; г) $\cos(-6) > 0$.

71. а) $\sin 10 < 0$; б) $\cos(-12) > 0$; в) $\sin(-15) < 0$; г) $\cos 8 < 0$.

72. а) $\sin 1 \cos 2 < 0$; б) $\sin \frac{\pi}{7} \cos \left(-\frac{7\pi}{5}\right) < 0$; в) $\cos 2 \sin(-3) > 0$; г) $\cos \left(-\frac{14\pi}{9}\right) \sin \frac{4\pi}{9} > 0$.

73. а) $\frac{\sin^2 t}{1 + \cos t} = \frac{1 - \cos^2 t}{1 + \cos t} = \frac{(1 - \cos t)(1 + \cos t)}{1 + \cos t} = 1 - \cos t$;

б) $\sin^4 t + \cos^4 t + 2 \sin^2 t \cos^2 t = (\sin^2 t + \cos^2 t)^2 = 1$;

в) $\frac{\cos^2 t}{1 + \sin t} + \sin t = 1 - \sin t + \sin t = 1$;

г) $\cos^4 t + \cos^2 t \sin^2 t - \cos^2 t + 1 = \cos^4 t (\sin^2 t - 1) + 1 = \cos^4 t - \cos^4 t + 1 = 1$.

74. а) $10 \sin t = \sqrt{75}$, $\sin t = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$, $t = (-1)^n \frac{\pi}{3} + \pi n$;

б) $\sqrt{8} \sin t + 2 = 0$, $\sin t = -\frac{\sqrt{2}}{2}$, $t = (-1)^{k+1} \frac{\pi}{4} + \pi k$;

в) $8 \cos t - \sqrt{32} = 0$, $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm \frac{\pi}{4} + 2\pi k$;

г) $8 \cos t = -\sqrt{48}$, $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm \frac{5\pi}{6} + 2\pi k$.

75. а) $\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} = \sin^2 \frac{\pi}{4} = \frac{1}{2}$, $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$;

б) $\sqrt{\frac{4}{3}} \cos t = \cos^2 1 + \sin^2 1$, $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm \frac{\pi}{6} + 2\pi n$.

$$76. \text{ а) } |\sin t| = 1, \quad \sin t = \pm 1, \quad t = \frac{\pi}{2} + \pi n;$$

$$\text{б) } \sqrt{1 - \sin^2 t} = \frac{1}{2}, \quad |\cos t| = \frac{1}{2}, \quad \cos t = \pm \frac{1}{2}, \quad t = \pm \frac{\pi}{3} + 2\pi k;$$

$$\text{в) } |\cos t| = 1, \quad \cos t = \pm 1, \quad t = \pi n;$$

$$\text{г) } \sqrt{1 - \cos^2 t} = \frac{\sqrt{2}}{2}, \quad |\sin t| = \frac{\sqrt{2}}{2}, \quad \sin t = \pm \frac{\sqrt{2}}{2}, \quad t = \frac{\pi}{4} + \frac{\pi n}{2}.$$

$$77. \text{ а) } \cos 1 + \cos(1 + \pi) + \sin t \left(-\frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{6}\right) = \cos 1 - \cos 1 - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0;$$

$$\text{б) } \sin 2 + \sin(2 + \pi) + \cos^2\left(-\frac{\pi}{12}\right) + \sin^2 \frac{\pi}{12} = \sin 2 - \sin 2 + 1 = 1.$$

$$78. \text{ а) } \sqrt{\sin 10, 2\pi} - \text{Да};$$

$$\text{б) } \sqrt{\cos 1, 3\pi} - \text{Нет};$$

$$\text{в) } \sqrt{\sin(-3, 4)\pi} - \text{Да};$$

$$\text{г) } \sqrt{\cos(-6, 9\pi)} - \text{Нет}.$$

$$79. \text{ а) } \cos 2(2x - 1) < 0, \quad x = \frac{1}{2}$$

$$\text{б) } \cos 3 \cos 5(x^2 - 4) < 0, \quad -x^2 + 4 < 0; \quad x \in (-\infty; -2) \cup (2; +\infty).$$

$$80. \text{ а) } (\cos t - 5)(3x - 1) \geq 0, \quad 3x - 1 \leq 0, \quad x \leq \frac{1}{3};$$

$$\text{б) } (2 + \sin t)(9 - x^2) \geq 0, \quad 9 - x^2 \geq 0, \quad x \in [-3; 3].$$

$$81. \text{ а) } a = \sin \frac{7\pi}{10}, \quad b = \sin \frac{5\pi}{6}, \quad a > b; \quad \text{б) } a = \cos 2, \quad b = \sin 2, \quad a < b;$$

$$\text{в) } a = \cos \frac{\pi}{8}, \quad b = \cos \frac{\pi}{3}, \quad a > b; \quad \text{г) } a = \sin 1, \quad b = \cos 1, \quad a > b.$$

$$82. \text{ а) } \sin \frac{2\pi}{9} - \sin \frac{10\pi}{9} > 0;$$

$$\text{б) } \sin 1 - \sin 1, 1 < 0;$$

$$\text{в) } \sin \frac{15\pi}{8} - \cos \frac{\pi}{4} < 0;$$

$$\text{г) } \cos 1 - \cos 0, 9 < 0.$$

$$83. \text{ а) } \sin \frac{4\pi}{3}, \sin \frac{7\pi}{7}, \sin \frac{\pi}{7}, \sin \frac{\pi}{5}, \sin \frac{2\pi}{3}; \quad \text{б) } \cos \frac{5\pi}{6}, \cos \frac{5\pi}{4}, \cos \frac{\pi}{3}, \cos \frac{7\pi}{4}, \cos \frac{\pi}{8}.$$

$$84. \text{ а) } \cos 4, \sin 3, \cos 5, \sin 2;$$

$$\text{б) } \cos 3, \cos 4, \cos 7, \cos 6;$$

$$\text{в) } \sin 4, \sin 6, \sin 3, \sin 7;$$

$$\text{г) } \cos 3, \sin 5, \sin 4, \cos 2.$$

$$85. \text{ а) } \sqrt{\sin^2 1 + \sin^2 2 - 2 \sin 1 \sin 2} + \sqrt{\frac{1}{4} - \sin 1 + \sin^2 1} + \sqrt{1 + \sin^2 2 - 2 \sin 2} =$$

$$= |\sin 1 - \sin 2| + \left| \sin 1 - \frac{1}{2} \right| + |\sin 2 - 1| = \sin 2 - \sin 1 - \frac{1}{2} - \sin 2 + 1 = \frac{1}{2};$$

$$\begin{aligned} \text{б)} \quad & \sqrt{\cos^2 6 + \cos^2 7 - 2 \cos 6 \cos 7} + \sqrt{\frac{1}{4} - \cos 7 + \cos^2 7} + \sqrt{1 + \cos^2 6 - 2 \cos 6} = \\ & = |\cos 6 - \cos 7| + \left| \cos 7 - \frac{1}{2} \right| + |\cos 6 - 1| = 1 - \cos 6 + \cos 6 - \cos 7 + \cos 7 - \frac{1}{2} = \frac{1}{2}. \end{aligned}$$

$$86. \text{ а)} \sin(\pi - t) = \sin t; \quad \sin(\pi - t) = -\sin(-t) = \sin t;$$

$$\text{б)} \sin(2\pi - t) = -\sin t; \quad \sin(2\pi - t) = \sin(-t) = -\sin t;$$

$$\text{в)} \cos(\pi - t) = -\cos t; \quad \cos(\pi - t) = -\cos(-t) = -\cos t;$$

$$\text{г)} \cos(2\pi - t) = \cos t; \quad \cos(2\pi - t) = \cos(-t) = \cos t.$$

$$87. \text{ а)} \sin t > 0, t \in (2\pi k; \pi + 2\pi k); \quad \text{б)} \sin t < \frac{\sqrt{3}}{2}, t \in \left(-\frac{4\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k\right);$$

$$\text{в)} \sin t < 0, t \in (-\pi + 2\pi k; 2\pi k); \quad \text{г)} \sin t > \frac{\sqrt{3}}{2}, t \in \left(\frac{\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k\right).$$

$$88. \text{ а)} \cos t > 0, t \in \left(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{2} + 2\pi k\right); \quad \text{б)} \cos t < \frac{\sqrt{2}}{2}, t \in \left(\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k\right);$$

$$\text{в)} \cos t < 0, t \in \left(\frac{\pi}{2} + 2\pi k; \frac{3\pi}{2} + 2\pi k\right); \quad \text{г)} \cos t > \frac{\sqrt{2}}{2}, t \in \left(-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k\right).$$

$$89. \text{ а)} \sin t < -\frac{1}{2}, t \in \left(\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k\right);$$

$$\text{б)} \sin t > -\frac{\sqrt{2}}{2}, t \in \left(-\frac{\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k\right);$$

$$\text{в)} \sin t > -\frac{1}{2}, t \in \left(-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k\right);$$

$$\text{г)} \sin t < -\frac{\sqrt{2}}{2}, t \in \left(-\frac{5\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k\right).$$

$$90. \text{ а)} \cos t > -\frac{\sqrt{3}}{2}, t \in \left(-\frac{5\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k\right);$$

$$\text{б)} \cos t < -\frac{1}{2}, t \in \left(\frac{2\pi}{3} + 2\pi k; \frac{4\pi}{3} + 2\pi k\right);$$

$$\text{в)} \cos t < -\frac{\sqrt{3}}{2}, t \in \left(\frac{5\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k\right);$$

$$\text{г)} \cos t > -\frac{1}{2}, t \in \left(-\frac{2\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k\right).$$

$$91. \text{ а)} \sin t \leq \frac{1}{2}, t \in \left[-\frac{7\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k\right]; \quad \text{б)} \cos t \geq -\frac{\sqrt{2}}{2}, t \in \left[-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k\right];$$

$$\text{в)} \sin t \geq -\frac{1}{2}, t \in \left[-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k\right]; \quad \text{г)} \cos t \leq \frac{\sqrt{2}}{2}, t \in \left[\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k\right].$$

§5. Тангенс и котангенс

92. а) $\operatorname{tg} \frac{5\pi}{4} = 1$; б) $\operatorname{ctg} \frac{4\pi}{3} = \frac{\sqrt{3}}{3}$; в) $\operatorname{tg} \frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$; г) $\operatorname{ctg} \frac{7\pi}{4} = -1$.

93. а) $\operatorname{tg}\left(-\frac{5\pi}{4}\right) = -1$; б) $\operatorname{ctg}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$; в) $\operatorname{tg}\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$; г) $\operatorname{ctg}\left(-\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$

94. а) $\operatorname{tg} \frac{\pi}{4} + \operatorname{ctg} \frac{5\pi}{4} = 1 + 1 = 2$; б) $\operatorname{ctg} \frac{\pi}{3} - \operatorname{tg} \frac{\pi}{6} = \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 0$;

в) $\operatorname{tg} \frac{\pi}{6} \cdot \operatorname{ctg} \frac{\pi}{6} = 1$; г) $\operatorname{tg} \frac{9\pi}{4} + \operatorname{ctg} \frac{\pi}{4} = 1 + 1 = 2$.

95. а) $\operatorname{tg} \frac{\pi}{4} \cdot \sin \frac{\pi}{3} \cdot \operatorname{ctg} \frac{\pi}{6} = 1 \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{3} = \frac{3}{2}$;

б) $2 \sin \frac{\pi}{3} \cos \frac{\pi}{6} - \frac{1}{2} \operatorname{tg} \frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \sqrt{3} = \frac{3 - \sqrt{3}}{2}$;

в) $2 \sin \pi + 3 \cos \pi + \operatorname{ctg} \frac{\pi}{2} = 0 - 3 + 0 = -3$;

г) $\operatorname{tg} 0 + 8 \cos \frac{3\pi}{2} - 6 \sin \frac{\pi}{3} = 0 + 0 - 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$.

96. а) $\operatorname{tg} \frac{\pi}{5} \cdot \operatorname{ctg} \frac{\pi}{5} = 1$; б) $\operatorname{tg} 2,3 \cdot \operatorname{ctg} 2,3 = 3$; в) $\operatorname{tg} \frac{\pi}{7} \cdot \operatorname{ctg} \frac{\pi}{7} = 1$; г) $7 \operatorname{ctg} \frac{\pi}{12} \cdot \operatorname{ctg} \frac{\pi}{12} = 7$.

97. а) $\operatorname{tg} 2,5 \cdot \operatorname{ctg} 2,5 + \cos^2 \pi - \sin^2 \frac{\pi}{8} - \cos^2 \frac{\pi}{8} = 1 + 1 - 1 = 1$;

б) $\sin^2 \frac{3\pi}{7} - 2 \operatorname{tg} 1 \cdot \operatorname{ctg} 1 + \cos^2 \left(-\frac{3\pi}{7}\right) + \sin^2 \frac{5\pi}{2} = 1 - 2 + 1 = 0$.

98. а) $\operatorname{tg} \frac{6\pi}{7} < 0$; б) $\operatorname{ctg} \frac{10\pi}{7} > 0$; в) $\operatorname{tg} \frac{8\pi}{7} > 0$; г) $\operatorname{ctg} \frac{11\pi}{7} < 0$.

99. а) $\sin t \cdot \operatorname{ctg} t = \cos t$, $\sin t \cdot \frac{\cos t}{\sin t} = \cos t$; б) $\frac{\sin t}{\operatorname{tg} t} = \cos t$, $\sin t \cdot \frac{\cos t}{\sin t} = \cos t$;

в) $\cos t \cdot \operatorname{tg} t = \sin t$; г) $\frac{\cos t}{\operatorname{ctg} t} = \sin t$, $\cos t \cdot \frac{\sin t}{\cos t} = \sin t$.

100. а) $\sin t \cdot \cos t \cdot \operatorname{tg} t = \sin t \cdot \cos t \cdot \frac{\sin t}{\cos t} = \sin^2 t$;

б) $\sin t \cdot \cos t \cdot \operatorname{ctg} t - 1 = \sin t \cdot \cos t \cdot \frac{\cos t}{\sin t} - 1 = \cos^2 t - 1 = -\sin^2 t$;

в) $\sin^2 t - \operatorname{tg} t \cdot \operatorname{ctg} t = \sin^2 t - 1 = -\cos^2 t$; г) $\frac{1 - \cos^2 t}{1 - \sin^2 t} = \frac{\sin^2 t}{\cos^2 t} = \operatorname{tg}^2 t$.

101. $\frac{\sin \frac{\pi}{4} - \cos \pi - \operatorname{tg} \frac{\pi}{4}}{2 \sin \frac{\pi}{6} - \sin \frac{3\pi}{2}} = \frac{\sqrt{2}}{4}$; $\frac{\frac{\sqrt{2}}{2} + 1 - 1}{1 + 1} = \frac{\sqrt{2}}{4}$.

$$102. \text{ а) } \cos \frac{5\pi}{9} - \operatorname{tg} \frac{25\pi}{18} < 0;$$

$$\text{б) } \operatorname{tg} 1 - \cos 2 > 0;$$

$$\text{в) } \sin \frac{7\pi}{10} - \operatorname{ctg} \frac{3\pi}{5} > 0;$$

$$\text{г) } \sin 2 - \operatorname{ctg} 5,5 > 0.$$

$$103. \text{ а) } \sin 1 \cdot \cos 2 \cdot \operatorname{tg} 3 \cdot \operatorname{ctg} 4 > 0;$$

$$\text{б) } \sin(-5) \cos(-6) \operatorname{tg}(-7) \operatorname{ctg}(-8) < 0.$$

$$104. \text{ а) } 1 + \operatorname{tg}^2 t = \cos^{-2} t, 1 + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t} = \cos^{-2} t;$$

$$\text{б) } 1 + \operatorname{ctg}^2 t = \sin^{-2} t, 1 + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t} = \sin^{-2} t;$$

$$\text{в) } \sin^2 t (1 + \operatorname{ctg}^2 t) = 1, \sin^2 t \cdot \sin^{-2} t = 1; \quad \text{г) } \cos^2 t (1 + \operatorname{tg}^2 t) = 1, \cos^2 t \cdot \cos^{-2} t = 1$$

$$105. \text{ а) } \operatorname{tg}(\pi - t) = -\operatorname{tg} t, \quad \operatorname{tg}(\pi - t) = \operatorname{tg}(-t) = -\operatorname{tg} t;$$

$$\text{б) } \operatorname{tg}(2\pi + t) = \operatorname{tg} t, \quad \operatorname{tg}(2\pi + t) = \operatorname{tg}(\pi + t) = \operatorname{tg} t;$$

$$\text{в) } \operatorname{ctg}(\pi - t) = -\operatorname{ctg} t, \quad \operatorname{ctg}(\pi - t) = \operatorname{ctg}(-t) = -\operatorname{ctg} t;$$

$$\text{г) } \operatorname{ctg}(2\pi + t) = \operatorname{ctg} t, \quad \operatorname{ctg}(2\pi + t) = \operatorname{ctg}(\pi + t) = \operatorname{ctg} t.$$

$$106. \text{ а) } \cos^2 t \cdot \operatorname{tg}^2 t - \sin^2 t \cdot \cos^2 t = \sin^2 t (1 - \cos^2 t) = \sin^4 t;$$

$$\text{б) } 1 - \cos^2 t + \operatorname{tg}^2 t \cdot \cos^2 t = \sin^2 t + \sin^2 t = 2 \sin^2 t;$$

$$\text{в) } (1 - \sin^2 t)(\operatorname{tg}^2 t + 1) = \cos^2 t \frac{1}{\cos^2 t} = 1; \quad \text{г) } (1 - \cos^2 t)(\operatorname{ctg}^2 t + 1) = \sin^2 t \frac{1}{\sin^2 t} = 1.$$

$$107. \text{ а) } \frac{\cos^2 t - \sin^2 t}{\cos t \sin t} = \frac{\frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t}}{\frac{\cos t \sin t}{\cos^2 t}} = \frac{1 - \operatorname{tg}^2 t}{\operatorname{tg} t};$$

$$\text{б) } \frac{\cos^2 t - \sin^2 t}{\sin t \cos t} = \frac{\frac{\cos^2 t}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t}}{\frac{\cos t \sin t}{\sin^2 t}} = \frac{\operatorname{ctg}^2 t - 1}{\operatorname{ctg} t}.$$

$$108. \text{ а) } \cos 1, \sin 1, 1, \operatorname{tg} 1. \quad \text{б) } \operatorname{ctg} 2, \cos 2, \sin 2, 2.$$

$$109. \text{ а) } \operatorname{ctg} 5(x-1) \geq 0, -x+1 \geq 0, x \leq 1;$$

$$\text{б) } \frac{\operatorname{tg} 7 \cdot \cos 1}{\sin 1} (2x^2 - 72) > 0, 2x^2 - 72 < 0, x^2 > 36, x \in (-6; 6);$$

$$\text{в) } (\operatorname{tg} 2 \cdot \sin 5)(7-5x) \leq 0, 7-5x \leq 0, x \geq \frac{7}{5};$$

$$\text{г) } \operatorname{tg} 1 \cdot \operatorname{ctg} 2 \cdot \operatorname{tg} 3 \cdot \operatorname{ctg} 4 \cdot (x^2 + 2) > 0, x^2 + 2 > 0, x \in \mathbb{R}.$$

§6. Тригонометрические функции числового аргумента

$$110. \text{ а) } 1 - \sin^2 t = \cos^2 t;$$

$$\text{б) } \cos^3 t - 1 = -\sin^2 t;$$

$$\text{в) } 1 - \cos^2 t = \sin^2 t;$$

$$\text{г) } \sin^2 t - 1 = -\cos^2 t.$$

111. а) $(1 - \sin t)(1 + \sin t) = 1 - \sin^2 t = \cos^2 t$; б) $\cos^2 t + 1 - \sin^2 t = \cos^2 t + \cos^2 t = 2\cos^2 t$;
 в) $(1 - \cos t)(1 + \cos t) = 1 - \cos^2 t = \sin^2 t$; г) $\sin^2 t + 2\cos^2 t - 1 = 1 + \cos^2 t - 1 = \cos^2 t$.
112. а) $\frac{1}{\cos^2 t} - 1 = \frac{1 - \cos^2 t}{\cos^2 t} = \operatorname{tg}^2 t$; б) $\frac{1 - \sin^2 t}{\cos^2 t} = 1$;
 в) $1 - \frac{1}{\sin^2 t} = \frac{\sin^2 t - 1}{\sin^2 t} = -\operatorname{ctg}^2 t$; г) $\frac{1 - \cos^2 t}{1 - \sin^2 t} = \frac{\sin^2 t}{\cos^2 t} = \operatorname{tg}^2 t$.
113. а) $\frac{(\sin t + \cos t)^2}{1 + 2\sin t \cos t} = \frac{1 + 2\sin t \cos t}{1 + 2\sin t \cos t} = 1$; б) $\frac{1 - 2\sin t \cos t}{(\cos t - \sin t)^2} = \frac{1 - 2\sin t \cos t}{1 - 2\sin t \cos t} = 1$.
114. а) $\frac{\cos^2 t}{1 - \sin t} - \sin t = 1$, $\frac{1 - \sin^2 t}{1 - \sin t} - \sin t = \frac{1 - \sin^2 t - \sin t + \sin^2 t}{1 - \sin t} = 1$;
 б) $\frac{\sin^2 t}{1 + \cos t} + \cos t = 1$, $\frac{1 - \cos^2 t}{1 + \cos t} + \cos t = \frac{\sin^2 t + \cos t + \cos^2 t}{1 + \cos t} = 1$.
115. а) $(\sin t + \cos t)^2 - 2\sin t \cos t = 1 + 2\sin t \cos t - 2\sin t \cos t = 1$;
 б) $\frac{2 - \sin^2 t - \cos^2 t}{3\sin^2 t + 3\cos^2 t} = \frac{2 - 1}{3} = \frac{1}{3}$; в) $\sin^4 t + \cos^4 t + 2\sin^2 t \cos^2 t = (\sin^2 t + \cos^2 t)^2 = 1$;
 г) $\frac{\sin^4 t - \cos^4 t}{\sin^2 t - \cos^2 t} = \frac{(\sin^2 t - \cos^2 t)(\sin^2 t + \cos^2 t)}{\sin^2 t - \cos^2 t} = \sin^2 t + \cos^2 t = 1$.
116. а) $\sin t = \frac{4}{5}$, $\frac{\pi}{2} < t < \pi$, $\cos t = -\frac{3}{5}$, $\operatorname{tg} t = -\frac{4}{3}$, $\operatorname{ctg} t = -\frac{3}{4}$;
 б) $\sin t = \frac{5}{13}$, $0 < t < \frac{\pi}{2}$, $\sqrt{1 - \frac{25}{169}} = \frac{12}{13}$, $\operatorname{tg} t = \frac{5}{12}$, $\operatorname{ctg} t = \frac{12}{5}$;
 в) $\sin t = -0,6$, $-\frac{\pi}{2} < t < 0$, $\cos t = \frac{4}{5}$, $\operatorname{tg} t = -\frac{3}{4}$, $\operatorname{ctg} t = -\frac{4}{3}$;
 г) $\sin t = -0,28$, $\pi < t < \frac{3\pi}{2}$, $\cos t = -\sqrt{1 - \frac{43}{625}} = -\frac{24}{25}$, $\operatorname{tg} t = \frac{7}{24}$, $\operatorname{ctg} t = \frac{24}{7}$.
117. а) $\cos t = 0,8$, $0 < t < \frac{\pi}{2}$, $\sin t = \frac{3}{5}$, $\operatorname{tg} t = \frac{3}{4}$, $\operatorname{ctg} t = \frac{4}{3}$;
 б) $\cos t = -\frac{5}{13}$, $\frac{\pi}{2} < t < \pi$, $\sin t = \frac{12}{13}$, $\operatorname{tg} t = -\frac{12}{5}$, $\operatorname{ctg} t = -\frac{5}{12}$;
 в) $\cos t = 0,6$, $\frac{3\pi}{2} < t < 2\pi$, $\sin t = -\frac{4}{5}$, $\operatorname{tg} t = -\frac{4}{3}$, $\operatorname{ctg} t = -\frac{3}{4}$;
 г) $\cos t = -\frac{24}{25}$, $\pi < t < \frac{3\pi}{2}$, $\sin t = -\frac{7}{25}$, $\operatorname{tg} t = \frac{7}{24}$, $\operatorname{ctg} t = \frac{24}{7}$.
118. а) $\operatorname{tg} t = \frac{3}{4}$, $0 < t < \frac{\pi}{2}$, $\operatorname{ctg} t = \frac{4}{3}$, $\sin t = \frac{3}{5}$, $\cos t = \frac{4}{5}$;
 б) $\operatorname{tg} t = 2,4$, $\pi < t < \frac{3\pi}{2}$, $\operatorname{ctg} t = \frac{5}{12}$, $\cos t = -\frac{5}{13}$, $\sin t = -\frac{12}{13}$;

$$\text{в)} \operatorname{tg} t = -\frac{3}{4}, \frac{\pi}{2} < t < \pi, \operatorname{ctg} t = -\frac{4}{3}, \sin t = \frac{3}{5}, \cos t = -\frac{4}{5};$$

$$\text{р)} \operatorname{tg} t = -\frac{1}{3}, \frac{3\pi}{2} < t < 2\pi, \operatorname{ctg} t = -3, \frac{\cos t}{\sqrt{1-\cos^2 t}} = -3, \cos^2 t = 9-9\cos^2 t,$$

$$\cos t = \frac{3}{\sqrt{10}}, \sin t = -\sqrt{1-\frac{9}{10}} = -\frac{1}{\sqrt{10}}.$$

$$119. \text{а)} \operatorname{ctg} t = \frac{12}{5}, \pi < t < \frac{3\pi}{2}, \operatorname{tg} t = \frac{5}{12}, \sin t = -\frac{5}{13}, \cos t = -\frac{12}{13};$$

$$\text{б)} \operatorname{ctg} t = \frac{7}{24}, 0 < t < \frac{\pi}{2}, \operatorname{tg} t = \frac{24}{7}, \sin t = \frac{24}{25}, \cos t = \frac{7}{25};$$

$$\text{в)} \operatorname{ctg} t = -\frac{5}{12}, \frac{3\pi}{2} < t < 2\pi, \operatorname{tg} t = -\frac{12}{5}, \cos t = \frac{5}{13}, \sin t = -\frac{12}{13};$$

$$\text{р)} \operatorname{ctg} t = -\frac{8}{15}, \frac{\pi}{2} < t < \pi, \operatorname{tg} t = -\frac{15}{8}, \frac{\cos t}{\sqrt{1-\cos^2 t}} = -\frac{8}{15}, \cos^2 t = \frac{64}{225} - \frac{64}{225} \cos^2 t$$

$$\cos t = -\frac{8}{17}, \sin t = \sqrt{1-\frac{64}{289}} = \frac{15}{17}.$$

$$120. \text{а)} f(x) = 1 - (\cos^2 t - \sin^2 t) = 2 \sin^2 t, f_{\max} = 2, f_{\min} = 0;$$

$$\text{б)} f(t) = 1 - \sin t \cdot \cos t \cdot \operatorname{tg} t = 1 - \sin t \cdot \cos t \cdot \frac{\sin t}{\cos t} = \cos^2 t, f_{\max} = 1, f_{\min} = 0.$$

$$\text{в)} f(t) = \cos^2 t \cdot \operatorname{tg}^2 t + 5 \cos^2 t - 1 = \cos^2 t \cdot \frac{\sin^2 t}{\cos^2 t} + 5 \cos^2 t - 1 = \sin^2 t + 5 \cos^2 t - 1 = 4 \cos^2 t, f_{\max} = 4, f_{\min} = 0;$$

$$\text{р)} f(t) = \sin t + 3 \sin^2 t + 3 \cos^2 t = \sin t + 3, f_{\max} = 4, f_{\min} = 2.$$

$$121. \text{а)} \operatorname{ctg} t - \frac{\cos t - 1}{\sin t} = \frac{\cos t - \cos t + 1}{\sin t} = \frac{1}{\sin t};$$

$$\text{б)} \operatorname{ctg}^2 t - \left(\frac{1}{\sin^2 t} - 1 \right) = \frac{\cos^2 t - 1 + \sin^2 t}{\sin^2 t} = 0;$$

$$\text{в)} \cos^2 t - (\operatorname{ctg}^2 t + 1) \sin^2 t = \cos^2 t - \cos^2 t - \sin^2 t = -\sin^2 t;$$

$$\text{р)} \frac{\sin^2 t - 1}{\cos^2 t - 1} + \operatorname{tg} t \cdot \operatorname{ctg} t = \operatorname{ctg}^2 t + 1 = \frac{\sin^2 t + \cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}.$$

$$122. \text{а)} \frac{\sin t}{1 + \cos t} + \frac{\sin t}{1 - \cos t} = \frac{\sin t - \sin t \cos t + \sin t + \cos t \sin t}{1 - \cos^2 t} = \frac{2 \sin t}{\sin^2 t} = \frac{2}{\sin t};$$

$$\text{б)} \operatorname{ctg}^2 t (\cos^2 t - 1) + 1 = -\cos^2 t + 1 = \sin^2 t;$$

$$\text{в)} \frac{\cos t}{1 + \sin t} + \frac{\cos t}{1 - \sin t} = \frac{\cos t - \sin t \cos t + \cos t + \sin t \cos t}{1 - \sin^2 t} = \frac{2 \cos t}{\cos^2 t} = \frac{2}{\cos t};$$

$$\text{р)} \frac{\operatorname{tg} t + 1}{1 + \operatorname{ctg} t} = \frac{\frac{\sin t + \cos t}{\sin t}}{\frac{\sin t + \cos t}{\sin t}} = \operatorname{tg} t.$$

$$123. \text{ а) } (3 \sin t + 4 \cos t)^2 + (4 \sin t - 3 \cos t)^2 = 9 \sin^2 t + 16 \cos^2 t +$$

$$+ 24 \sin t \cos t + 16 \sin^2 t + 9 \cos^2 t - 24 \sin t \cos t = 25;$$

$$\text{ б) } (\operatorname{tg} t + \operatorname{ctg} t)^2 - (\operatorname{tg} t - \operatorname{ctg} t)^2 = \operatorname{tg}^2 t + \operatorname{ctg}^2 t + 2 - \operatorname{tg}^2 t - \operatorname{ctg}^2 t + 2 = 4;$$

$$\text{ в) } \sin t \cdot \cos t (\operatorname{tg} t + \operatorname{ctg} t) = \sin t \cdot \cos t \frac{\sin^2 t + \cos^2 t}{\sin t \cos t} = 1;$$

$$\text{ р) } \sin^2 t \cdot \cos^2 t (\operatorname{tg}^2 t + \operatorname{ctg}^2 t + 2) = \sin^2 t \cdot \cos^2 t (\operatorname{tg} t + \operatorname{ctg} t)^2 = \\ = \sin^2 t \cdot \cos^2 t \frac{1}{\sin^2 t \cos^2 t} = 1.$$

$$124. \text{ а) } \frac{1 - \sin^2 t}{1 - \cos^2 t} + \operatorname{tg} t \cdot \operatorname{ctg} t = \frac{\cos^2 t}{\sin^2 t} + 1 = \operatorname{ctg}^2 t + 1 = \frac{1}{\sin^2 t};$$

$$\text{ б) } \frac{\cos^2 t - \operatorname{ctg}^2 t}{\sin^2 t - \operatorname{tg}^2 t} = \frac{\frac{\cos^2 t \cdot \sin^2 t - \cos^2 t}{\sin^2 t}}{\frac{\sin^2 t \cdot \cos^2 t - \sin^2 t}{\cos^2 t}} = \frac{\cos^2 t (\sin^2 t - 1) \cos^2 t}{\sin^2 t \sin^2 t (\cos^2 t - 1)} = \frac{-\cos^6 t}{-\sin^6 t} = \operatorname{ctg}^6 t.$$

$$125. \text{ а) } \frac{\operatorname{tg} t}{\operatorname{tg} t + \operatorname{ctg} t} = \sin^2 t, \quad \frac{\operatorname{tg} t}{1} = \frac{\sin t}{\cos t} = \sin t \cdot \cos t = \sin^2 t;$$

$$\text{ б) } \frac{1 + \operatorname{tg} t}{1 + \operatorname{ctg} t} = \operatorname{tg} t, \quad \frac{\frac{\sin t + \cos t}{\cos t}}{\frac{\sin t + \cos t}{\sin t}} = \frac{\sin t}{\cos t} = \operatorname{tg} t;$$

$$\text{ в) } \frac{\operatorname{ctg} t}{\operatorname{tg} t + \operatorname{ctg} t} = \cos^2 t, \quad \frac{\operatorname{ctg} t}{1} = \operatorname{ctg} t \cdot \sin t \cdot \cos t = \cos^2 t;$$

$$\text{ р) } \frac{1 - \operatorname{ctg} t}{1 - \operatorname{tg} t} = -\operatorname{ctg} t, \quad \frac{\frac{\sin t - \cos t}{\sin t}}{\frac{-\sin t + \cos t}{\cos t}} = -\frac{\cos t}{\sin t} = -\operatorname{ctg} t.$$

$$126. \text{ а) } 1 + \sin t = \frac{\cos t + \operatorname{ctg} t}{\operatorname{ctg} t}, \quad \frac{\frac{\cos t \sin t + \cos t}{\sin t}}{\frac{\cos t}{\sin t}} = \frac{\cos t \sin t + \cos t}{\cos t} = \sin t + 1;$$

$$\text{ б) } \frac{\sin t + \operatorname{tg} t}{\operatorname{tg} t} = 1 + \cos t, \quad \sin t \frac{\cos t}{\sin t} + 1 = \cos t + 1;$$

$$\text{ в) } \frac{1 - \sin t}{\cos t} = \frac{\cos t}{1 + \sin t}, \quad \frac{1 - \sin^2 t}{\cos t (1 + \sin t)} = \frac{\cos t}{1 + \sin t};$$

$$\text{ р) } \frac{\sin t}{1 - \cos t} = \frac{1 + \cos t}{\sin t}, \quad \frac{\sin t (1 + \cos t)}{1 - \cos^2 t} = \frac{1 + \cos t}{\sin t}.$$

$$127. \text{ а) } \frac{(\sin t + \cos t)^2 - 1}{\operatorname{ctg} t - \sin t \cos t} = 2 \operatorname{tg}^2 t, \quad \frac{2 \sin t \cos t}{\cos t - \sin^2 t \cos t} \sin t = \frac{2 \sin^2 t \cos t}{\cos t (1 - \sin^2 t)} = 2 \operatorname{tg}^2 t;$$

$$б) \sin^3 t(1 + \operatorname{ctg} t) + \cos^3 t(1 + \operatorname{tg} t) = \sin t + \cos t,$$

$$\sin^3 t \frac{\sin t + \cos t}{\sin t} + \cos^3 t \frac{\sin t + \cos t}{\cos t} = (\sin t + \cos t)(\sin^2 t + \cos^2 t) = \sin t + \cos t;$$

$$в) \frac{(\sin t + \cos t)^2}{\operatorname{tg} t - \sin t \cos 2t} = 2 \operatorname{ctg}^2 t = \frac{2 \sin t \cos t}{\sin t - \sin t \cos^2 t} \cos t = \frac{2 \sin t \cos^2 t}{\sin^3 t} = 2 \operatorname{ctg}^2 t;$$

$$г) \frac{1 - 4 \sin^2 t \cos^2 t}{(\sin t + \cos t)^2} + 2 \sin t \cdot \cos t = 1,$$

$$\frac{(1 - 2 \sin t \cos t)(1 + 2 \sin t \cos t)}{1 + 2 \sin t \cos t} + 2 \sin t \cos t = 1 - 2 \sin t \cos t + 2 \sin t \cos t = 1.$$

$$128. а) \sin(4\pi + t) = \frac{3}{5}, \quad 0 < t < \frac{\pi}{2}, \quad \cos t = \frac{4}{5}, \quad \operatorname{tg} t = \frac{3}{4}, \quad \operatorname{tg}(-t) = -\frac{3}{4}, \quad \operatorname{tg}(\pi - t) = -\frac{3}{4};$$

$$б) \cos(2\pi + t) = \frac{12}{13}, \quad \frac{3\pi}{2} < t < 2\pi, \quad \sin t = -\frac{5}{13}, \quad \operatorname{ctg} t = -\frac{12}{5}, \quad \operatorname{ctg}(-t) = \frac{12}{5}, \quad \operatorname{ctg}(\pi - t) = \frac{12}{5}.$$

$$129. а) \cos t = -\frac{5}{13}, \quad 8,5\pi < t < 9\pi, \quad \sin t = \frac{12}{13}, \quad \sin(-t) = -\frac{12}{13};$$

$$б) \sin t = \frac{4}{5}, \quad \frac{9\pi}{2} < t < 5\pi, \quad \cos t = -\frac{3}{5}, \quad \cos(-t) = -\frac{3}{5},$$

$$\sin(-t) = -\frac{4}{5}, \quad \cos(-t) + \sin(-t) = -\frac{7}{5}.$$

$$130. а) \sin t + \cos t = 0,8, \quad (\sin t + \cos t)^2 = \frac{16}{25}, \quad 2 \cos t \cdot \sin t = -\frac{9}{25}, \quad \cos t \cdot \sin t = -\frac{9}{50};$$

$$б) \sin t - \cos t = \frac{1}{3}, \quad (\cos t - \sin t)^2 = \frac{1}{9}, \quad -2 \sin t \cdot \cos t = -\frac{8}{9}, \quad 9 \sin t \cdot \cos t = 4.$$

$$131. \operatorname{tg} t + \operatorname{ctg} t = 2,3, \quad (\operatorname{tg} t + \operatorname{ctg} t)^2 = \operatorname{tg}^2 t + 2 \operatorname{tg} t \cdot \operatorname{ctg} t + \operatorname{ctg}^2 t = 5,29, \quad \operatorname{tg}^2 t + \operatorname{ctg}^2 t = 3,29.$$

$$132. \sin t \cos t = -\frac{1}{2}, \quad \sin^4 t + \cos^4 t = 1 - 2 \sin^2 t \cdot \cos^2 t = 1 - \frac{1}{2} = \frac{1}{2}.$$

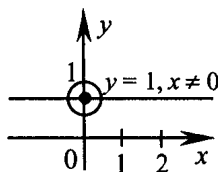
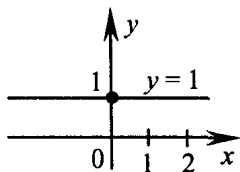
$$133. \operatorname{tg} t - \frac{1}{\operatorname{tg} t} = -\frac{7}{12}, \quad 0 < t < \frac{\pi}{2}, \quad 12 \operatorname{tg}^2 t + 7 \operatorname{tg} t - 12 = 0,$$

$$\operatorname{tg} t = \frac{-7 \pm \sqrt{49 - 4 \cdot 12 \cdot (-12)}}{24} = \frac{-7 \pm 25}{24}, \quad \operatorname{tg} t = -\frac{4}{3} \text{ не подходит, т.к. } 0 < t < \frac{\pi}{2},$$

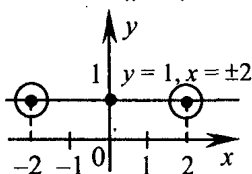
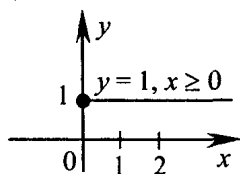
$$\operatorname{tg} t = \frac{3}{4} \Rightarrow \cos t = \frac{4}{5}, \quad \sin t = \frac{3}{5} \Rightarrow \sin t + \cos t = \frac{7}{5}.$$

$$134. а) y = \cos^2 t + \sin^2 t = 1;$$

$$б) y = \cos^2 \frac{1}{x} + \sin^2 \frac{1}{x} = 1 \quad (x \neq 0);$$

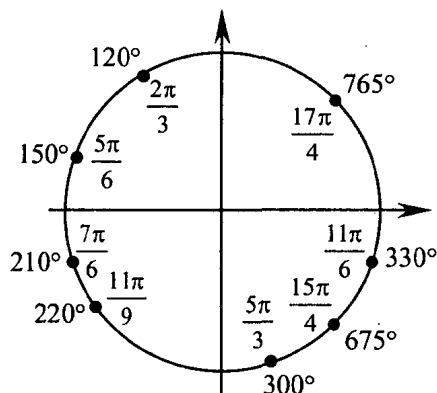


$$\text{в)} y = \sin^2 \sqrt{x} + \cos^2 \sqrt{x} = 1 \quad (x \geq 0); \text{г)} y = \sin^2 \frac{1}{x^2 - 4} + \cos^2 \frac{1}{x^2 - 4} = 1 \quad (x \neq \pm 2).$$



§7. Тригонометрические функции углового аргумента

135 – 138 см. рис.



$$135. \text{ а) } 120^\circ = \frac{2\pi}{3}; \quad \text{б) } 220^\circ = \frac{11\pi}{9}; \quad \text{в) } 300^\circ = \frac{5\pi}{3}; \quad \text{г) } 765^\circ = \frac{17\pi}{4}.$$

$$136. \text{ а) } 210^\circ = \frac{7\pi}{6}; \quad \text{б) } 150^\circ = \frac{5\pi}{6}; \quad \text{в) } 330^\circ = \frac{11\pi}{6}; \quad \text{г) } 675^\circ = \frac{15\pi}{4}.$$

$$137. \text{ а) } \frac{3\pi}{4} = 135^\circ; \quad \text{б) } \frac{11\pi}{3} = 660^\circ; \quad \text{в) } \frac{6\pi}{5} = 216^\circ; \quad \text{г) } \frac{46\pi}{9} = 920^\circ.$$

$$138. \text{ а) } \frac{5\pi}{8} = 112,5^\circ; \quad \text{б) } \frac{7\pi}{12} = 105^\circ; \quad \text{в) } \frac{11\pi}{12} = 165^\circ; \quad \text{г) } \frac{47\pi}{9} = 940^\circ.$$

$$139. \text{ а) } \sin 90^\circ = 1, \quad \cos 90^\circ = 0, \quad \operatorname{tg} 90^\circ = \text{не сущ.}, \quad \operatorname{ctg} 90^\circ = 0;$$

$$\text{б) } \sin 180^\circ = 0, \quad \cos 180^\circ = -1, \quad \operatorname{tg} 180^\circ = 0, \quad \operatorname{ctg} 180^\circ = \text{не сущ.};$$

$$\text{в) } \sin 270^\circ = -1, \quad \cos 270^\circ = 0, \quad \operatorname{tg} 270^\circ = \text{не сущ.}, \quad \operatorname{ctg} 270^\circ = 0;$$

$$\text{г) } \sin 360^\circ = 0, \quad \cos 360^\circ = 1, \quad \operatorname{tg} 360^\circ = 0, \quad \operatorname{ctg} 360^\circ = \text{не сущ.}.$$

$$140. \text{ а) } \sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3}, \quad \operatorname{ctg} 30^\circ = \sqrt{3};$$

$$\text{б) } \sin 150^\circ = \frac{1}{2}, \quad \cos 150^\circ = -\frac{\sqrt{3}}{2}, \quad \operatorname{tg} 150^\circ = -\frac{\sqrt{3}}{3}, \quad \operatorname{ctg} 150^\circ = -\sqrt{3};$$

$$\text{в) } \sin 210^\circ = -\frac{1}{2}, \cos 210^\circ = -\frac{\sqrt{3}}{2}, \operatorname{tg} 210^\circ = \frac{\sqrt{3}}{3}, \operatorname{ctg} 210^\circ = \sqrt{3};$$

$$\text{г) } \sin 240^\circ = -\frac{\sqrt{3}}{2}, \cos 240^\circ = -\frac{1}{2}, \operatorname{tg} 240^\circ = \sqrt{3}, \operatorname{ctg} 240^\circ = \frac{\sqrt{3}}{3}.$$

$$141. \sin 160^\circ, \sin 40^\circ, \sin 120^\circ, \sin 80^\circ.$$

$$142. \cos 160^\circ, \cos 120^\circ, \cos 80^\circ, \cos 40^\circ.$$

$$143. \sin 210^\circ, \sin 20^\circ, \sin 400^\circ, \sin 110^\circ.$$

$$144. \text{а) } \operatorname{tg} \alpha = \frac{x}{2}, x = 2 \operatorname{tg} \alpha;$$

$$\text{б) } \cos \alpha = \frac{x}{4}, x = 4 \cos \alpha;$$

$$\text{в) } \cos \alpha = \frac{3}{x}, x = \frac{3}{\cos \alpha};$$

$$\text{г) } \operatorname{ctg} \alpha = x.$$

$$145. \text{а) } \sin 30^\circ = \frac{2}{x}, x = \frac{2}{\frac{1}{2}} = 4;$$

$$\text{б) } x = \frac{\sqrt{2}}{2};$$

$$\text{в) } \frac{\sqrt{3}}{2} = \frac{2}{x}, x = \frac{4}{\sqrt{3}};$$

$$\text{г) } \frac{x}{2} = \cos 60^\circ = \frac{1}{2}, x = 1.$$

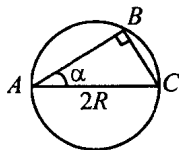
$$146. \text{а) } c = 12, \alpha = 60^\circ, a = 6, b = 6\sqrt{3}, S = 18\sqrt{3}, R = 6;$$

$$\text{б) } c = 6, \alpha = 45^\circ, a = b = 3\sqrt{2}, S = 9, R = 3;$$

$$\text{в) } c = 4, \alpha = 30^\circ, a = 2, b = 2\sqrt{3}, S = 2\sqrt{3}, R = 2;$$

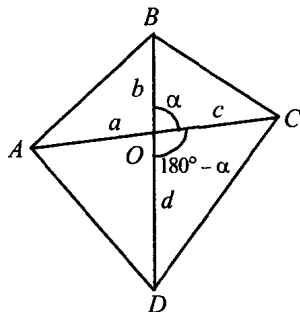
$$\text{г) } c = 60, \alpha = 60^\circ, a = 30, b = 30\sqrt{3}, S = 450\sqrt{3}, R = 30.$$

$$147. \cos \alpha = \frac{AB}{AC}, AB = AC \cdot \cos \alpha = 2R \cos \alpha.$$



$$148. S_{ABCD} = \frac{1}{2}ab \sin \alpha + \frac{1}{2}bc \sin \alpha + \frac{1}{2}cd \sin \alpha + \frac{1}{2}da \sin \alpha =$$

$$= \frac{1}{2} \sin \alpha (ab + bc + cd + da) = \frac{1}{2} \sin \alpha (b(a+c) + d(c+a)) = \frac{1}{2} \sin \alpha (b+d)(a+c).$$



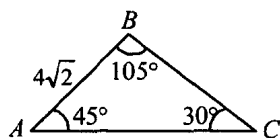
$$149. \frac{AB}{\sin C} = \frac{BC}{\sin A}, \quad \frac{4\sqrt{2}}{1/2} = \frac{BC}{\sqrt{2}/2} \Rightarrow BC = 8,$$

$$\angle B = 180^\circ - 45^\circ - 30^\circ = 105^\circ,$$

$$\frac{AC}{\sin 105^\circ} = \frac{16}{\sqrt{2}}, \quad AC = \frac{16}{\sqrt{2}} \sin 105^\circ; \quad \sin 105^\circ = \sin 75^\circ = \sin(45^\circ + 30^\circ),$$

$$AC = \frac{16}{\sqrt{2}} \left(\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} \right) = \frac{16}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4} = 4\sqrt{3} + 4;$$

$$S = \frac{1}{2} \cdot AC \cdot AB \cdot \sin \angle C; \quad S = \frac{1}{2} \cdot \frac{1}{2} (4\sqrt{3} + 4) 8 = 8(\sqrt{3} + 1);$$

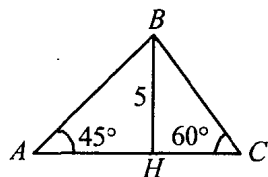


Ответ: $BC = 8$, $AC = 4\sqrt{3} + 4$, $S = 8(\sqrt{3} + 1) \text{ (см}^2\text{)}$.

$$150. AH = BH = 5 \text{ (т.к. } \angle A = \angle ABH = 45^\circ)$$

$$\operatorname{tg} 60^\circ = \frac{5}{HC}, \quad HC = \frac{5}{\operatorname{tg} 60^\circ} = \frac{5}{\sqrt{3}};$$

$$S = \frac{1}{2} \cdot 5 \cdot \left(5 + \frac{5}{\sqrt{3}} \right) = \frac{25(3 + \sqrt{3})}{6}.$$



§8. Формулы приведения

$$151. \text{ а) } \sin\left(\frac{\pi}{2} - t\right) = \cos t;$$

$$\text{ б) } \cos(2\pi - t) = \cos t;$$

$$\text{ в) } \cos\left(\frac{3\pi}{2} + t\right) = -\sin t;$$

$$\text{ г) } \sin(\pi + t) = -\sin t.$$

$$152. \text{ а) } \sin(\pi - t) = \sin t;$$

$$\text{ б) } \cos\left(\frac{\pi}{2} + t\right) = -\sin t;$$

$$\text{ в) } \cos(2\pi + t) = \cos t;$$

$$\text{ г) } \sin\left(\frac{3\pi}{2} - t\right) = -\cos t.$$

$$153. \text{ а) } \cos(90^\circ - \alpha) = \sin \alpha;$$

$$\text{ б) } \sin(360^\circ - \alpha) = -\sin \alpha;$$

$$\text{ в) } \sin(270^\circ + \alpha) = -\cos \alpha;$$

$$\text{ г) } \cos(180^\circ + \alpha) = -\cos \alpha.$$

$$154. \text{ а) } \operatorname{tg}(90^\circ - \alpha) = \operatorname{ctg} \alpha;$$

$$\text{ б) } \operatorname{ctg}(180^\circ - \alpha) = -\operatorname{ctg} \alpha;$$

$$\text{ в) } \operatorname{tg}(180^\circ + \alpha) = \operatorname{ctg} \alpha;$$

$$\text{ г) } \operatorname{ctg}(360^\circ + \alpha) = \operatorname{ctg} \alpha.$$

$$155. \text{ а) } \sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2};$$

$$\text{ б) } \operatorname{tg} 300^\circ = \operatorname{tg} 120^\circ = -\operatorname{tg} 60^\circ = -\sqrt{3};$$

$$\text{ в) } \cos 330^\circ = \cos(-30^\circ) = \frac{\sqrt{3}}{2};$$

$$\text{ г) } \operatorname{ctg} 315^\circ = -\operatorname{ctg} 45^\circ = -1.$$

$$156. \text{ а) } \cos \frac{5\pi}{3} = \frac{1}{2}; \quad \text{ б) } \sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2}; \quad \text{ в) } \sin \frac{7\pi}{6} = -\frac{1}{2}; \quad \text{ г) } \cos\left(-\frac{7\pi}{3}\right) = \frac{1}{2}.$$

$$157. \text{ а) } \cos 630^\circ - \sin 1470^\circ - \operatorname{ctg} 1125^\circ = -\frac{1}{2} - 1 = -\frac{3}{2};$$

$$\text{ б) } \sin(-7\pi) + 2\cos\frac{31\pi}{3} - \operatorname{tg}\frac{7\pi}{4} = 1 + 1 = 2;$$

$$\text{ в) } \operatorname{tg} 1800^\circ - \sin 495^\circ + \cos 945^\circ = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2};$$

$$\text{ г) } \cos(-9\pi) + 2\sin\left(-\frac{49\pi}{6}\right) - \operatorname{ctg}\left(-\frac{21\pi}{4}\right) = -1 - 1 + 1 = -1.$$

$$158. \text{ а) } \sin(90^\circ - \alpha) + \cos(180^\circ + \alpha) + \operatorname{tg}(270^\circ + \alpha) + \operatorname{ctg}(360^\circ + \alpha) = \\ = \cos \alpha - \cos \alpha - \operatorname{ctg} \alpha + \operatorname{ctg} \alpha = 0;$$

$$\text{ б) } \sin\left(\frac{\pi}{2} + t\right) - \cos(\pi - t) + \operatorname{tg}(\pi - t) + \operatorname{ctg}\left(\frac{5\pi}{2} - t\right) = \cos t + \cos t - \operatorname{tg} t + \operatorname{tg} t = 2\cos t;$$

$$159. \text{ а) } \frac{\cos(180^\circ + \alpha) \cdot \sin(-\alpha)}{\sin(-\alpha) \cdot \sin(90^\circ + \alpha)} = \frac{-\cos \alpha \cos \alpha}{-\sin \alpha \cos \alpha} = \operatorname{ctg} \alpha;$$

$$\text{ б) } \frac{\sin(\pi - t) \cos(2\pi - t)}{\operatorname{tg}(\pi - t) \cos(\pi - t)} = \frac{\sin t \cos t}{-\operatorname{tg} t \cdot (-\cos t)} = \cos t;$$

$$\text{ в) } \frac{\sin(-\alpha) \operatorname{ctg}(-\alpha)}{\cos(360^\circ - \alpha) \operatorname{tg}(180^\circ + \alpha)} = \frac{\cos \alpha}{\cos \alpha \operatorname{tg} \alpha} = \operatorname{ctg} \alpha;$$

$$\text{ г) } \frac{\sin(\pi + t) \sin(2\pi + t)}{\operatorname{tg}(\pi + t) \cos\left(\frac{3\pi}{2} + t\right)} = \frac{-\sin t \sin t}{\frac{\sin t}{\cos t} \sin t} = -\cos t.$$

$$160. \text{ а) } \frac{\cos(\pi - t) + \cos\left(\frac{\pi}{2} - t\right)}{\sin(2\pi - t) - \sin\left(\frac{3\pi}{2} + t\right)} = \frac{-\cos t + \sin t}{-\sin t + \cos t} = -1; \quad \text{ б) } \frac{\sin^2(\pi - t) + \sin^2\left(\frac{\pi}{2} - t\right)}{\sin(\pi - t)}.$$

$$161. \text{ а) } \frac{\operatorname{tg}(\pi - t) \cdot \sin\left(\frac{3\pi}{2} + t\right)}{\cos(\pi + t) \cdot \operatorname{tg}\left(\frac{3\pi}{2} + t\right)} = \operatorname{tg}^2 t, \quad \frac{-\operatorname{tg} t}{-\cos t} \cos\left(\frac{3\pi}{2} + t\right) = \operatorname{tg}^2 t;$$

$$\text{ б) } \frac{\sin(\pi - t) \cdot \operatorname{ctg}\left(\frac{\pi}{2} - t\right)}{\operatorname{tg}(\pi + t) \cdot \operatorname{tg}\left(\frac{\pi}{2} + t\right)} \cdot \frac{\cos(2\pi - t)}{\sin(-t)} = \sin t, \quad \frac{\sin t}{\operatorname{tg} t} \cdot \frac{\operatorname{tg} t}{-\operatorname{ctg} t} \cdot \frac{\cos t}{-\sin t} = \operatorname{tg} t \cdot \cos t = \sin t$$

$$162. \text{ а) } \frac{\cos^2(\pi - t) + \sin^2\left(\frac{\pi}{2} - t\right) + \cos(\pi + t) \cos(2\pi - t)}{\operatorname{tg}^2\left(t - \frac{\pi}{2}\right) \operatorname{ctg}^2\left(\frac{3\pi}{2} + t\right)} = \cos^2 t,$$

$$\frac{\cos^2 t + \cos^2 t - \cos^2 t}{c \operatorname{tg}^2 t \operatorname{tg}^2 t} = \cos^2 t;$$

$$\text{б) } \frac{\sin^2\left(t - \frac{3\pi}{2}\right) \cos(2\pi - t)}{\operatorname{tg}^2\left(t - \frac{\pi}{2}\right) \cos\left(t - \frac{3\pi}{2}\right)} = \cos t, \quad \frac{\cos^2 t \cos t}{\operatorname{ctg}^2 t \sin^2 t} = \cos t.$$

$$163. \text{ а) } \frac{11 \cos 287^\circ - 25 \sin 557^\circ}{\sin 17^\circ} = \frac{-11 \cos 167^\circ + 25 \sin 17^\circ}{\sin 17^\circ} = \frac{11 \sin 17^\circ + 25 \sin 17^\circ}{\sin 17^\circ} = 36;$$

$$\text{б) } \frac{13 \sin 469^\circ - 8 \cos 341^\circ}{\cos 19^\circ} = \frac{13 \sin 109^\circ - 8 \cos 19^\circ}{\cos 19^\circ} = \frac{13 \cos 109^\circ - 8 \cos 19^\circ}{\cos 19^\circ} = 5.$$

$$164. \text{ а) } \frac{2 \cos \frac{11\pi}{5} + 8 \sin \frac{13\pi}{10}}{\cos \frac{\pi}{5}} = \frac{2 \cos \frac{\pi}{5} + 8 \sin \frac{3\pi}{10}}{\cos \frac{\pi}{5}} = \frac{2 \cos \frac{\pi}{5} - 8 \sin \frac{\pi}{5}}{\cos \frac{\pi}{5}} = -6;$$

$$\text{б) } \frac{5 \sin \frac{5\pi}{7} + 2 \cos \frac{25\pi}{14}}{\sin \frac{2\pi}{7}} = \frac{5 \sin \frac{2\pi}{7} - 2 \cos \frac{11\pi}{14}}{\sin \frac{2\pi}{7}} = \frac{5 \sin \frac{2\pi}{7} + 2 \sin \frac{4\pi}{14}}{\sin \frac{2\pi}{7}} = 7.$$

$$165. \text{ а) } 2 \cos(2\pi + t) + \sin\left(\frac{\pi}{2} + t\right) = 3, \quad 2 \cos t + \cos t = 3, \quad \cos t = 1, \quad t = 2\pi n;$$

$$\text{б) } \sin(\pi + t) + 2 \cos\left(\frac{\pi}{2} + t\right) = 3, \quad -\sin t - 2 \sin t = 3, \quad \sin t = -1, \quad t = -\frac{\pi}{2} + 2\pi n;$$

$$\text{в) } 2 \sin(\pi + t) + \cos\left(\frac{\pi}{2} - t\right) = -\frac{1}{2}, \quad -2 \sin t + \sin t = -\frac{1}{2}, \quad \sin t = \frac{1}{2}, \quad t = (-1)^k \frac{\pi}{6} + 2\pi k;$$

$$\text{г) } 3 \sin\left(\frac{\pi}{2} + t\right) - \cos(2\pi + t) = 1, \quad 3 \cos t - \cos t = 1, \quad \cos t = \frac{1}{2}, \quad t = \pm \frac{\pi}{3} + 2\pi n.$$

$$166. \text{ а) } 5 \sin\left(\frac{\pi}{2} + t\right) - \sin\left(\frac{3\pi}{2} + t\right) - 8 \cos(2\pi - t) = 1,$$

$$5 \cos t + \cos t - 8 \cos t = 1, \quad \cos t = -\frac{1}{2}, \quad t = \pm \frac{2\pi}{3} + 2\pi n;$$

$$\text{б) } \sin(2\pi + t) - \cos\left(\frac{\pi}{2} - t\right) + \sin(\pi - t) = 1,$$

$$\sin t - \sin t + \sin t = 1, \quad \sin t = 1, \quad t = \frac{\pi}{2} + 2\pi n.$$

$$167. \text{ а) } \sin^2(\pi + t) + \cos^2(2\pi - t) = 0, \quad \sin^2 t + \cos^2 t = 0 \text{ корней нет;}$$

$$\text{б) } \sin^2(\pi + t) + \cos^2(2\pi - t) = 1, \quad \sin^2 t + \cos^2(2\pi - t) = 1, \quad \sin^2 t + \cos^2 t = 1, \quad t \in \mathbb{R}.$$

§9. Функция $y = \sin x$, ее свойства и график

168. а) $\sin \pi = 0$; б) $\sin\left(-\frac{\pi}{2}\right) = -1$; в) $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; г) $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

169. а) $f(x) = \sin x$, $f(-x) = -\sin x$; б) $f(x) = \sin x$, $f(2x) = \sin 2x$;
в) $f(x) = \sin x$, $f(x+1) = \sin(x+1)$; г) $f(x) = \sin x$, $f(x) - 5 = \sin x - 5$.

170. а) $y = 2\sin\left(x - \frac{\pi}{6}\right) + 1$, $x = \frac{4\pi}{3}$, $y = 2\sin \frac{7\pi}{6} + 1 = -1 + 1 = 0$;

б) $y = -\sin\left(x + \frac{\pi}{4}\right)$, $x = -\frac{\pi}{2}$, $y = -\sin\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$;

в) $y = 2\sin\left(x - \frac{\pi}{6}\right) + 1$, $x = \frac{7\pi}{6}$, $y = 2\sin \pi + 1 = 1$;

г) $y = -\sin\left(x + \frac{\pi}{4}\right)$, $x = -\frac{15\pi}{4}$, $y = -\sin\left(\frac{14\pi}{4}\right) = \sin \frac{7\pi}{4} = -1$.

171. а) $y = \sin x$, $\sin\left(-\frac{\pi}{2}\right) = -1$, $\left(-\frac{\pi}{2}; -1\right)$ принадлежит;

б) $y = \sin x$, $\frac{1}{2} \neq \sin \frac{\pi}{2}$, $\left(\frac{\pi}{2}; \frac{1}{2}\right)$ не принадлежит;

в) $y = \sin x$, $1 \neq \sin \pi$, $(\pi; 1)$ не принадлежит;

г) $y = \sin x$, $-1 = \sin \frac{3\pi}{2}$, $\left(\frac{3\pi}{2}; -1\right)$ принадлежит.

172. а) $y = \sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin \frac{\pi}{6} + 2 = \frac{3}{2}$, $\left(0; \frac{3}{2}\right)$ принадлежит;

б) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin \frac{\pi}{3} + 2 = -\frac{\sqrt{3}}{2} + 2$, $\left(\frac{\pi}{6}; -\frac{\sqrt{3}}{2} + 2\right)$ принадлежит;

в) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2$, $\frac{3}{2} = -\sin \frac{5\pi}{6} + 2 = -\frac{1}{2} + 2$, $\left(\frac{2\pi}{3}; \frac{3}{2}\right)$ принадлежит;

г) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2$, $-\sin\left(4\pi + \frac{\pi}{6}\right) + 2 = -\frac{1}{2} + 2 \neq 2,5$, $(4\pi; 2,5)$ не принадлежит.

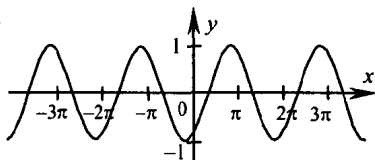
173. а) $y = \sin x$, $x \in \left[\frac{\pi}{4}; \frac{2\pi}{3}\right]$, $f_{\max} = 1$, $f_{\min} = \frac{\sqrt{2}}{2}$;

б) $y = \sin x$, $x \in \left[\frac{\pi}{4}; \infty\right)$, $f_{\max} = 1$, $f_{\min} = -1$;

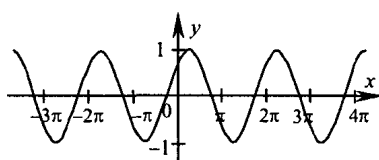
в) $y = \sin x$, $x \in \left[-\frac{3\pi}{2}; \frac{3\pi}{4}\right]$, $f_{\max} = 1$, $f_{\min} = -1$;

г) $y = \sin x$, $x \in \left[-\pi; \frac{\pi}{3}\right]$, $f_{\max} = \frac{\sqrt{3}}{2}$, $f_{\min} = -1$.

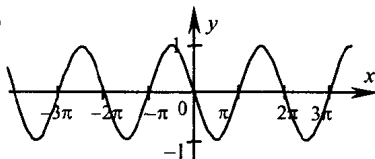
174. а)



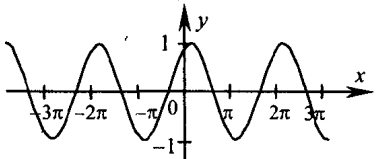
б)



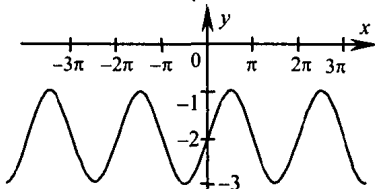
в)



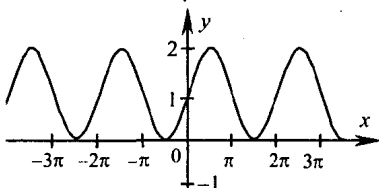
г)



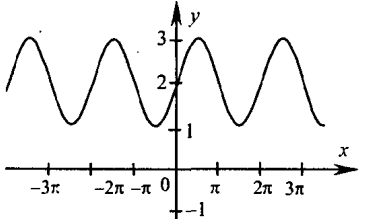
175. а)



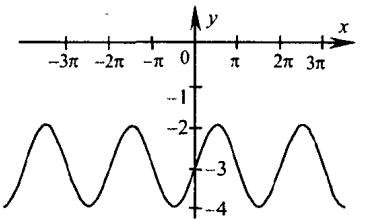
б)



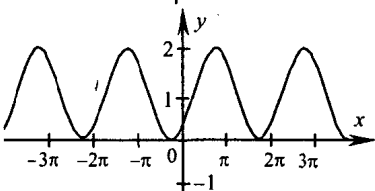
в)



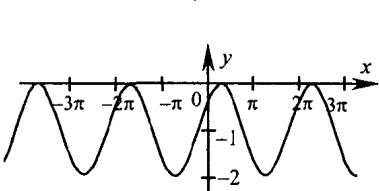
г)



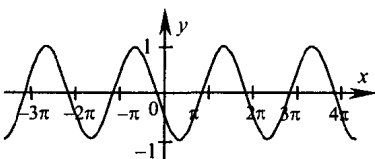
176. а)



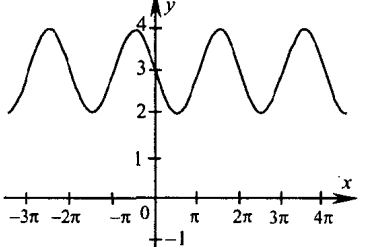
б)



177. а)

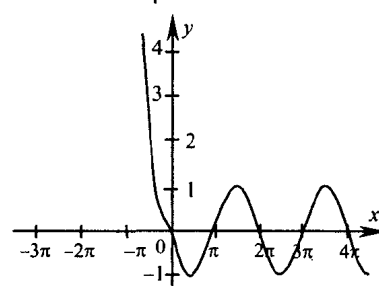


б)



$$178. f(x) = \begin{cases} x^2, & x < 0 \\ \sin x, & x \geq 0 \end{cases}$$

- а) 1) Область определения $D(f) = R$
- 2) Область значений $E(f) = [-1; +\infty)$
- 3) При $x > 0$ функция периодична, $T = 2\pi$
- 4) Функция ни четная, ни нечетная
- 5) $f(x) = 0$ при $x = \pi n, n \geq 0, x = 0$ при $y = 0$



6) Промежутки знакопостоянства:

$$f(x) > 0 \text{ при } x < 0, x \in (2\pi n, \pi + 2\pi n), n \geq 0$$

$$f(x) < 0 \text{ при } x \in (2\pi n - \pi, 2\pi n), n \geq 0$$

7) $f_{\min} = -1, f_{\max} = +\infty$

8) Функция убывает при $x \leq 0$ и $x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \geq 0$

возрастает при $x \in \left[0; \frac{\pi}{2}\right] \cup \left[2\pi n - \frac{\pi}{2}; 2\pi n + \frac{\pi}{2}\right], n \geq 1$

б) 1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = [-1; +\infty)$

3) При $x < 0$ функция периодична, $T = 2\pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = \pi n, n \leq 0, x = 0$ при $y = 0$

6) Промежутки знакопостоянства:

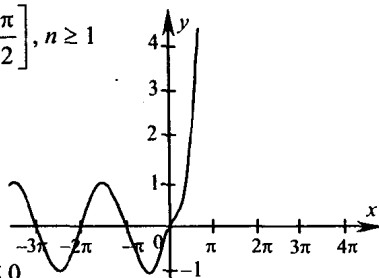
$$f(x) > 0 \text{ при } x > 0, x \in (2\pi n, \pi + 2\pi n), n \leq 0$$

$$f(x) < 0 \text{ при } x \in (2\pi n - \pi, 2\pi n), n \leq 0$$

7) $f_{\min} = -1, f_{\max} = +\infty$

8) Функция убывает при $x \leq 0$ и $x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \leq 0$

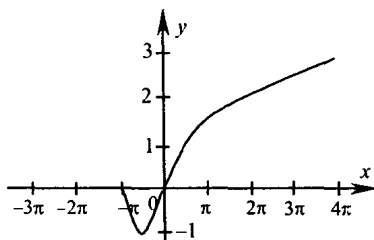
возрастает при $x \in \left[2\pi n - \frac{\pi}{2}; 2\pi n + \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}; +\infty\right), n < 0$



$$179. f(x) = \begin{cases} \sin x, & -\pi \leq x \leq 0 \\ x\sqrt{x}, & x \geq 0 \end{cases}$$

а) $f\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1, f(0) = 0, f(1) = 1, f(\pi^2) = \pi$

б)



в) 1) Область определения $D(f) = [-\pi; +\infty)$

2) Область значений $E(f) = [-1; +\infty)$

3) Функция неперидична

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = 0$

6) Промежутки знакопостоянства:

$$f(x) > 0 \text{ при } x > 0; f(x) < 0 \text{ при } x \in [-\pi; 0)$$

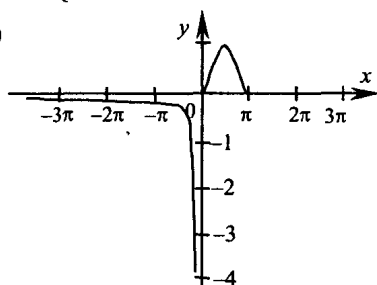
7) $f_{\min} = -1, f_{\max} = +\infty$

8) Функция убывает при $x \leq 0$ и $x \in \left[-\pi; -\frac{\pi}{2}\right]$, возрастает при $x \geq -\frac{\pi}{2}$.

180. $f(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$

а) $f(-2) = -\frac{1}{2}$, $f(0) = 0$, $f(1) = \sin 1$;

б)



в) 1) Область определения $D(f) = (-\infty; \pi]$

2) Область значений $E(f) = (-\infty; 1]$

3) Функция неперiodичная

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = 0$, $x = \pi$

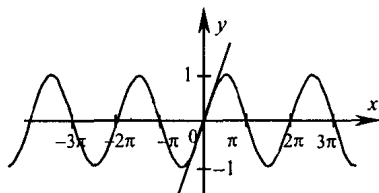
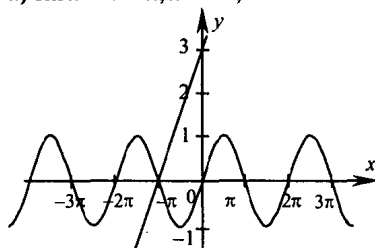
6) Промежутки знакопостоянства: $f(x) < 0$ при $x < 0$; $f(x) > 0$ при $x \in (0; \pi)$

7) $f_{\min} = -\infty$, $f_{\max} = 1$

8) Функция убывает при $x < 0$ и $x \in \left[-\frac{\pi}{2}; \pi\right]$, возрастает при $x \geq \left[0; \frac{\pi}{2}\right]$.

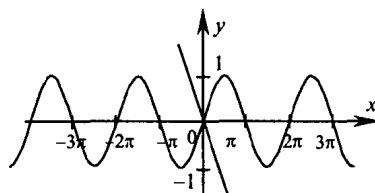
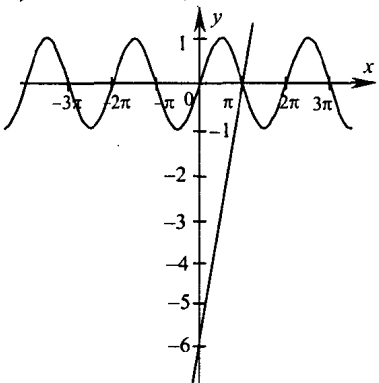
181. а) $\sin x = x + \pi$, $x = -\pi$;

б) $\sin x = 2x$, $x = 0$;

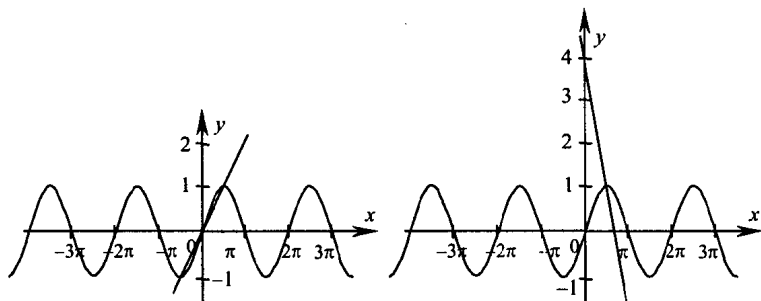


в) $\sin x = -x$, $x = 0$;

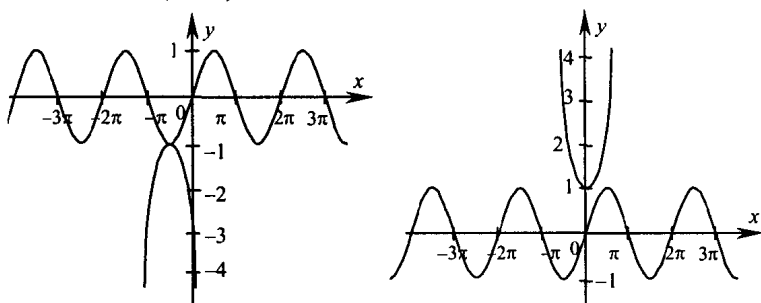
г) $\sin x = 2x - 2\pi$, $x = \pi$.



182. а) $\sin x = \frac{2}{\pi}x$, $x = 0$, $x = \pm \frac{\pi}{2}$; б) $\sin x = -\frac{4}{\pi}x + 3$, $x = \frac{\pi}{2}$.



183. а) $\sin x + 1 = -\left(x + \frac{\pi}{2}\right)^2$, $x = -\frac{\pi}{2}$; б) $\sin x = x^2 + 1$. решений нет.



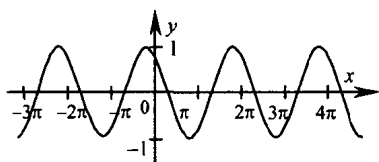
184. а) $\begin{cases} y = \sin x \\ y = x^2 + 4x - 1 \end{cases}$; $\begin{cases} y = \sin x \\ y = (x + 2)^2 - 3 \end{cases}$ система имеет 2 решения;

б) $\begin{cases} y = \sin x \\ y = \frac{1}{x} \end{cases}$ система имеет бесконечное множество решений;

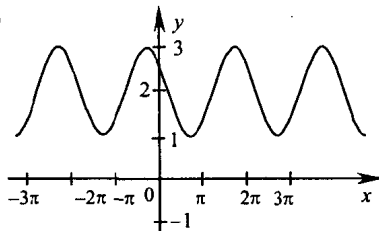
в) $\begin{cases} y = \sin x \\ y = -3x^2 - 2 \end{cases}$ система не имеет решений;

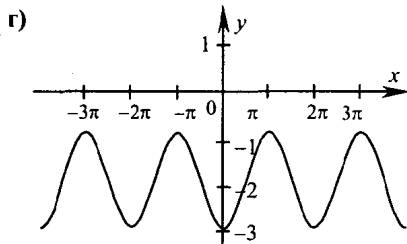
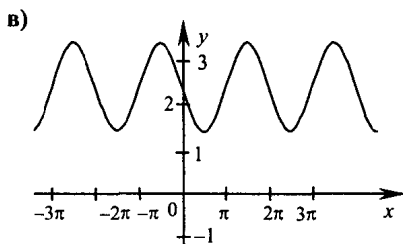
г) $\begin{cases} y = \sin x \\ |x| - y = 0 \end{cases}$ система имеет одно решение.

185. а)



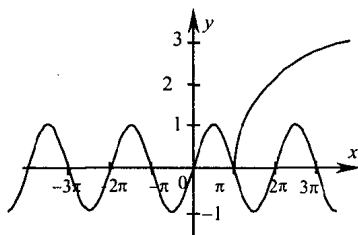
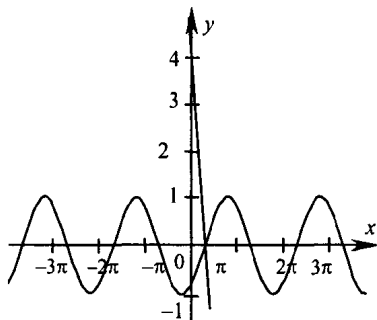
б)





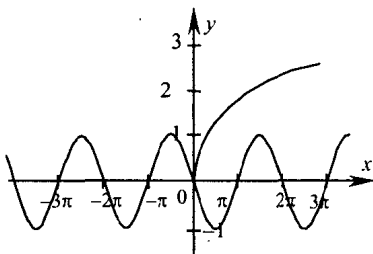
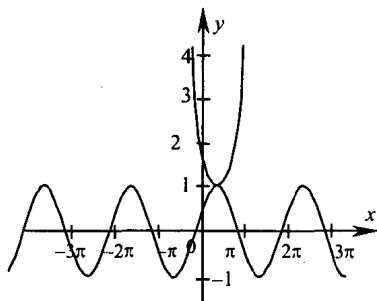
186. а) $\sin\left(x - \frac{\pi}{3}\right) = \pi - 3x$, $x = \frac{\pi}{3}$;

б) $\sin x = \sqrt{x - \pi}$, $x = \pi$;



в) $\sin\left(x + \frac{\pi}{6}\right) - 1 = \left(x - \frac{\pi}{3}\right)^2$, $x = \frac{\pi}{3}$;

г) $-\sin x = \sqrt{x}$, $x = 0$.



187. $y = \sin\left(x - \frac{\pi}{4}\right) + \frac{1}{2}$

а) $x \in \left[\frac{\pi}{4}; \frac{3\pi}{4}\right]$, $y_{\max} = \frac{3}{2}$, $y_{\min} = \frac{1}{2}$;

б) $\left(\frac{3\pi}{4}; \frac{9\pi}{4}\right)$, $y_{\min} = -\frac{1}{2}$;

в) $[0; \pi)$, $y_{\max} = \frac{3}{2}$, $y_{\min} = \frac{1 - \sqrt{2}}{2}$;

г) $\left[\frac{\pi}{4}; \infty\right)$, $y_{\max} = \frac{3}{2}$, $y_{\min} = -\frac{1}{2}$.

188. а) $f(x) = x^5 \sin \frac{x}{2}$, $f(-x) = -x^5(-1) \sin \frac{x}{2} = f(x)$;

б) $f(x) = \frac{\sin^2 x}{x^2 - 1}$, $f(-x) = \frac{\sin^2(-x)}{(-x^2) - 1} = \frac{\sin^2 x}{x^2 - 1} = f(x)$;

$$\text{в) } f(x) = \frac{2 \sin \frac{x}{2}}{x^3}, \quad f(-x) = \frac{-2 \sin \frac{x}{2}}{-x^3} = \frac{2 \sin \frac{x}{2}}{x^3} = f(x);$$

$$\text{г) } f(x) = \sin^2 x - x^4, \quad f(-x) = \sin^2(x) - (-x^4) = f(x).$$

$$189. \text{ а) } f(x) = -x - \sin x, \quad f(-x) = -(-x) - \sin(-x) = -(x - \sin x) = -f(x);$$

$$\text{б) } f(x) = x^3 \sin x^2, \quad f(-x) = -x^3 \sin(-x^2) = -x^3 \sin x^2 = -f(x);$$

$$\text{в) } f(x) = \frac{x^2 \sin x}{x^2 - 9}, \quad f(-x) = -\frac{x^2 \sin x}{x^2 - 9} = -f(x);$$

$$\text{г) } f(x) = x^3 - \sin x, \quad f(-x) = -x^3 + \sin x = -f(x).$$

$$190. f(x) = 2x^2 - x + 1;$$

$$f(\sin x) = 2 \sin^2 x - \sin x + 1 = 2 - 2 \cos^2 x - \sin x + 1 = 3 - 2 \cos^2 x - \sin x.$$

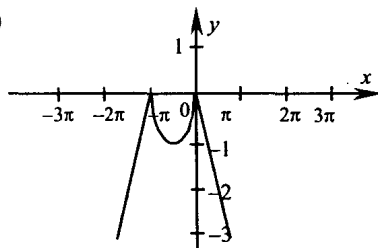
$$191. f(x) = 3x^2 + 2x - 7;$$

$$f(\sin x) = 3 \sin^2 x + 2 \sin x - 7 = 3 - 3 \cos^2 x + 2 \sin x - 7 = -4 - 3 \cos^2 x + 2 \sin x.$$

$$192. f(x) = \begin{cases} 2x + 2\pi, & x \leq -\pi \\ \sin x, & -\pi < x \leq 0 \\ -2x, & x > 0 \end{cases}$$

$$\text{а) } f(-\pi - 2) = -2\pi - 4 + 2\pi = -4, \quad f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \quad f(2) = -4;$$

б)



в) 1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = (-\infty; 0]$

3) Функция непериодичная

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\pi, x = 0$

6) Промежутки знакопостоянства:

$f(x) < 0$ при $x < -\pi, x \in (-\pi; 0), x > 0$

7) $f_{\min} = -\infty, f_{\max} = 0$

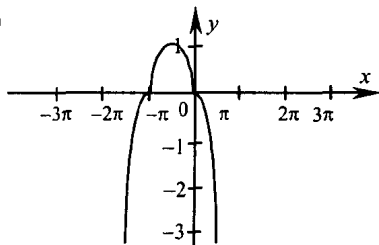
8) Функция возрастает при $x \in (-\infty; -\pi) \cup \left[-\frac{\pi}{2}; 0\right]$,

убывает при $x \in \left[-\pi; \frac{\pi}{2}\right] \cup [0; +\infty)$.

$$193. f(x) = \begin{cases} -x^2, & x < 0 \\ \sin x, & 0 \leq x \leq \pi \\ -(x - \pi)^2, & x > \pi \end{cases}$$

а) $f(-3) = -9$, $f\left(\frac{\pi}{2}\right) = 1$, $f(2\pi - 3) = -(\pi - 3)^2 = -\pi^2 + 6\pi - 9$

б)



в) 1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = (-\infty; 1]$

3) Функция неперiodическая

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = 0$, $x = \pi$

6) Промежутки знакопостоянства:

$$f(x) > 0 \text{ при } x \in (0; \pi), x > 0$$

$$f(x) < 0 \text{ при } x < 0; x > \pi$$

7) $f_{\min} = -\infty$, $f_{\max} = 1$

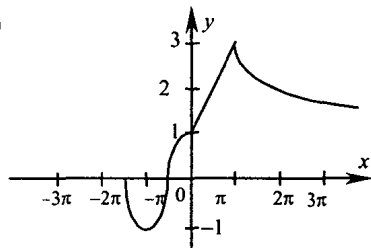
8) Функция возрастает при $x \in \left(-\infty; \frac{\pi}{2}\right]$,

убывает при $x \in \left[\frac{\pi}{2}; +\infty\right)$.

$$194. f(x) = \begin{cases} \sin\left(x + \frac{\pi}{2}\right), & -\frac{3\pi}{2} \leq x \leq 0 \\ x+1, & 0 < x < 2 \\ -\sqrt{x-2}+3, & x \geq 2 \end{cases}$$

а) $f(0) = 1$, $f(6) = 1$, $f(-\pi - 2)$ не определено, т.к. $(-\pi - 2) < -\frac{3\pi}{2}$,

б)



в) 1) Область определения $D(f) = \left[-\frac{3\pi}{2}; +\infty\right)$

2) Область значений $E(f) = [-1; 3]$

3) Функция неперiodическая

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\frac{\pi}{2}$, $x = 11$

6) Промежутки знакопостоянства:

$$f(x) > 0 \text{ при } x \in \left(-\frac{\pi}{2}; 11\right); f(x) < 0 \text{ при } x \in \left(-\frac{3\pi}{2}; -\frac{\pi}{2}\right) \cup (11; +\infty),$$

7) $f_{\min} = -\infty$, $f_{\max} = 3$

8) Функция возрастает при $x \in [-\pi; 2]$
убывает при $x \in [2; +\infty)$.

§10. Функция $y = \cos x$, ее свойства и график

195. а) $\cos \frac{\pi}{2} = 0$; б) $\cos(-\pi) = -1$; в) $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; г) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$.

196. а) $f(x) = \cos x$, $f(-x) = \cos x$; б) $f(x) = \cos x$, $f(3x) = \cos 3x$;
в) $f(x) = \cos x$, $f(x+2) = \cos(x+2)$; г) $f(x) = \cos x$, $f(x) - 6 = \cos x - 6$.

197. а) $y = 2 \sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = -2$; б) $y = 2 \sin \frac{\pi}{6} + \cos \frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2}$.

198. а) $y = \cos\left(-\frac{\pi}{3}\right) - \left(-\frac{\pi}{3}\right) = \frac{1}{2} - \frac{\pi^2}{9}$; б) $y = \cos \pi - \pi^2 = -1 - \pi^2$.

199. а) $\frac{1}{\cos \frac{2\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$; б) $\frac{1}{\cos \frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$.

200. а) $y = 2 \cos\left(-\frac{\pi}{2} - \frac{\pi}{4}\right) - 1 = -\sqrt{2} - 1$; б) $y = 2 \cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) - 1 = 1$.

201. а) $y = \cos x$, $\cos \frac{\pi}{3} = \frac{1}{2}$, $\left(\frac{\pi}{3}; \frac{1}{2}\right)$ принадлежит;

б) $y = \cos x$, $\frac{1}{2} \neq \cos \frac{\pi}{6}$, $\left(\frac{\pi}{6}; \frac{1}{2}\right)$ не принадлежит;

в) $y = \cos x$, $-\frac{1}{2} = \cos \frac{2\pi}{3}$, $\left(\frac{2\pi}{3}; -\frac{1}{2}\right)$ принадлежит;

г) $y = \cos x$, $-\frac{\sqrt{3}}{2} = \cos \frac{5\pi}{6}$, $\left(\frac{5\pi}{6}; -\frac{\sqrt{3}}{2}\right)$ принадлежит.

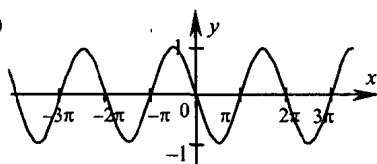
202. а) $\sqrt{3} + 1 = 2 \cos\left(-\frac{\pi}{6}\right) + 1$, $(0; \sqrt{3} + 1)$ — принадлежит;

б) $1 \neq 2 \cos\left(\frac{\pi}{6} - \frac{\pi}{6}\right) + 1$, $\left(\frac{\pi}{6}; 1\right)$ — не принадлежит;

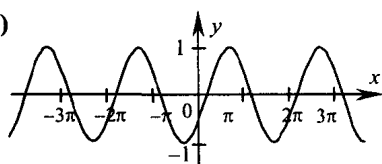
в) $2 = 2 \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + 1$, $\left(\frac{\pi}{2}; 2\right)$ — принадлежит;

г) $3 = 2 \cos\left(\frac{\pi}{6} - \frac{\pi}{6}\right) + 1$, $\left(\frac{\pi}{6}; 3\right)$ – принадлежит.

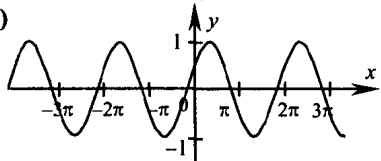
203. а)



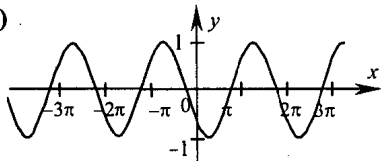
б)



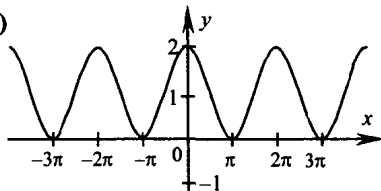
в)



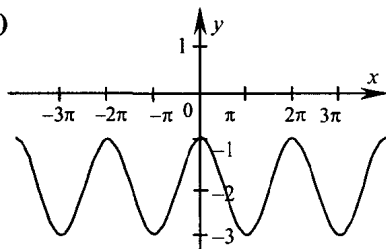
г)



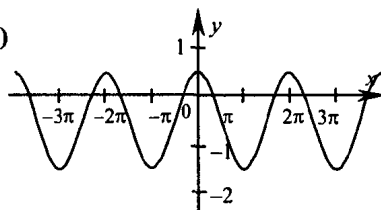
204. а)



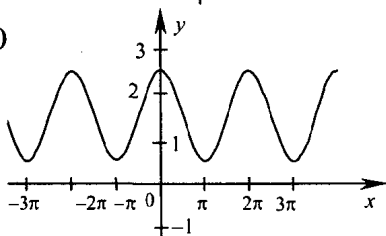
б)



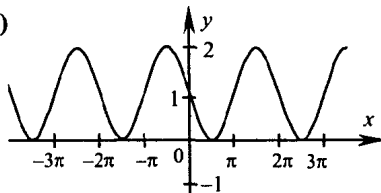
в)



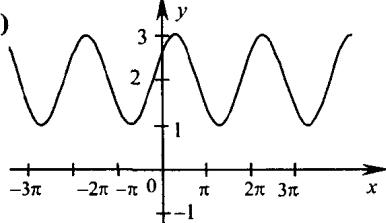
г)



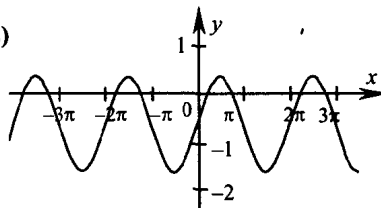
205. а)



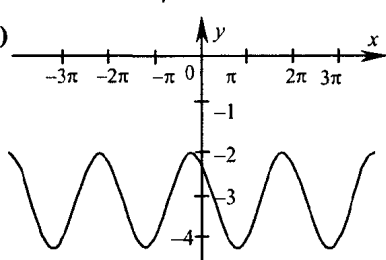
б)



в)



г)



206. $y = \cos x$.

а) $x \in \left[\frac{\pi}{6}; \frac{2\pi}{3}\right], y_{\min} = -\frac{1}{2}, y_{\max} = \frac{\sqrt{3}}{2};$

б) $x \in \left(-\pi; \frac{\pi}{4}\right), y_{\min} = \text{не существует}, y_{\max} = 1;$

в) $x \in \left[\frac{\pi}{4}; +\infty\right), y_{\min} = -1, y_{\max} = 1;$

г) $x \in \left[-\frac{\pi}{3}; \frac{3\pi}{2}\right), y_{\min} = -1, y_{\max} = 1.$

207. а) $f(x) = \begin{cases} x+2, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$

1) Область определения $D(f) = \mathbb{R}$ 2) Область значений $E(f) = (-\infty; 2]$ 3) При $x \geq 0$ функция периодична, $T = 2\pi$

4) Функция ни четная, ни нечетная

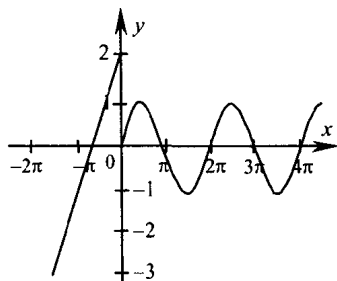
5) $f(x) = 0$ при $x = -2, x = \frac{\pi}{2} + \pi n, n \geq 0$

6) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in (-\infty; -2) \cup \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \geq 0$$

$$f(x) > 0 \text{ при } x \in \left(-2; \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right), n \geq 1$$

7) $f_{\min} = -\infty, f_{\max} = 2$

8) Функция возрастает при $x \in (-\infty; 0) \cup (-\pi + 2\pi n; 2\pi n), n \geq 0$
убывает при $x \in (2\pi n; 2\pi n + \pi), n \geq 0.$ 

б) $f(x) = \begin{cases} \cos x, & x \leq \frac{\pi}{2} \\ \sin x, & x > \frac{\pi}{2} \end{cases}$

1) Область определения $D(f) = \mathbb{R}$ 2) Область значений $E(f) = [-1; 1]$ 3) Функция периодична на промежутках $\left(-\infty; \frac{\pi}{2}\right]$ и $\left(\frac{\pi}{2}; +\infty\right), T = 2\pi$

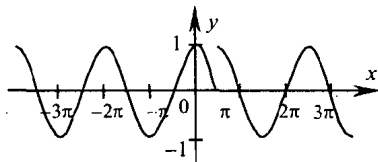
4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = \frac{\pi}{2} - \pi n, n \geq 0, x = \pi(1 + k), k \geq 0$

6) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \leq -1, x \in (-\pi + 2\pi k; 2\pi k), k \geq 1,$$

7) $f_{\min} = -1, f_{\max} = 1$



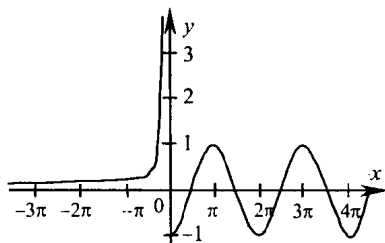
8) Функция возрастает при $x \in [-\pi + 2\pi n; 2\pi n)$, $n \leq 0$

$$x \in \left[-\frac{\pi}{2} + 2\pi k; \frac{\pi}{2} + 2\pi k\right], k \geq 1$$

убывает при $x \in [2\pi n; \pi + 2\pi n)$, $n \leq -1$, $x \in \left[0; \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2} + 2\pi k; \frac{3\pi}{2} + 2\pi k\right]$, $k \geq 0$.

$$\text{в) } f(x) = \begin{cases} -\frac{2}{x}, & x < 0 \\ -\cos x, & x \geq 0 \end{cases}$$

- 1) Область определения $D(f) = R$
- 2) Область значений $E(f) = [-1; +\infty)$
- 3) При $x \geq 0$ функция периодична, $T = 2\pi$
- 4) Функция ни четная, ни нечетная



$$5) f(x) = 0 \text{ при } x = \frac{\pi}{2} + \pi n, n \geq -1$$

6) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left[0; \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right), n \geq 1$$

$$f(x) > 0 \text{ при } x \in (-\infty; 0) \cup \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \geq 0$$

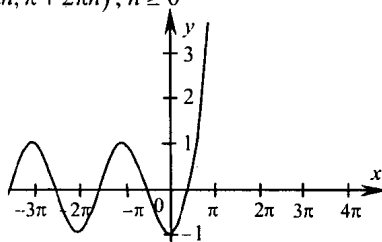
$$7) f_{\min} = -1, f_{\max} = +\infty$$

8) Функция возрастает при $x \in (-\infty; 0) \cup [2\pi n; \pi + 2\pi n)$, $n \geq 0$

убывает при $x \in [2\pi k - \pi; 2\pi k)$, $k \geq 1$.

$$\text{г) } f(x) = \begin{cases} -\cos x, & x < 0 \\ 2x^2 - 1, & x \geq 0 \end{cases}$$

- 1) Область определения $D(f) = R$
- 2) Область значений $E(f) = [-1; +\infty)$
- 3) При $x < 0$ функция периодична, $T = 2\pi$
- 4) Функция ни четная, ни нечетная



$$5) f(x) = 0 \text{ при } x = \frac{\pi}{2} - \pi n, n \geq 1, x = \frac{\sqrt{2}}{2}$$

6) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left(-\frac{\pi}{2} - 2\pi n; \frac{\pi}{2} - 2\pi n\right) \cup \left(-\frac{\pi}{2}; \frac{\sqrt{2}}{2}\right), n \geq 1$$

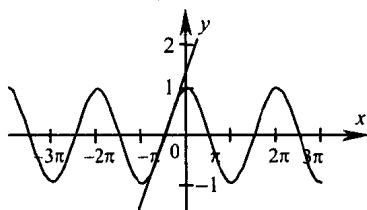
$$f(x) > 0 \text{ при } x \in \left(\frac{\pi}{2} - 2\pi k; \frac{3\pi}{2} - 2\pi k\right) \cup \left(\frac{\sqrt{2}}{2}; +\infty\right), k \geq 1$$

$$7) f_{\min} = -1, f_{\max} = +\infty$$

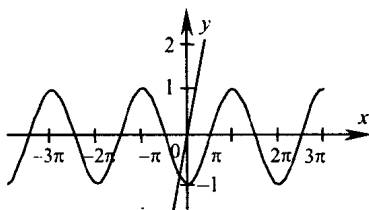
8) Функция возрастает при $x \in [-2\pi n; -2\pi n + \pi) \cup [0; +\infty)$, $n \geq 1$

убывает при $x \in [-2\pi n - \pi; -2\pi n)$, $n \geq 0$.

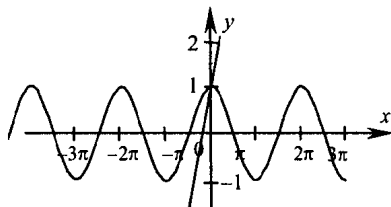
208. а) $\cos x = x + \frac{\pi}{2}$; $x = -\frac{\pi}{2}$;



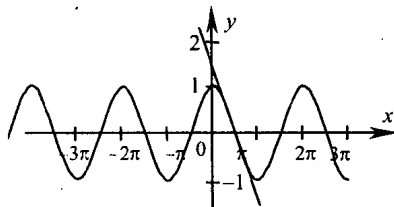
б) $-\cos x = 3x - 1$; $x = 0$;



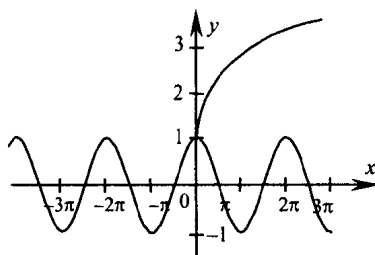
в) $\cos x = 2x + 1$; $x = 0$;



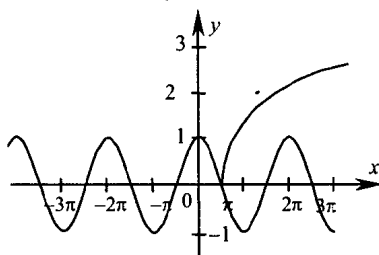
г) $\cos x = -x + \frac{\pi}{2}$; $x = \frac{\pi}{2}$.



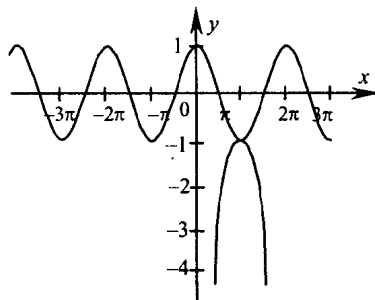
209. а) $\cos x = \sqrt{x} + 1$; $x = 0$;



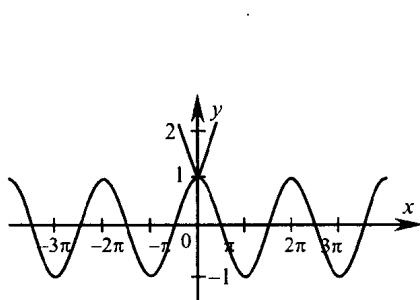
б) $\cos x = \sqrt{x - \frac{\pi}{2}}$; $x = \frac{\pi}{2}$;



в) $\cos x + 1 = -(x - \pi)^2$; $x = \pi$;



г) $\cos x = |x| + 1$; $x = 0$.



210. а) $\begin{cases} y = \cos x \\ y = -x^2 + 2x - 3 \end{cases}$; $\begin{cases} y = \cos x \\ y = -(x - 1)^2 - 2 \end{cases}$ решений нет;

б) $\begin{cases} y = \cos x \\ y = 2 \end{cases}$ бесконечное множество решений;

$$\text{в) } \begin{cases} y = \cos x \\ y = x^2 - 3 \end{cases} \quad 2 \text{ решения}$$

$$\text{г) } \begin{cases} y = \cos x \\ |x| - y = 0 \end{cases}; \quad \begin{cases} y = \cos x \\ y = |x| \end{cases} \quad 2 \text{ решения.}$$

$$211. \text{ а) } f(x) = x^2 \cos x, \quad f(-x) = (-x)^2 \cos(-x) = x^2 \cos x = f(x);$$

$$\text{б) } f(x) = \frac{\cos x^3}{4 - x^2}, \quad f(-x) = \frac{\cos(-x)^3}{4 - (-x)^2} = \frac{\cos x^3}{4 - x^2} = f(x);$$

$$\text{в) } f(x) = \frac{\cos 5x + 1}{|x|}, \quad f(-x) = \frac{\cos(-5x) + 1}{|-x|} = \frac{\cos 5x + 1}{|x|} = f(x);$$

$$\text{г) } f(x) = (4 + \cos x)(\sin^6 x - 1), \\ f(x) = (4 + \cos(-x))(\sin^6(-x) - 1) = (4 + \cos x)(\sin^6 x - 1) = f(x).$$

$$212. \text{ а) } f(x) = \sin x \cos x, \quad f(-x) = -\sin(-x)\cos(-x) = -\sin x \cos x = -f(x);$$

$$\text{б) } f(x) = x^5 \cos 3x, \quad f(-x) = (-x)^5 \cos(-3x) = -x^5 \cos 3x = -f(x);$$

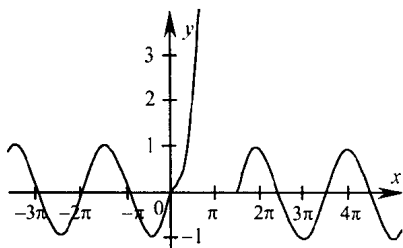
$$\text{в) } f(x) = \frac{\cos x^3}{x(25 - x^2)}, \quad f(-x) = \frac{\cos(-x)^3}{-x(25 - (-x)^2)} = -\frac{\cos x^3}{x(25 - x^2)} = -f(x);$$

$$\text{г) } f(x) = x^{11} \cdot \cos x + \sin x, \\ f(-x) = (-x)^{11} \cdot \cos(-x) + \sin(-x) = (-x)^{11} \cdot \cos x - \sin x = f(-x).$$

$$213. -f(\cos x) = -2 \cos^2 x + 3 \cos x + 2 = 2(1 - \cos^2 x) + 3 \cos x = 2 \sin^2 x + 3 \cos x$$

$$214. f(\cos x) = 5 \cos^2 x + \cos x + 4 = 5 - 5 \sin^2 x + \cos x - 4 = -5 \sin^2 x + \cos x + 9$$

$$215. f(x) = \begin{cases} \sin x, & x \leq 0 \\ x^2, & 0 < x < \frac{\pi}{2} \\ \cos x, & x \geq \frac{\pi}{2} \end{cases}$$



1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = [-1; +\infty)$

3) Функция ни четная, ни нечетная

4) При $x \leq 0$ и $x \geq \frac{\pi}{2}$ функция периодична, $T = 2\pi$

5) $f(x) = 0$ при $x = -\pi n, n \geq 0, x = \frac{\pi}{2} + \pi k, k \geq 0$

6) Промежутки знакопостоянства:

$$f(x) > 0 \text{ при } x \in (-2\pi n; -2\pi n + \pi) \cup \left(0; \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2} + 2\pi k; \frac{\pi}{2} + 2\pi k\right), n \geq 1, k \geq 1$$

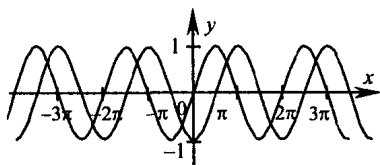
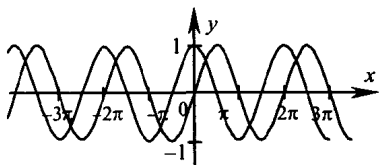
7) $f_{\min} = -1, f_{\max} = +\infty$

8) Функция возрастает при $x \in \left[-\frac{\pi}{2} - 2\pi n; \frac{\pi}{2} - 2\pi n\right], n \geq 1$

$$x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \cup \left[2\pi; \frac{5\pi}{2}\right] \cup [\pi + 2\pi n; 2\pi + 2\pi n], n \geq 1.$$

216. а) $\sin x = \cos x, x = \frac{\pi}{4} + \pi k;$

б) $\sin x = -\cos x, x = -\frac{\pi}{4} + \pi k.$



§11. Периодичность функции $y = \sin x, y = \cos x$

217. См. рис. 73. 218. См. рис. 74. 219. См. рис. 75. 220. См. рис. 76.

221. 32π является периодом функции $y = \sin x, y = \cos x$, но не основным.

222. а) $\sin 50, 5\pi = \sin \frac{\pi}{2} = 1;$ б) $\cos 51, 75\pi = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2};$

в) $\sin 25, 25\pi = -\sin \frac{3\pi}{4} = -\frac{\sqrt{2}}{2};$ г) $\sin 30, 5\pi = \sin \frac{\pi}{2} = 1.$

223. а) $\sin 390^\circ = \sin 30^\circ = \frac{1}{2};$ б) $\cos 750^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2};$

в) $\sin 540^\circ = \sin 180^\circ = 0;$ г) $\cos 930^\circ = \cos 150^\circ = -\frac{\sqrt{3}}{2}.$

224. а) $\sin^2(x - 8\pi) = 1 - \cos^2(16\pi - x),$
 $\sin^2(x - 8\pi) = \sin^2 x; 1 - \cos^2(16\pi - x) = 1 - \cos^2 x = \sin^2 x;$

б) $\cos^2(4\pi + x) = 1 - \sin^2 x(22\pi - x), \cos^2(4\pi + x) = \cos^2 x,$
 $10\sin^2 x(22\pi - x) = 1 - \sin^2 x = \cos^2 x.$

225. а) $y = \sin 2x, T = \pi, y(x + T) = \sin(2x + 2\pi) = \sin 2x = y(x);$

б) $y = \cos 3x, T = \frac{2\pi}{3}, y(x + T) = \cos(3x + 2\pi) = \cos 3x = y(x);$

в) $y = \sin \frac{x}{2}, T = 4\pi, y(x + T) = \sin\left(\frac{x}{2} + 2\pi\right) = \sin \frac{x}{2} = y(x);$

г) $y = \cos \frac{3x}{4}, T = \frac{8\pi}{3}, y(x + T) = \cos\left(\frac{3x}{4} + 2\pi\right) = \cos \frac{3x}{4} = y(x).$

226. а) $\sin 8 = \sin(8 - 2\pi);$

б) $\cos(-10) = \cos(-10 + 4\pi);$

в) $\sin(-25) = \sin(-25 + 8\pi);$

г) $\cos 35 = \cos(35 - 10\pi).$

227. а) $\cos(t + 4\pi) = ? \quad \cos(2\pi - t) = -\frac{3}{5}, \quad \cos t = -\frac{3}{5}, \quad \cos(t + 4\pi) = \cos t = -\frac{3}{5};$

б) $\sin(32\pi - t) = ? \quad \sin(2\pi - t) = \frac{5}{13}, \quad \sin(32\pi - t) = \sin(2\pi - t) = \frac{5}{13}.$

228. а) $\sin(t + 2\pi) + \sin(t - 4\pi) = 1$, $\sin t + \sin t = 1$, $\sin t = \frac{1}{2}$; $t = (-1)^k \frac{\pi}{6} + \pi k$;

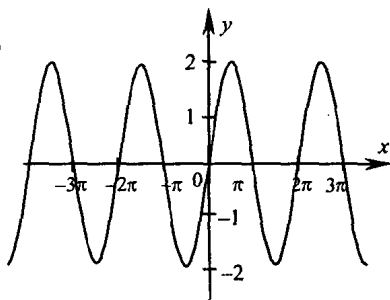
б) $3 \cos(2\pi + t) + \cos(t - 2\pi) + 2 = 0$, $4 \cos t = -2$, $\cos t = -\frac{1}{2}$, $t = \pm \frac{2\pi}{3} + 2\pi n$;

в) $\sin(t + 4\pi) + \sin(t - 6\pi) = \sqrt{3}$, $2 \sin t = \sqrt{3}$, $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k \frac{\pi}{3} + \pi k$;

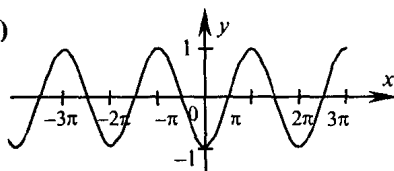
г) $\cos(t + 2\pi) + \cos(t - 8\pi) = \sqrt{2}$, $2 \cos t = \sqrt{2}$, $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm \frac{\pi}{4} + 2\pi k$.

§12. Как построить график функции $y = mf(x)$, если известен график функции $y = f(x)$

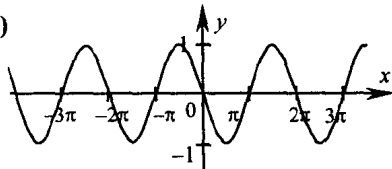
229. а)



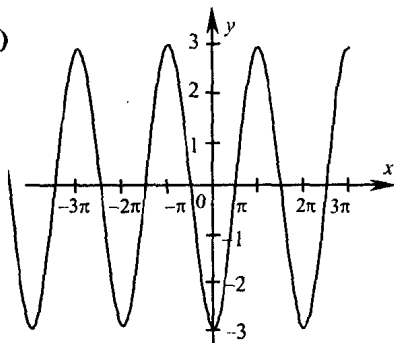
б)



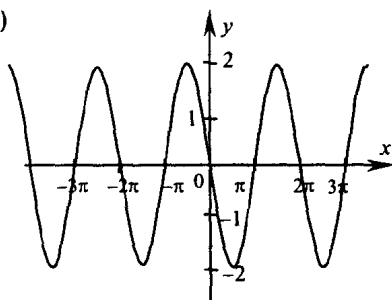
в)



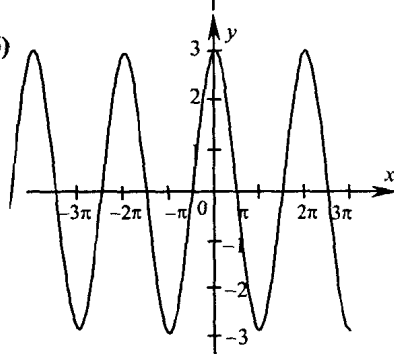
г)



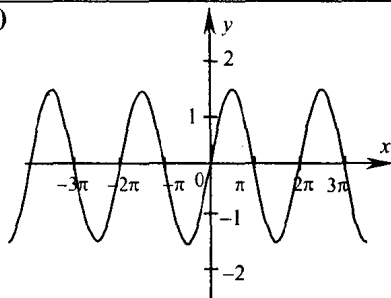
230. а)



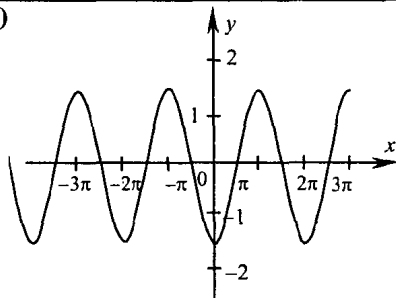
б)



в)



г)



231. а) $x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, $y_{\max} = 2$, $y_{\min} = 0$; б) $x \in \left(0; \frac{3\pi}{2}\right)$, $y_{\max} = 0$, $y_{\min} = -2$;

в) $x \in \left[\frac{\pi}{3}; \frac{3\pi}{2}\right]$, $y_{\max} = 1$, $y_{\min} = -2$; г) $x \in \left[-\frac{3\pi}{2}; -\frac{\pi}{4}\right]$, $y_{\max} = \sqrt{2}$, $y_{\min} = -2$.

232. а) $x \in [0; +\infty)$, $y_{\max} = 3$, $y_{\min} = -3$; б) $x \in \left(-\infty; \frac{\pi}{2}\right)$, $y_{\max} = 3$, $y_{\min} = -3$;

в) $x \in \left[\frac{\pi}{4}; +\infty\right)$, $y_{\max} = 3$, $y_{\min} = -3$; г) $x \in (-\infty; 0)$, $y_{\max} = 3$, $y_{\min} = -3$.

233. а) $f(-x) = -3 \sin x$;

б) $2f(x) = 6 \sin x$;

в) $2f(x) + 1 = 6 \sin x + 1$;

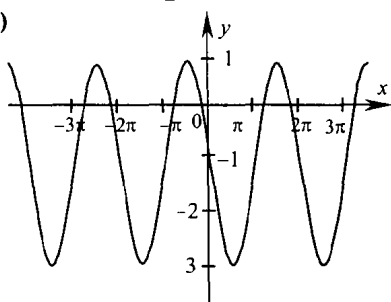
г) $f(-x) + f(x) = -3 \sin x + 3 \sin x = 0$.

234. $f(x) = -\frac{1}{2} \cos x$; а) $f(-x) = -\frac{1}{2} \cos x$; б) $2f(x) = -\cos x$;

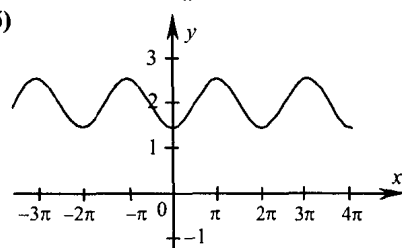
в) $f(x + 2\pi) = -\frac{1}{2} \cos x$;

г) $f(-x) - f(x) = -\frac{1}{2} \cos x + \frac{1}{2} \cos x = 0$.

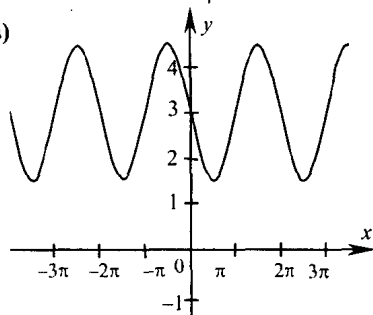
235. а)



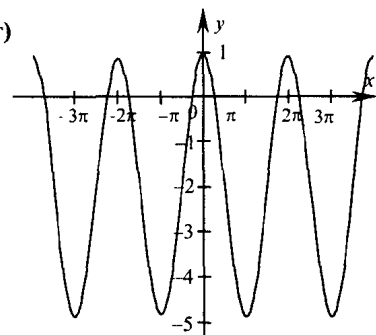
б)



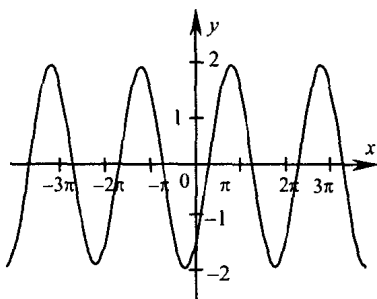
в)



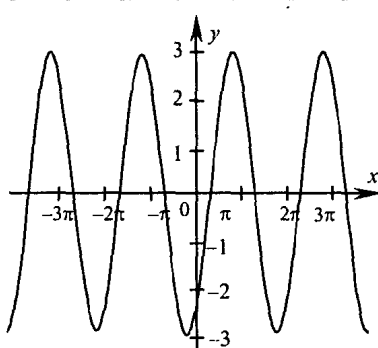
г)



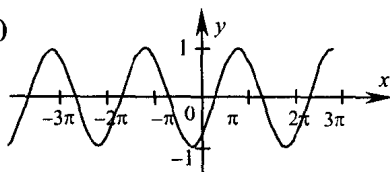
236. а)



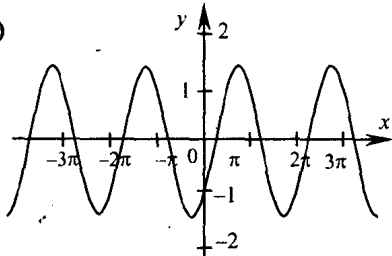
б)



в)



г)



$$237. \text{ а) } \begin{cases} x^2, & x < 0 \\ \frac{1}{2} \sin x, & 0 \leq x \leq \pi \end{cases}$$

$$\text{ б) } \begin{cases} 1,5 \cos x, & x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \\ x - \frac{\pi}{2}, & x > \frac{\pi}{2} \end{cases}$$

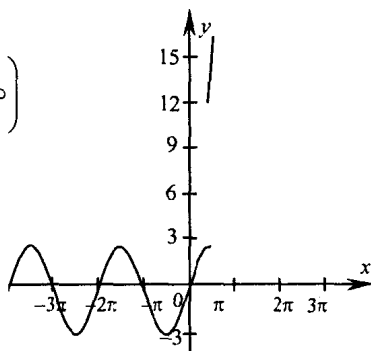
$$238. \text{ а) } f(x) = \begin{cases} 3 \sin x, & x < \frac{\pi}{2} \\ 3x^3, & x \geq \frac{\pi}{2} \end{cases}$$

1) Область определения $D(f) = R$ 2) Область значений $E(f) = [-3; 3] \cup \left[\frac{3\pi^3}{8}; +\infty\right)$ 3) Функция периодична при и $x < \frac{\pi}{2}$, $T = 2\pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\pi n$, $n \geq 0$ 6) $f_{\min} = -3$, $f_{\max} = +\infty$

7) Промежутки знакопостоянства:

 $f(x) < 0$ при $x \in (-2\pi n - \pi; -2\pi n)$, $n \geq 0$, $f(x) > 0$ при $x \in (-2\pi n; -2\pi n + \pi)$, $n \geq 0$,8) Функция возрастает при $x \in \left[-2\pi n - \frac{\pi}{2}; -2\pi n + \frac{\pi}{2}\right]$, $n \geq 0$ убывает при $x \in \left[-2\pi n + \frac{\pi}{2}; -2\pi n + \frac{3\pi}{2}\right]$, $n \geq 0$.

$$6) f(x) = \begin{cases} -2\cos x, & x < 0 \\ \frac{1}{2}x^4, & x \geq 0 \end{cases}$$

1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = [-2; +\infty)$

3) Функция периодична при $x < 0$, $T = 2\pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\frac{\pi}{2} - \pi n$, $n \geq 1$, $x = 0$

6) $f_{\min} = -3$, $f_{\max} = +\infty$

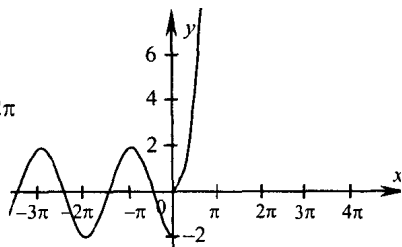
7) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left(-\frac{\pi}{2} - 2\pi n; \frac{\pi}{2} - 2\pi n\right) \cup \left(-\frac{\pi}{2}; 0\right), n \geq 1,$$

$$f(x) > 0 \text{ при } x \in \left(\frac{\pi}{2} - 2\pi n; \frac{3\pi}{2} - 2\pi n\right), n \geq 1,$$

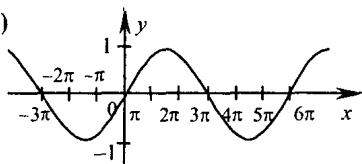
8) Функция возрастает при $x \in [-2\pi n; -2\pi n + \pi]$, $n \geq 1$, $x \geq 0$,

убывает при $x \in [-2\pi n - \pi; -2\pi n]$, $n \geq 0$

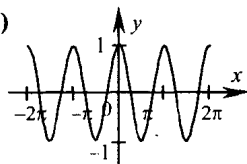


§13. Как построить график функции $y = f(kx)$, если известен график функции $y = f(x)$

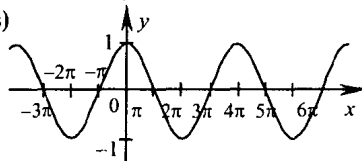
239. а)



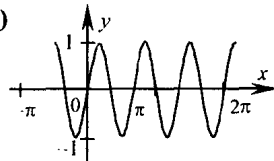
б)



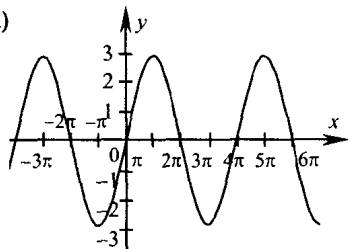
в)



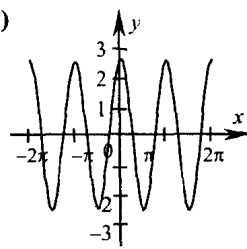
г)

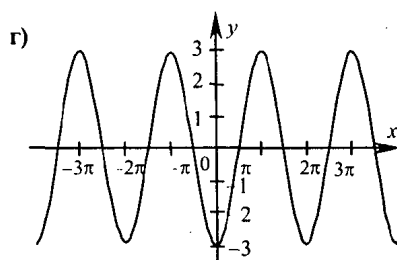
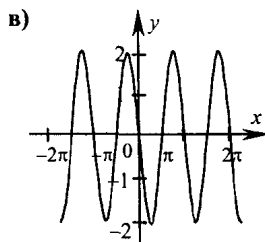
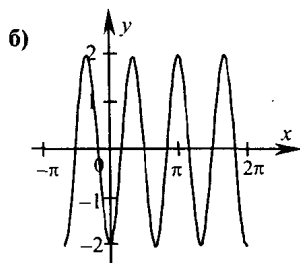
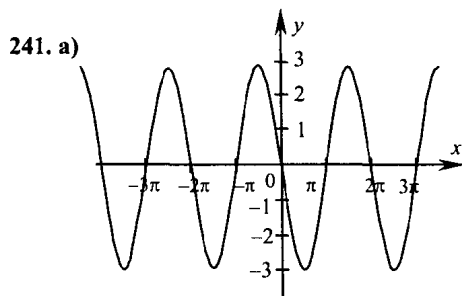
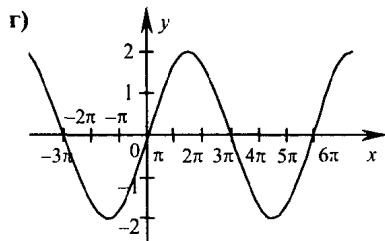
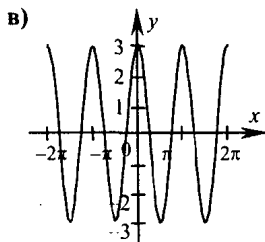


240. а)



б)





242. а) $x \in \left[-\frac{\pi}{2}; 0\right]$, $y_{\max} = 0$, $y_{\min} = -1$;

б) $x \in \left(-\frac{\pi}{4}; \frac{\pi}{2}\right]$, $y_{\max} = 1$, $y_{\min} = -1$;

в) $x \in \left[-\frac{\pi}{4}; \frac{\pi}{4}\right]$, $y_{\max} = 1$, $y_{\min} = -1$;

г) $x \in (0; \pi]$, $y_{\max} = 1$, $y_{\min} = -1$.

243. а) $x \in [0; +\infty)$, $y_{\max} = 1$, $y_{\min} = -1$;

б) $x \in (-\infty; \pi)$, $y_{\max} = 1$, $y_{\min} = -1$;

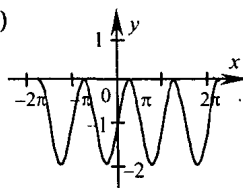
в) $x \in \left(-\infty; \frac{\pi}{2}\right]$, $y_{\max} = 1$, $y_{\min} = -1$;

г) $x \in \left(\frac{\pi}{3}; +\infty\right)$, $y_{\max} = 1$, $y_{\min} = -1$.

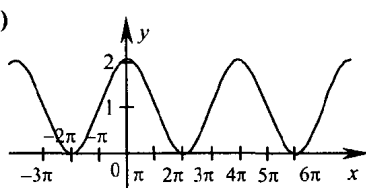
244. а) $f(-x) = \cos \frac{x}{3}$; б) $3f(x) = 3 \cos \frac{x}{3}$; в) $f(-3x) = \cos x$; г) $f(-x) - f(x) = 0$.

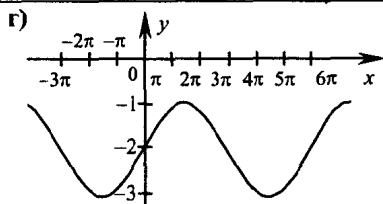
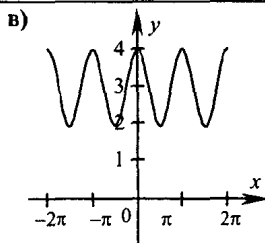
245. а) $f(-x) = -\sin 2x$; б) $2f(x) = 2 \sin 2x$; в) $f(-3x) = -\sin 6x$; г) $f(-x) + f(x) = 0$.

246. а)



б)





247. а) $f(x) = \begin{cases} \cos 2x, & x \leq \pi \\ -\frac{1}{2}, & x > \pi \end{cases}$

1) Область определения $D(f) = R$

2) Область значений $E(f) = [-1; 1]$

3) Функция периодична при $x \leq \pi$, $T = \pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\frac{\pi}{4} - \frac{\pi n}{2}$, $n \geq 0$, $x = \frac{3\pi}{4}$;

6) $f_{\min} = -1$, $f_{\max} = 1$

7) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left(\frac{\pi}{4} - \pi n; \frac{2\pi}{4} - \pi n \right) \cup (\pi; +\infty), n \geq 0,$$

$$f(x) > 0 \text{ при } x \in \left(\frac{\pi}{4} - \pi n; \frac{\pi}{4} - \pi n \right) \cup \left(\frac{3\pi}{4}; \pi \right], n \geq 0,$$

8) Функция возрастает при $x \in \left[\frac{\pi}{2} - \pi n; \pi - \pi n \right]$, $n \geq 0$

убывает при $x \in \left[-\pi n; \frac{\pi}{2} - \pi n \right]$, $n \geq 0$.

б) $f(x) = \begin{cases} -\sin 3x, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

1) Область определения $D(f) = R$

2) Область значений $E(f) = [-1; +\infty)$

3) Функция периодична при $x < 0$, $T = \frac{2}{3}\pi$

4) Функция ни четная, ни нечетная

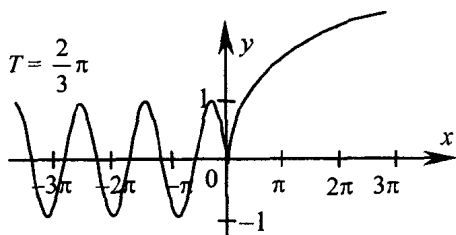
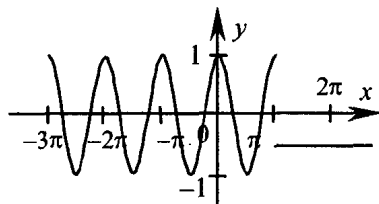
5) $f(x) = 0$ при $x = -\frac{\pi n}{3}$, $n \geq 0$,

6) $f_{\min} = -1$, $f_{\max} = +\infty$

7) Промежутки знакопостоянства:

$$f(x) < 0 \text{ при } x \in \left(-\frac{2\pi}{3}; \frac{\pi}{3} - \frac{2\pi n}{3} \right), n \geq 1,$$

$$f(x) > 0 \text{ при } x \in \left(-\frac{\pi}{3} - \frac{2\pi n}{3}; -\frac{2\pi n}{3} \right), n \geq 0, x \geq 0$$



8) Функция возрастает при $x \in \left[\frac{\pi}{6} - \frac{2\pi n}{3}; \frac{\pi}{3} - \frac{2\pi n}{3} \right], n \geq 1,$

убывает при $x \in \left[-\frac{\pi}{6} - \frac{3\pi n}{2}; \frac{\pi}{6} - \frac{3\pi n}{2} \right], n \geq 1, x \in \left[-\frac{\pi}{3}; 0 \right).$

248. а) $f(x) = \begin{cases} -2 \sin x, & x < 0 \\ \sqrt{2x}, & x \geq 0 \end{cases}$

1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = [-2; +\infty)$

3) Функция периодична при $x \leq 0, T = 2\pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = -\pi n, n \geq 0;$

6) $f_{\min} = -2, f_{\max} = +\infty$

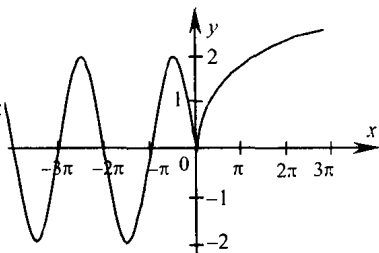
7) Промежутки знакопостоянства:

$f(x) < 0$ при $x \in (-2\pi n; -2\pi n + \pi), n \geq 1,$

$f(x) > 0$ при $x \in (-2\pi n - \pi; -2\pi n), n \geq 0,$

8) Функция возрастает при $x \in \left[-2\pi n + \frac{\pi}{2}; -2\pi n + \frac{3\pi}{2} \right] \cup [0; +\infty), n \geq 1$

убывает при $x \in \left[-2\pi n - \frac{\pi}{2}; -2\pi n + \frac{\pi}{2} \right] \cup \left[-\frac{\pi}{2}; 0 \right), n \geq 1.$



б) $f(x) = \begin{cases} \sqrt{-x}, & x \leq 0 \\ 3 \cos x - 3, & x > 0 \end{cases}$

1) Область определения $D(f) = \mathbb{R}$

2) Область значений $E(f) = [-6; +\infty)$

3) Функция периодична при $x \geq 0, T = 2\pi$

4) Функция ни четная, ни нечетная

5) $f(x) = 0$ при $x = 2\pi n, n \geq 0,$

6) $f_{\min} = -6, f_{\max} = +\infty$

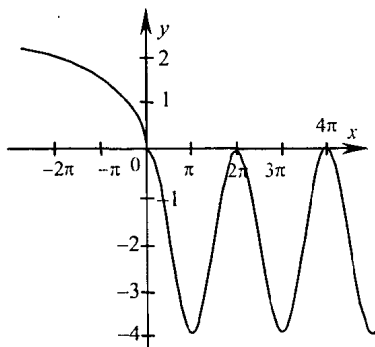
7) Промежутки знакопостоянства:

$f(x) < 0$ при $x \neq 2\pi n, n \geq 0,$

$f(x) > 0$ при $x < 0;$

8) Функция возрастает при $x \in (-2\pi n - \pi; 2\pi n), n \geq 1,$

убывает при $x \in (2\pi n; 2\pi n + \pi), n \geq 0, x \leq 0.$



249. а) $y = \begin{cases} -x, & x < 0 \\ \sin 2x, & x \geq 0 \end{cases};$

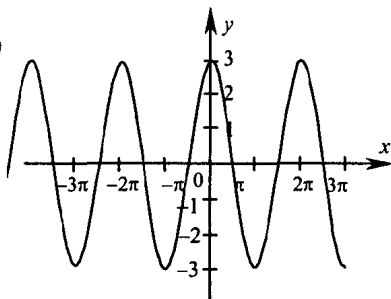
б) $y = \begin{cases} \cos 3x, & x \in \left[-\frac{\pi}{6}; \frac{\pi}{3} \right]; \\ -1, & x > \frac{\pi}{3} \end{cases}$

в) $y = \begin{cases} \sin 2x, & x < 0 \\ 2 \cos x, & x > 0 \end{cases};$

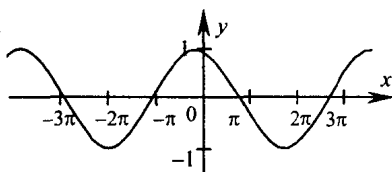
г) $y = \begin{cases} -2 \sin x, & x \in [-2\pi; 0] \\ \cos \frac{x}{2}, & x \in (0; 3\pi] \end{cases}$

§14. График гармонического колебания

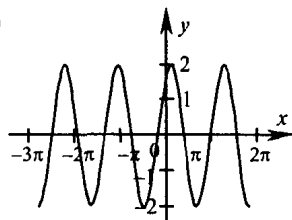
250. а)



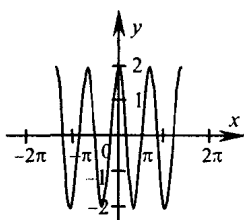
б)



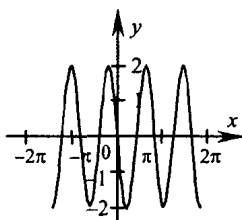
251. а)



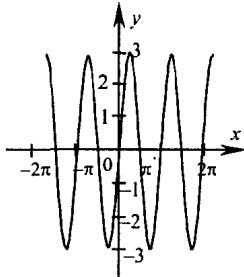
б)



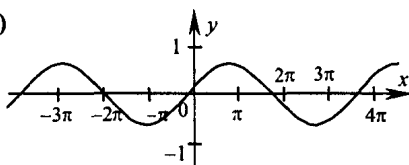
252. а)



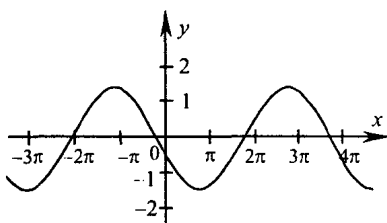
б)



253. а)



б)



§15. Функции $y = \operatorname{tg} x$, $y = \operatorname{ctg} x$, их свойства и графики

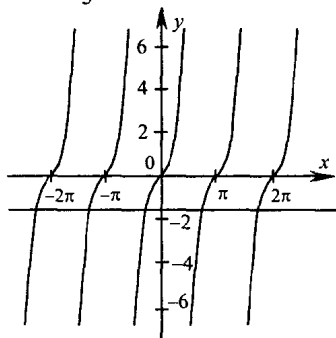
254. а) $\operatorname{tg} \frac{\pi}{4} = 1$; б) $\operatorname{tg} \frac{2\pi}{3} = -\sqrt{3}$; в) $\operatorname{tg} \frac{3\pi}{4} = -1$; г) $\operatorname{tg} \pi = 0$.

255. $t = \operatorname{tg} x$

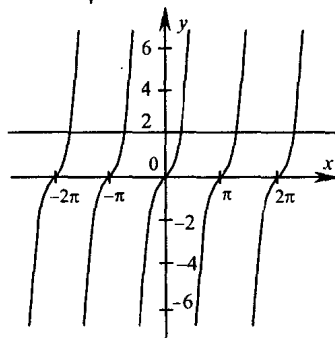
а) $x \in \left(\frac{\pi}{2}; \frac{3\pi}{2} \right)$, $y_{\max} = -$, $y_{\min} = -$; б) $x \in \left(\frac{3\pi}{4}; \pi \right]$, $y_{\max} = 0$, $y_{\min} = -$;

в) $x \in \left[-\frac{\pi}{4}; \frac{\pi}{6} \right]$, $y_{\max} = \frac{\sqrt{3}}{3}$, $y_{\min} = -1$; г) $x \in \left[\pi; -\frac{3\pi}{2} \right)$, $y_{\max} = -$, $y_{\min} = 0$.

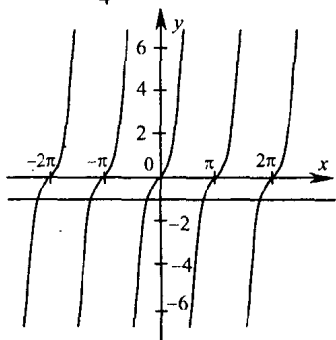
256. а) $x = \frac{\pi}{3} + \pi n$, $n \geq 0$;



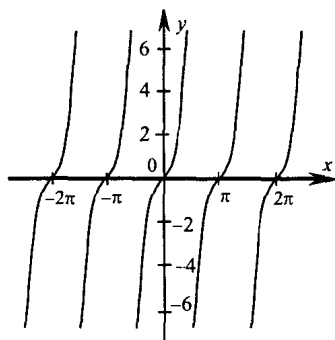
б) $x = \frac{\pi}{4} + \pi n$;



в) $x = -\frac{\pi}{4} - \pi n$;



г) $x = \pi n$.



257. а) $\operatorname{ctg} \frac{\pi}{4} = 1$;

б) $x = \operatorname{ctg} \frac{\pi}{3} = \frac{\sqrt{3}}{3}$;

в) $\operatorname{ctg} 2\pi = -$;

г) $\operatorname{ctg} \frac{\pi}{2} = 0$.

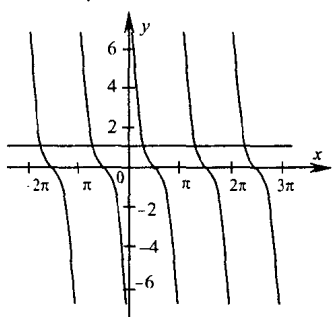
258. а) $x \in \left[\frac{\pi}{4}; \frac{\pi}{2}\right]$, $y_{\max} = 1$, $y_{\min} = 0$;

б) $x \in \left[\frac{\pi}{2}; \pi\right]$, $y_{\max} = -$, $y_{\min} = 0$;

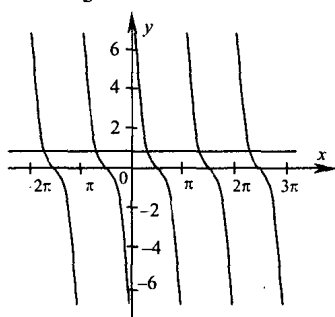
в) $x \in [-\pi; 0]$, $y_{\max} = -$, $y_{\min} = -$;

г) $x \in \left[\frac{\pi}{6}; \frac{3\pi}{4}\right]$, $y_{\max} = \sqrt{3}$, $y_{\min} = -1$.

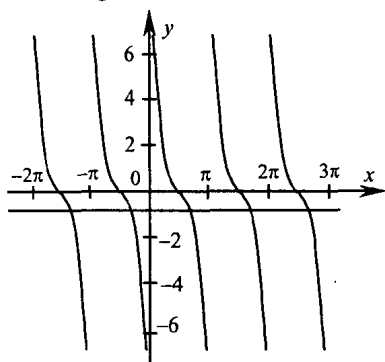
259. а) $x = \frac{\pi}{4} + \pi k$;



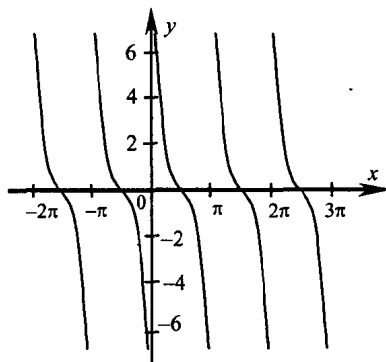
б) $x = \frac{\pi}{3} + \pi k$;



в) $x = -\frac{\pi}{3} + \pi k$;



г) $x = \frac{\pi}{2} + \pi k$.



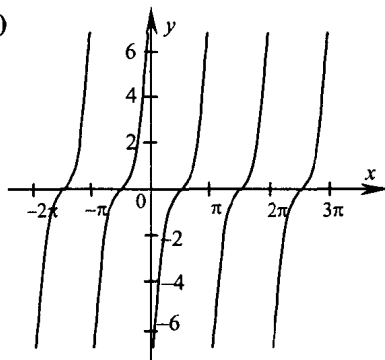
260. а) $f(x) = \operatorname{tg} x - \operatorname{ctg} x$, $f(-x) = -\operatorname{tg} x - \cos x$, ни четная, ни нечетная;

б) $f(x) = \operatorname{tg} x + x$, $f(-x) = -\operatorname{tg} x - x = -f(x)$, нечетная;

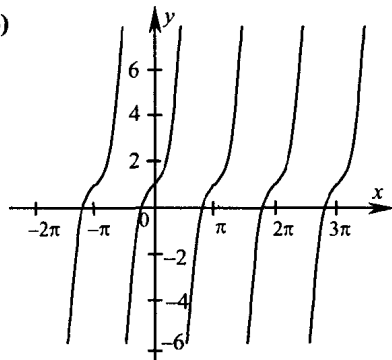
в) $f(x) = \operatorname{ctg}^2 x - x^4$, $f(-x) = \operatorname{ctg}^2 x - x^4$, четная;

г) $f(x) = x^3 - \operatorname{ctg} x$, $f(-x) = -x^3 + \operatorname{ctg} x = -f(x)$, нечетная.

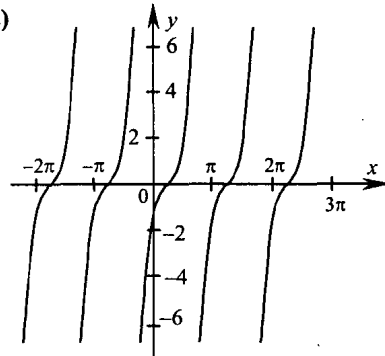
261. а)



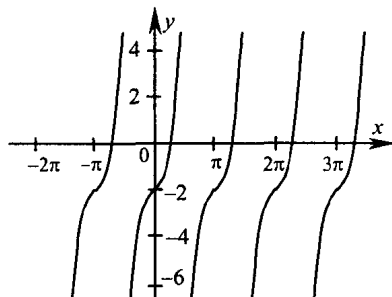
б)



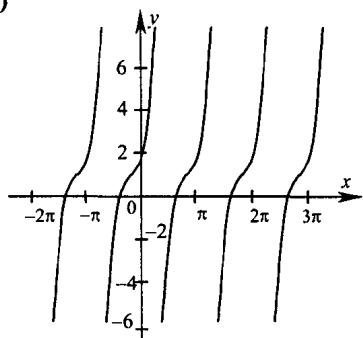
в)



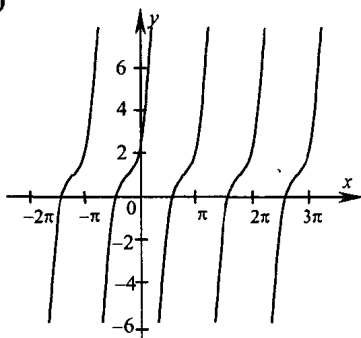
г)



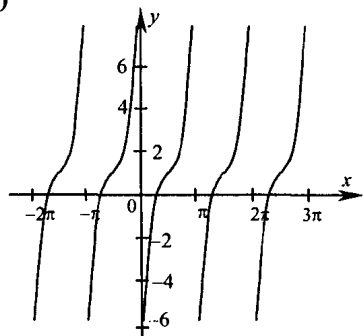
262. а)



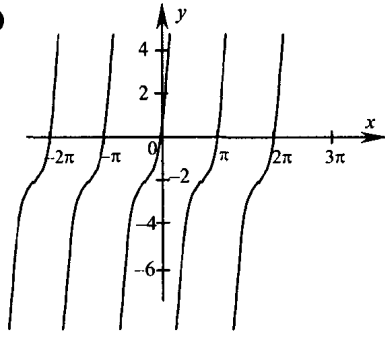
б)



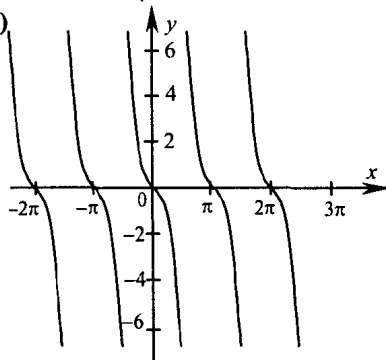
в)



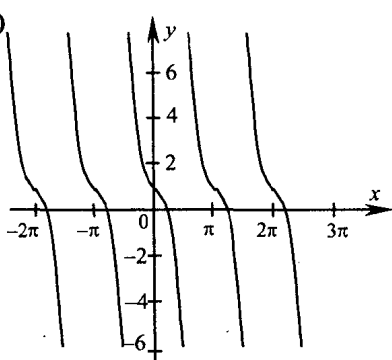
г)



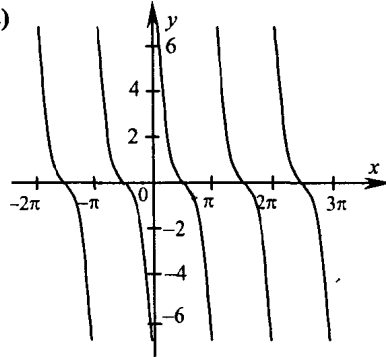
263. а)



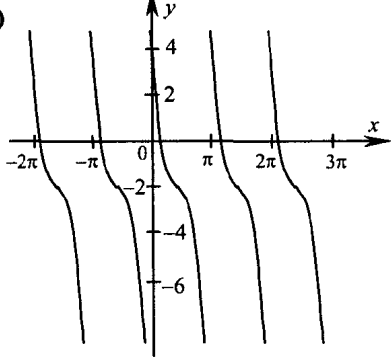
б)

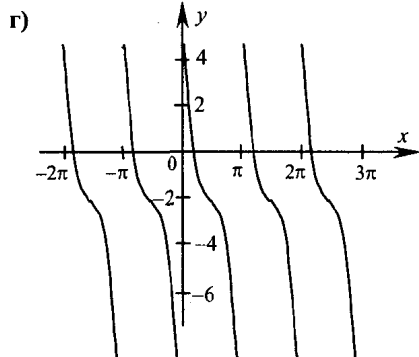
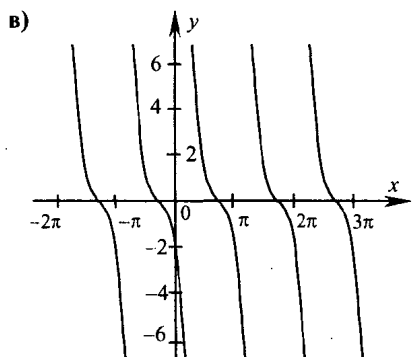
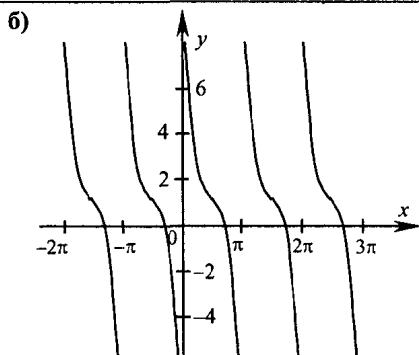
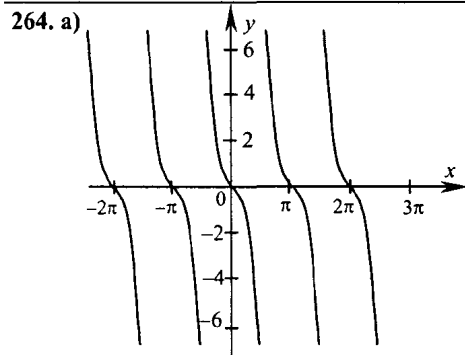


в)



г)





265. а) $y = \operatorname{tg} 2x$, $T = \frac{\pi}{2}$, $y(x+T) = \operatorname{tg}(2x+\pi) = \operatorname{tg} x$;

б) $y = \operatorname{tg} \frac{x}{3}$, $T = 3\pi$, $y(x+T) = \operatorname{tg}\left(\frac{x}{3} + \pi\right) = \operatorname{tg} \frac{x}{3}$;

в) $y = \operatorname{tg} 5x$, $T = \frac{\pi}{5}$, $y(x+T) = \operatorname{tg}(5x+\pi) = \operatorname{tg} 5x$;

г) $y = \frac{2x}{5}$, $T = \frac{5\pi}{2}$, $y(x+T) = \operatorname{tg}\left(\frac{2x}{5} + \pi\right) = \operatorname{tg} \frac{2x}{5}$.

266. а) $y = \operatorname{tg} x + \sin 2x - \operatorname{tg} 3x - \cos 4x$, $T = \pi$,
 $y(x+\pi) = \operatorname{tg}(x+\pi) + \sin(2x+2\pi) - \operatorname{tg}(3x+3\pi) - \cos(4x+4\pi) = y(x)$;

б) $y = \sin 3x + \cos 5x + \operatorname{ctg} x - 2\operatorname{tg} 2x$, $T = \pi$,
 $y(x+\pi) = \sin(3x+3\pi) + \cos(5x+5\pi) + \operatorname{ctg}(\pi+x) - 2\operatorname{tg}(2x+\pi) =$
 $= -\sin 3x - \cos 5x + \operatorname{ctg} x - 2\operatorname{tg} 2x \neq y(x)$, $\Rightarrow \pi$ не есть период.

267. $\operatorname{tg}(9\pi-x) = -\frac{3}{4}$; $\operatorname{tg}(9\pi-x) = -\operatorname{tg} x$; $\operatorname{tg} x = \frac{3}{4}$, $\operatorname{ctg} x = \frac{4}{3}$.

268. $\operatorname{ctg}(7\pi-x) = \frac{5}{7}$; $\operatorname{ctg} x = -\frac{5}{7}$; $\operatorname{tg} = -\frac{7}{5}$.

269. а) $\operatorname{tg} 200^\circ - \operatorname{tg} 201^\circ < 0$;

в) $\operatorname{tg} 2,2 - \operatorname{tg} 2,1 > 0$;

б) $\operatorname{tg} 1 - \operatorname{tg} 1,01 < 0$;

г) $\operatorname{tg} \frac{3\pi}{5} - \operatorname{tg} \frac{6\pi}{5} < 0$.

270. а) $f(x) = \operatorname{tg} x \sin^2 x$, $f(-x) = -\operatorname{tg} x \cdot \sin^2 x = -f(x)$, нечетная;

б) $f(x) = \frac{\operatorname{tg}^2 x}{x^2 - 1}$, $f(-x) = \frac{\operatorname{tg}^2 x}{x^2 - 1} = f(x)$, четная;

в) $f(x) = x^5 \operatorname{tg} x$,

$f(-x) = x^5 \operatorname{tg} x = f(x)$, четная;

г) $f(x) = x^2 + \sin x + \operatorname{tg} x$,

$f(-x) = x^2 - \sin x - \operatorname{tg} x$, ни четная, ни нечетная.

271. а) $f(x) = \sin x + \operatorname{ctg} x$,

$f(-x) = -\sin x - \operatorname{ctg} x = -f(x)$, нечетная;

б) $f(x) = \frac{2 \operatorname{ctg} x}{x^3}$,

$f(-x) = \frac{-2 \operatorname{ctg} x}{-x^3} = f(x)$, четная;

в) $f(x) = \frac{x^4 \operatorname{ctg} x}{x^2 - 4}$,

$f(-x) = -\frac{x^4 \operatorname{ctg} x}{x^2 - 4} = -f(x)$, нечетная;

г) $f(x) = \operatorname{ctg} x - x \cos x$,

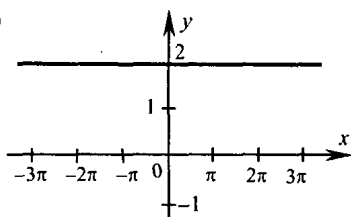
$f(-x) = -\operatorname{ctg} x + x \cos x = -f(x)$, нечетная.

272. $f(x) = \operatorname{tg} x$, $f(2x + 2\pi) + f(7\pi - 2x) = \operatorname{tg}(2x + 2\pi) + \operatorname{tg}(7\pi - 2x) = \operatorname{tg} 2x - \operatorname{tg} 2x = 0$.

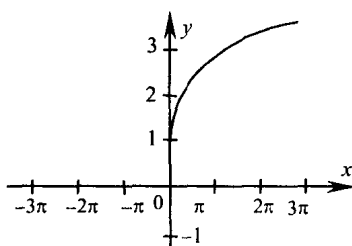
273. $f(x) = x^2 + 1$, $f(\operatorname{tg} x) = \operatorname{tg}^2 x + 1 = \frac{1}{\cos^2 x}$.

274. $f(x) = x^2 + 1$, $f(\operatorname{ctg} x) = \operatorname{ctg}^2 x + 1 = \frac{1}{\sin^2 x}$.

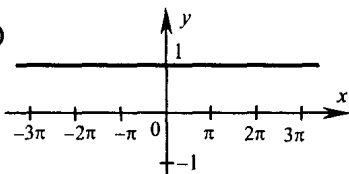
275. а)



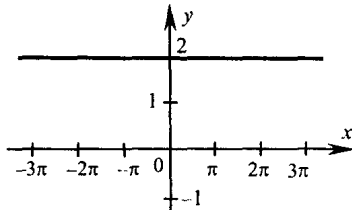
б)



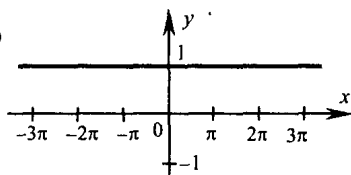
276. а)



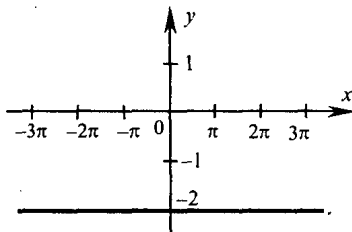
б)



277. а)



б)



Глава 2. Тригонометрические уравнения

§16. Первые представления о решении тригонометрических уравнений

278. а) $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm \frac{\pi}{4} + 2\pi n$;

б) $\sin t = -\frac{1}{2}$, $t = (-1)^{k+1} \frac{\pi}{6} + \pi k$;

в) $\cos t = -\frac{1}{2}$, $t = \pm \frac{2\pi}{3} + 2\pi n$;

г) $\sin t = \frac{\sqrt{2}}{2}$, $t = (-1)^k \frac{\pi}{4} + \pi k$.

279. а) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm \frac{\pi}{6} + 2\pi k$;

б) $\sin t = -\frac{\sqrt{3}}{2}$, $t = (-1)^{k+1} \frac{\pi}{6} + \pi k$;

в) $\cos t = -\frac{\sqrt{3}}{2}$, $t = \pm \frac{5\pi}{6} + 2\pi k$;

г) $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k \frac{\pi}{3} + \pi k$.

280. а) $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi k$;

б) $\cos t = 2$, решений нет;

в) $\cos t = -1$, $t = \pi + 2\pi k$;

г) $\sin t = -3$, решений нет.

281. а) $\operatorname{tg} t = \sqrt{3}$, $t = \frac{\pi}{3} + \pi n$;

б) $\operatorname{ctg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{3} + 2\pi n$;

в) $\operatorname{tg} t = -\frac{\sqrt{3}}{3}$, $t = -\frac{\pi}{6} + \pi n$;

г) $\operatorname{ctg} t = \sqrt{3}$, $t = \frac{\pi}{6} + \pi n$.

282. а) $\sin t(2 \cos t + 1) = 0$, $\begin{cases} \sin t = 0 \\ \cos t = -\frac{1}{2} \end{cases}$, $t = \pi n$, $t = \pm \frac{2\pi}{3} + 2\pi n$;

б) $(\sin t - 1)(\cos t + 1) = 0$, $\begin{cases} \sin t = 1 \\ \cos t = -1 \end{cases}$, $t = \frac{\pi}{2} + 2\pi n$, $t = \pi + 2\pi n$;

в) $\cos t \cdot (2 \sin t + 1) = 0$, $\begin{cases} \cos t = 0 \\ \sin t = -\frac{1}{2} \end{cases}$, $t = \frac{\pi}{2} + \pi n$, $t = (-1)^{n+1} \frac{\pi}{6} + \pi n$;

г) $(2 \sin t - \sqrt{2})(2 \cos t + 1) = 0$, $\begin{cases} \sin t = \frac{\sqrt{2}}{2} \\ \cos t = -\frac{1}{2} \end{cases}$, $t = (-1)^{n+1} \frac{\pi}{4} + \pi n$, $t = \pm \frac{2\pi}{3} + 2\pi n$.

283. а) $\cos\left(\frac{\pi}{2} - t\right) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$; б) $\cos(t - \pi) = 1$, $-\cos t = 1$, $t = \pi + 2\pi n$;

в) $\sin t(\pi - t) = 1$, $\sin t = 1$, $t = \frac{\pi}{2} + 2\pi n$; г) $\sin\left(t - \frac{\pi}{2}\right) = 1$, $\cos t = -1$, $t = \pi + 2\pi n$.

284. а) $3 - 4 \sin^2 t = 0$, $\sin t = \pm \frac{\sqrt{3}}{2}$, $t = (-1)^k \frac{\pi}{3} + \pi k$, $t = (-1)^{k+1} \frac{\pi}{3} + \pi k$;

б) $\sin^2 t - \sin t = 0$, $\begin{cases} \sin t = 0 \\ \sin t = 1 \end{cases}$, $t = \pi n$, $t = \frac{\pi}{2} + 2\pi n$;

$$\text{в)} 4\sin^2 t - 1 = 0, \sin t = \pm \frac{1}{2}, t = (-1)^k \frac{\pi}{6} + \pi k, t = (-1)^{k+1} \frac{\pi}{6} + \pi k;$$

$$\text{г)} 2\sin^2 t + \sin t = 0, \begin{cases} \sin t = 0 \\ \sin t = -\frac{1}{2} \end{cases}, t = \pi n, t = (-1)^{k+1} \frac{\pi}{6} + \pi k.$$

$$285. \text{а)} 3 - 4\cos^2 t = 0 \cos t = \pm \frac{\sqrt{3}}{2}, t = \pm \frac{\pi}{6} + \pi n;$$

$$\text{б)} 2\cos^2 t - \cos t = 0, \begin{cases} \cos t = 0 \\ \cos t = \frac{1}{2} \end{cases}, t = \frac{\pi}{2} + \pi n, t = \pm \frac{\pi}{3} + 2\pi n;$$

$$\text{в)} 4\cos^2 t - 1 = 0, \cos t = \pm \frac{1}{2}, t = \pm \frac{\pi}{3} + 2\pi n, t = \pm \frac{2\pi}{3} + 2\pi n;$$

$$\text{г)} 2\cos^2 t + \cos t = 0, \begin{cases} \cos t = 0 \\ \cos t = -\frac{1}{2} \end{cases}, t = \frac{\pi}{2} + \pi n, t = \pm \frac{2\pi}{3} + 2\pi n.$$

$$286. \text{а)} 2\sin^2 t + 3\sin t - 2 = 0, \sin t = \frac{-3 + \sqrt{9 - 4 \cdot 2(-2)}}{4} = \frac{1}{2},$$

$$t = (-1)^k \frac{\pi}{6} + \pi k, \sin t = -2 \text{ не подходит};$$

$$\text{б)} 2\cos^2 t - 5\cos t + 2 = 0, \cos t = \frac{5+3}{4} = 2 - \text{не подходит}, \cos t = \frac{1}{2}, t = \pm \frac{\pi}{3} + 2\pi n;$$

$$\text{в)} 2\sin^2 t + \sin t - 1 = 0, \sin t = \frac{-1+3}{4} = \frac{1}{2}, t = (-1)^k \frac{\pi}{6} + \pi k,$$

$$\sin t = -1, t = -\frac{\pi}{2} + 2\pi n;$$

$$\text{г)} 4\cos^2 t + 9\cos t + 5 = 0, \cos t = \frac{-9+1}{8} = -1, t = \pi + 2\pi n,$$

$$\cos t = \frac{-9-1}{8} - \text{не подходит}.$$

$$287. \text{а)} 2\cos^2 t + \sin t + 1 = 0, 2 - 2\sin^2 t + \sin t + 1 = 0, 2\sin^2 t - \sin t - 3 = 0,$$

$$\sin t = \frac{1+5}{4} \text{ не подходит}, \sin t = \frac{1-5}{4} = -1, t = -\frac{\pi}{2} + 2\pi k;$$

$$\text{б)} \sin^2 t + \cos t - 3 = 0, \cos^2 t - 3\cos t + 2 = 0, \cos t = 2 \text{ не подходит}, \cos t = 1, t = 2\pi n.$$

$$288. \text{а)} \sin\left(\frac{\pi}{2} + t\right) - \cos(\pi + t) = 1, \cos t + \cos t = 1, \cos t = \frac{1}{2}, t = \pm \frac{\pi}{3} + 2\pi n;$$

$$\text{б)} \sin(\pi + t) + \sin(2\pi - t) - \cos\left(\frac{3\pi}{2} + t\right) + 1,5 = 0,$$

$$-\sin t - \sin t - \sin t = -\frac{3}{2}, \quad \sin t = \frac{1}{2}, \quad t = (-1)^k \frac{\pi}{6} + \pi k;$$

$$\text{в)} \cos\left(\frac{\pi}{2} - t\right) - \sin(\pi + t) = \sqrt{2}, \quad \sin t + \sin t = \sqrt{2}, \quad \sin t = \frac{\sqrt{2}}{2}, \quad t = (-1)^k \frac{\pi}{4} + \pi k;$$

$$\text{г)} \sin(\pi + t) + \cos\left(\frac{\pi}{2} + t\right) = \sqrt{3}, \quad -\sin t - \sin t = \sqrt{3}, \quad \sin t = -\frac{\sqrt{3}}{2}, \quad t = (-1)^{k+1} \frac{\pi}{3} + \pi k.$$

§17. Арккосинус и решение уравнения $\cos t = a$

$$289. \text{ а) } \arccos 0 = \frac{\pi}{2}; \quad \text{б) } \arccos 1 = 0; \quad \text{в) } \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}; \quad \text{г) } \arccos \frac{1}{2} = \frac{\pi}{3}.$$

$$290. \text{ а) } \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}; \quad \text{б) } \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6};$$

$$\text{в) } \arccos(-1) = \pi; \quad \text{г) } \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}.$$

$$291. \text{ а) } \arccos(-1) + \arccos 0 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}; \quad \text{б) } \arccos \frac{1}{2} - \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6};$$

$$\text{в) } \arccos\left(-\frac{\sqrt{2}}{2}\right) + \arccos \frac{\sqrt{2}}{2} = \frac{3\pi}{4} + \frac{\pi}{4} = \pi; \quad \text{г) } \arccos\left(-\frac{1}{2}\right) - \arccos \frac{1}{2} = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}.$$

$$292. \text{ а) } \sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}; \quad \text{б) } \operatorname{tg}\left(\arccos \frac{\sqrt{3}}{2}\right) = \operatorname{tg} \frac{\pi}{6} = \frac{\sqrt{3}}{3};$$

$$\text{в) } \operatorname{ctg}(\arccos 0) = \operatorname{ctg} \frac{\pi}{2} = 0; \quad \text{г) } \sin\left(\arccos \frac{\sqrt{2}}{2}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

$$293. \text{ а) } \cos t = \frac{1}{2}, \quad t = \pm \frac{\pi}{3} + 2\pi n; \quad \text{б) } \cos t = \frac{\sqrt{2}}{2}, \quad t = \pm \frac{\pi}{4} + 2\pi n;$$

$$\text{в) } \cos t = 1, \quad t = 2\pi n; \quad \text{г) } \cos t = \frac{\sqrt{3}}{2}, \quad t = \pm \frac{\pi}{6} + 2\pi n.$$

$$294. \text{ а) } \cos t = -1, \quad t = \pi + 2\pi n; \quad \text{б) } \cos t = -\frac{\sqrt{3}}{2}, \quad t = \pm \frac{5\pi}{6} + 2\pi n;$$

$$\text{в) } \cos t = -\frac{1}{2}, \quad t = \pm \frac{2\pi}{3} + 2\pi n; \quad \text{г) } \cos t = -\frac{\sqrt{2}}{2}, \quad t = \pm \frac{3\pi}{4} + 2\pi n.$$

$$295. \text{ а) } \cos t = \frac{1}{3}, \quad t = \pm \arccos \frac{1}{3} + 2\pi n; \quad \text{б) } \cos t = -1, \text{ решений нет;}$$

$$\text{в) } \cos t = -\frac{3}{7}, \quad t = \pm \arccos\left(-\frac{3}{7}\right) + 2\pi n; \quad \text{г) } \cos t = 2,04, \text{ решений нет.}$$

$$296. \text{ а) } \cos t \left(2 \arccos \frac{1}{2} - 3 \arccos 0 - \arccos \left(-\frac{1}{2} \right) \right) = \cos \left(\frac{2\pi}{3} - \frac{3\pi}{2} - \frac{2\pi}{3} \right) = \cos \frac{3\pi}{2} = 0;$$

$$\text{ б) } \frac{1}{3} \left(\arccos \frac{1}{3} + \arccos \left(-\frac{1}{3} \right) \right) = \frac{1}{3} \pi = \frac{\pi}{3}.$$

$$297. \text{ а) } x \in [-1; 1]; \quad \text{ б) } |x| \leq \frac{1}{2}; \quad \text{ в) } x \in [0; 2]; \quad \text{ г) } x \in [1; 2].$$

$$298. \text{ а) } \sqrt{5} > 1, \text{ нет}; \quad \text{ б) } s \frac{\sqrt{2}}{3} < 1, \text{ да}; \quad \text{ в) } \frac{\pi}{5} < 1, \text{ да}; \quad \text{ г) } -\sqrt{3} < -1, \text{ нет}.$$

$$299. \operatorname{tg}(\arccos 0,1 + \arccos(-0,1) + x) = \operatorname{tg} x, \quad \operatorname{tg}(\pi + x) = \operatorname{tg} x.$$

$$300. \text{ а) } \frac{8 \cos t - 3}{3 \cos t + 2} = 1, \quad \frac{8 \cos t - 3 - 3 \cos t - 2}{3 \cos t + 2} = 0, \quad \begin{cases} 5 \cos t - 5 = 0 \\ \cos t \neq -\frac{2}{3} \end{cases}, \quad \cos t = 1, \quad t = 2\pi n;$$

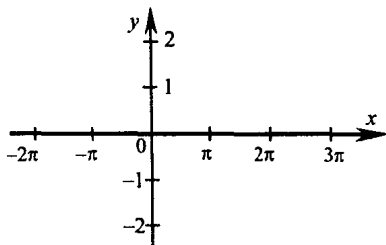
$$\text{ б) } \frac{3 \cos t + 1}{2} + \frac{5 \cos t - 1}{3} = 1 \frac{3}{4}, \quad 9 \cos t + 3 + 10 \cos t - 2 = \frac{7}{4} \cdot 6 = \frac{21}{2},$$

$$19 \cos t = \frac{19}{2}, \quad \cos t = \frac{1}{2}, \quad t = \pm \frac{\pi}{3} + 2\pi n.$$

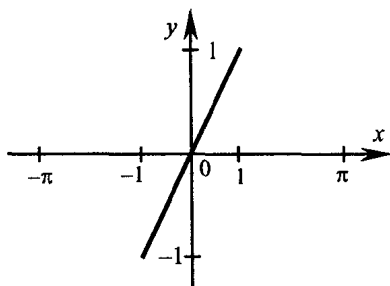
$$301. 6 \cos^2 t + 5 \cos t + 1 = 0, \quad \cos t = \frac{-5+1}{12} = -\frac{1}{3}, \quad t = \pm \arccos \left(-\frac{1}{3} \right) + 2\pi n,$$

$$\cos t = \frac{-5-1}{12} = -\frac{1}{2}, \quad t = \pm \frac{2\pi}{3} + 2\pi n.$$

302. а)



б)



$$303. \text{ а) } \cos t > \frac{1}{2}, \quad t \in \left(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k \right);$$

$$\text{ б) } \cos t \leq -\frac{\sqrt{2}}{2}, \quad t \in \left[\frac{3\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k \right];$$

$$\text{ в) } \cos t \geq -\frac{\sqrt{2}}{2}, \quad t \in \left[-\frac{5\pi}{4} + 2\pi k; -\frac{3\pi}{4} + 2\pi k \right];$$

$$\text{ г) } \cos t < \frac{1}{2}, \quad t \in \left(\frac{\pi}{3} + 2\pi k; \frac{5\pi}{3} + 2\pi k \right).$$

$$304. \text{ а) } \cos t < \frac{2}{3}, \quad t \in \left(\arccos \frac{2}{3} + 2\pi k; 2\pi - \arccos \frac{2}{3} + 2\pi k \right);$$

$$\text{ б) } \cos t > -\frac{1}{7}, \quad t \in \left(-\arccos \left(-\frac{1}{7} \right) + 2\pi k; \arccos \left(-\frac{1}{7} \right) + 2\pi k \right);$$

$$\text{ в) } \cos t > \frac{2}{3}, \quad t \in \left(-\arccos \frac{2}{3} + 2\pi k; \arccos \frac{2}{3} + 2\pi k \right);$$

$$\text{ г) } \cos t < -\frac{1}{7}, \quad t \in \left(\arccos \left(-\frac{1}{7} \right) + 2\pi k; 2\pi - \arccos \left(-\frac{1}{7} \right) + 2\pi k \right).$$

$$305. \text{ а) } 3 \cos^2 t - 4 \cos t \geq 4, \quad 3 \cos^2 t - 4 \cos t - 4 = 0.$$

Найдем корни квадратного уравнения:

$$\cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{4 \pm 8}{6}, \quad \cos t = -\frac{2}{3}, \quad \cos t = 2 \text{ не подходит,}$$

$$\cos t \leq -\frac{2}{3}, \quad t \in \left(\arccos \left(-\frac{2}{3} \right) + 2\pi k; 2\pi - \arccos \left(-\frac{2}{3} \right) + 2\pi k \right);$$

$$\text{ б) } 6 \cos^2 t + 1 > 5 \cos t.$$

Найдем корни квадратного уравнения:

$$6 \cos^2 t - 5 \cos t + 1 = 0, \quad \cos t = \frac{5+1}{12} = \frac{1}{2}, \quad \cos t = \frac{1}{3},$$

$$t \in \left(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k \right) \cup \left(\arccos \frac{1}{3} + 2\pi k; 2\pi - \arccos \frac{1}{3} + 2\pi k \right);$$

$$\text{ в) } 3 \cos^2 t - 4 \cos t < 4, \quad 3 \cos^2 t - 4 \cos t - 4 < 0.$$

Найдем корни квадратного уравнения:

$$3 \cos^2 t - 4 \cos t - 4 = 0, \quad \cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{2 \pm 4}{3},$$

$$\cos t = 2 \text{ — не подходит, } \cos t = -\frac{2}{3} \rightarrow \cos t > -\frac{2}{3},$$

$$t \in \left(-\arccos \left(-\frac{2}{3} \right) + 2\pi k; -\left(-\frac{2}{3} \right) + 2\pi k \right);$$

$$\text{ г) } 6 \cos^2 t + 1 \leq 5 \cos t, \quad 6 \cos^2 t - 5 \cos t + 1 \leq 0.$$

Найдем корни квадратного уравнения:

$$6 \cos^2 t - 5 \cos t + 1 = 0, \quad \cos t = \frac{5 \pm \sqrt{25 - 4 \cdot 6 \cdot 1}}{12} = \frac{5 \pm 1}{12}, \quad \cos t = \frac{1}{2}, \quad \cos t = \frac{1}{3},$$

$$t \in \left(-\arccos \frac{1}{3} + 2\pi k; -\frac{\pi}{3} + 2\pi k \right) \cup \left(\frac{\pi}{3} + 2\pi k; \arccos \frac{1}{3} + 2\pi k \right).$$

$$306. \text{ а) } 4 \cos^2 t < 1, \quad \cos^2 t < \frac{1}{4}, \quad \cos t \in \left(-\frac{1}{2}; \frac{1}{2} \right), \quad t \in \left(\frac{\pi}{3} + \pi k; \frac{2\pi}{3} + \pi k \right);$$

$$\text{ б) } 3 \cos^2 t < \cos t, \quad \cos t(3 \cos t - 1) < 0, \quad \cos t \in \left(0; \frac{1}{3} \right),$$

$$t \in \left(-\frac{\pi}{2} + 2\pi n; -\arccos \frac{1}{3} + 2\pi n\right) \cup \left(\arccos \frac{1}{3} + 2\pi n; \frac{\pi}{2} + 2\pi n\right).$$

$$307. \text{ а) } \sin\left(\arccos \frac{3}{5}\right) = \sqrt{1 - \cos^2\left(\arccos \frac{3}{5}\right)} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5};$$

$$\text{ б) } \sin\left(\arccos\left(-\frac{4}{5}\right)\right) = \sqrt{1 - \cos^2\left(\arccos\left(-\frac{4}{5}\right)\right)} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}.$$

$$308. \text{ а) } \operatorname{tg}\left(\arccos\left(-\frac{15}{13}\right)\right) = \frac{\sqrt{1 - \cos^2\left(\arccos\left(-\frac{5}{13}\right)\right)}}{\cos\left(\arccos\left(-\frac{5}{13}\right)\right)} = \frac{\sqrt{1 - \frac{25}{169}}}{-\frac{5}{13}} = -\frac{12}{5};$$

$$\text{ б) } \operatorname{ctg}\left(\arccos \frac{4}{5}\right) = \frac{\cos\left(\arccos \frac{4}{5}\right)}{\sqrt{1 - \cos^2\left(\arccos \frac{4}{5}\right)}} = \frac{4 \cdot 5}{5 \cdot 3} = \frac{4}{3}.$$

§18. Арксинус и решение уравнения $\sin t = a$

$$309. \text{ а) } \arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}; \quad \text{ б) } \arcsin 1 = \frac{\pi}{2}; \quad \text{ в) } \arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}; \quad \text{ г) } \arcsin 0 = 0.$$

$$310. \text{ а) } \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}; \quad \text{ б) } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6};$$

$$\text{ в) } \arcsin(-1) = -\frac{\pi}{2}; \quad \text{ г) } \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}.$$

$$311. \text{ а) } \arcsin 0 + \arccos 0 = \frac{\pi}{2}; \quad \text{ б) } \arcsin \frac{\sqrt{3}}{2} + \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{2};$$

$$\text{ в) } \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \arccos \frac{1}{2} = \frac{\pi}{12}; \quad \text{ г) } \arcsin(-1) + \arccos \frac{\sqrt{3}}{2} = -\frac{\pi}{3}.$$

$$312. \text{ а) } \arccos\left(-\frac{1}{2}\right) + \arcsin\left(-\frac{1}{2}\right) = \frac{\pi}{2}; \quad \text{ б) } \arccos\left(-\frac{\sqrt{2}}{2}\right) - \arcsin(-1) = \frac{5\pi}{4};$$

$$\text{ в) } \arccos\left(-\frac{\sqrt{3}}{2}\right) + \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2}; \quad \text{ г) } \arccos \frac{\sqrt{2}}{2} - \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \frac{7\pi}{12}.$$

$$313. \text{ а) } \sin t = \frac{\sqrt{3}}{2}, \quad t = (-1)^k \frac{\pi}{3} + \pi k;$$

$$\text{ б) } \sin t = \frac{\sqrt{2}}{2}, \quad t = (-1)^k \frac{\pi}{4} + \pi k;$$

$$\text{ в) } \sin t = 1, \quad t = \frac{\pi}{2} + 2\pi n;$$

$$\text{ г) } \sin t = \frac{1}{2}, \quad t = (-1)^k \frac{\pi}{6} + \pi k.$$

$$314. \text{ а) } \sin t = -1, \quad t = -\frac{\pi}{2} + 2\pi n;$$

$$\text{б) } \sin t = -\frac{\sqrt{2}}{2}, \quad t = (-1)^{k+1} \frac{\pi}{4} + \pi k;$$

$$\text{в) } \sin t = -\frac{1}{2}, \quad t = (-1)^{k+1} \frac{\pi}{6} + \pi k;$$

$$\text{г) } \sin t = -\frac{\sqrt{3}}{2}, \quad t = (-1)^{k+1} \frac{\pi}{3} + \pi k.$$

$$315. \text{ а) } \sin t = \frac{1}{4}, \quad t = (-1)^k \arcsin \frac{1}{4} + \pi k;$$

$$\text{б) } \sin t = 1,02, \text{ решения нет;}$$

$$\text{в) } \sin t = -\frac{1}{7}, \quad t = (-1)^k \arcsin \left(-\frac{1}{7}\right) + \pi k; \quad \text{г) } \sin \frac{\pi}{3}, \text{ решения нет.}$$

$$316. \text{ а) } \sin(\arccos x + \arccos(-x)) = 0, \quad \sin \pi = 0;$$

$$\text{б) } \cos(\arcsin x + \arcsin(-x)) = 1, \quad \cos 0 = 1.$$

$$317. \text{ а) } \sin \left(2 \arcsin \frac{1}{2} - 3 \arccos \left(-\frac{1}{2} \right) \right) = \sin \left(\frac{\pi}{3} - 2\pi \right) = \frac{\sqrt{3}}{2};$$

$$\text{б) } \cos \left(\frac{1}{2} \arcsin 1 + \arcsin \left(-\frac{\sqrt{2}}{2} \right) \right) = \cos \left(\frac{\pi}{4} - \frac{\pi}{4} \right) = 1.$$

$$318. \text{ а) } \operatorname{tg} \left(\arcsin \frac{\sqrt{3}}{2} + 2 \arccos \frac{\sqrt{2}}{2} \right) = \operatorname{tg} \left(\frac{\pi}{3} + \frac{\pi}{2} \right) = -\frac{\sqrt{3}}{3};$$

$$\text{б) } \operatorname{ctg} \left(3 \arccos(-1) - \arcsin \left(-\frac{1}{2} \right) \right) = \operatorname{ctg} \left(3\pi + \frac{\pi}{6} \right) = \sqrt{3}.$$

$$319. \text{ а) } \arcsin x, \quad x \in [-1; 1];$$

$$\text{б) } \arcsin(5 - 2x), \quad x \in [2; 3];$$

$$\text{в) } \arcsin \frac{x}{2}, \quad x \in [-2; 2];$$

$$\text{г) } \arcsin(x^2 - 3), \quad x \in [-2; -\sqrt{2}] \cup [\sqrt{2}; 2].$$

$$320. \text{ а) } -\frac{2}{3} > -1, \text{ да; } \quad \text{б) } 1,5 > 1, \text{ нет; } \quad \text{в) } 3 - \sqrt{20} < -1, \text{ нет; } \quad \text{г) } 4 - \sqrt{20} < 1, \text{ да.}$$

$$321. \text{ а) } (2 \cos x + 1)(2 \sin x - \sqrt{3}) = 0,$$

$$\begin{cases} \cos x = -\frac{1}{2} \\ \sin x = \frac{\sqrt{3}}{2} \end{cases}, \quad x = \pm \frac{2\pi}{3} + 2\pi k, \quad x = (-1)^k \frac{\pi}{3} + 2\pi k;$$

$$\text{б) } 2 \cos x - 3 \sin x \cos x = 0, \quad \cos x(2 - 3 \sin x) = 0,$$

$$\begin{cases} \cos x = 0 \\ \sin x = \frac{2}{3} \end{cases}, \quad x = \frac{\pi}{2} + \pi n, \quad x = (-1)^k \arcsin \frac{2}{3} + \pi k;$$

$$\text{в) } 4 \sin^2 x - 3 \sin x = 0, \quad \sin x(4 \sin x - 3) = 0, \quad \sin x = 0, \quad \sin x = \frac{3}{4},$$

$$x = \pi n, \quad x = (-1)^k \arcsin \frac{3}{4} + \pi k;$$

$$r) 2\sin^2 x - 1 = 0, \quad \sin x = \pm \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{4} + \frac{\pi n}{2}.$$

$$322. a) 6\sin^2 x + \sin x - 2 = 0, \quad \sin x = \frac{-1+7}{12} = \frac{1}{2}, \quad (-1)^n \arcsin \frac{\pi}{6} + \pi n,$$

$$\sin x = -\frac{2}{3}, \quad (-1)^{k+1} \arcsin \frac{2}{3} + \pi k;$$

$$б) 3\cos^2 x = 7(\sin x + 1), \quad 3 - 3\sin^2 x = 7\sin x + 7, \quad 3\sin^2 x + 7\sin x + 4 = 0,$$

$$\sin x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot 4}}{6} = \frac{-7 \pm 1}{6},$$

$$\sin x = \frac{-8}{6} - \text{не подходит}, \quad \sin x = -1, \quad x = -\frac{\pi}{2} + 2\pi n.$$

$$323. a) \sin t > \frac{\sqrt{3}}{2}, \quad t \in \left(\frac{\pi}{3} + 2\pi k; \frac{2\pi}{3} + 2\pi k \right); \quad б) \sin t > -\frac{1}{2}, \quad t \in \left(-\frac{\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k \right);$$

$$в) \sin t < \frac{\sqrt{3}}{2}, \quad t \in \left(-\frac{4\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k \right); \quad r) \sin t \leq -\frac{1}{2}, \quad t \in \left(\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k \right).$$

$$324. a) \sin t < \frac{1}{3}, \quad t \in \left(\pi - \arcsin \frac{1}{3} + 2\pi k; \arcsin \frac{1}{3} + 2\pi k \right);$$

$$б) \sin t \geq -\frac{3}{5}, \quad t \in \left(-\arcsin \frac{3}{5} + 2\pi k; \pi + \arcsin \frac{3}{5} + 2\pi k \right);$$

$$в) \sin t \geq \frac{1}{3}, \quad t \in \left(\arcsin \frac{1}{3} + 2\pi k; \pi - \arcsin \frac{1}{3} + 2\pi k \right);$$

$$r) \sin t < -\frac{3}{5}, \quad t \in \left(\pi + \arcsin \frac{3}{5} + 2\pi k; 2\pi - \arcsin \frac{3}{5} + 2\pi k \right).$$

$$325. a) 5\sin^2 t > 11\sin t + 12, \quad 5\sin^2 t - 11\sin t - 12 = 0,$$

$$\sin t = \frac{11+19}{10}, \quad \text{не подходит}, \quad \sin t = -\frac{8}{10},$$

$$t \in \left(\pi + \arcsin \frac{4}{5} + 2\pi k; 2\pi - \arcsin \frac{4}{5} + 2\pi k \right);$$

$$б) 5\sin^2 t \leq 11t + 12, \quad 5\sin^2 t - 11t - 12 = 0,$$

$$\sin t = -\frac{4}{5}, \quad t \in \left(-\arcsin \frac{4}{5} + 2\pi k; \pi + \arcsin \frac{4}{5} + 2\pi k \right).$$

$$326. a) 6\cos^2 t + \sin t > 4, \quad 6 - 6\sin^2 t + \sin t - 4 > 0, \quad 6\sin^2 t - \sin t - 2 < 0,$$

$$\sin t = \frac{1+7}{12} = \frac{3}{4}, \quad \sin t = -\frac{1}{2},$$

$$t \in \left(-\frac{\pi}{6} + 2\pi k; \arcsin \frac{2}{3} + 2\pi k\right) \cup \left(\pi - \arcsin \frac{2}{3} + 2\pi k; \frac{7\pi}{6} + 2\pi k\right);$$

$$\text{б) } 6 \cos^2 t + \sin t \leq 4, \quad 6 \sin^2 t - \sin t - 2 = 0, \quad \sin t = \frac{3}{4}, \quad \sin t = -\frac{1}{2},$$

$$t \in \left[\arcsin \frac{2}{3} + 2\pi k; \pi - \arcsin \frac{2}{3} + 2\pi k\right], \quad t \in \left[\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k\right].$$

$$327. \text{ а) } \cos\left(\arcsin\left(-\frac{5}{13}\right)\right) = \sqrt{1 - \sin^2\left(\arcsin\left(-\frac{5}{13}\right)\right)} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13};$$

$$\text{б) } \operatorname{tg}\left(\arcsin \frac{3}{5}\right) = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}; \quad \text{в) } \cos\left(\arcsin \frac{8}{17}\right) = \sqrt{1 - \sin^2\left(\arcsin\left(-\frac{8}{17}\right)\right)} = \frac{15}{17}$$

$$\text{г) } \operatorname{ctg}\left(\arcsin\left(-\frac{4}{5}\right)\right) = \frac{\sqrt{1 - \sin^2\left(\arcsin\left(-\frac{4}{5}\right)\right)}}{\sin\left(\arcsin\left(-\frac{4}{5}\right)\right)} = -\frac{3}{5} \cdot \frac{5}{4} = -\frac{3}{4}.$$

§19. Арктангенс и решение уравнения $\operatorname{tg} x = a$. Аркотангенс и решение уравнения $\operatorname{ctg} x = a$

$$328. \text{ а) } \operatorname{arctg} \frac{\sqrt{3}}{3} = \frac{\pi}{6}; \quad \text{б) } \operatorname{arctg} 1 = \frac{\pi}{4}; \quad \text{в) } \operatorname{arctg} \sqrt{3} = \frac{\pi}{3}; \quad \text{г) } \operatorname{arctg} 0 = 0.$$

$$329. \text{ а) } \operatorname{arctg}(-1) = -\frac{\pi}{4}; \quad \text{б) } \operatorname{arctg}(-\sqrt{3}) = -\frac{\pi}{3}; \quad \text{в) } \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}; \quad \text{г) } \operatorname{arctg}\left(-\frac{1}{3}\right) = -\frac{\pi}{6}.$$

$$330. \text{ а) } \operatorname{arctg} 1 - \operatorname{arctg} \sqrt{3} = -\frac{\pi}{12}; \quad \text{б) } \operatorname{arctg}(-\sqrt{3}) + \operatorname{arctg} 0 = -\frac{\pi}{3};$$

$$\text{в) } \operatorname{arctg} 1 - \operatorname{arctg}(-1) = \frac{\pi}{2}; \quad \text{г) } \operatorname{arctg} \frac{\sqrt{3}}{3} + \operatorname{arctg} \sqrt{3} = \frac{\pi}{2}.$$

$$331. \text{ а) } \operatorname{arccotg} \frac{\sqrt{3}}{3} = \frac{\pi}{3}; \quad \text{б) } \operatorname{arccotg} 1 = \frac{\pi}{4}; \quad \text{в) } \operatorname{arccotg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{3}; \quad \text{г) } \operatorname{arccotg} 0 = \frac{\pi}{2}.$$

$$332. \text{ а) } \operatorname{arccotg}(-1) - \operatorname{arctg}(-1) = \frac{\pi}{2}; \quad \text{б) } \arcsin\left(\frac{\sqrt{2}}{2}\right) + \operatorname{arctg}(-\sqrt{3}) = \frac{7\pi}{12};$$

$$\text{в) } \operatorname{arccotg}\left(-\frac{\sqrt{3}}{3}\right) - \operatorname{arctg} \frac{\sqrt{3}}{3} = \frac{\pi}{2}; \quad \text{г) } \arccos\left(-\frac{1}{2}\right) - \operatorname{arccotg}(-\sqrt{3}) = -\frac{\pi}{6}.$$

$$333. \text{ а) } \operatorname{tg} x = 1, \quad x = \frac{\pi}{4} + \pi n; \quad \text{б) } \operatorname{tg} x = -\frac{\sqrt{3}}{3}, \quad x = -\frac{\pi}{6} + \pi n;$$

$$\text{в) } \operatorname{tg} x = -1, \quad x = -\frac{\pi}{4} + \pi n; \quad \text{г) } \operatorname{tg} x = \frac{\sqrt{3}}{3}, \quad x = \frac{\pi}{6} + \pi n.$$

334. а) $\operatorname{tg} x = 0$, $x = \pi n$;

б) $\operatorname{tg} x = -2$, $x = -\operatorname{arctg} 2 + \pi n$;

в) $\operatorname{tg} x = -3$, $x = -\operatorname{arctg} 3 + \pi n$;

г) $\operatorname{tg} x = \frac{1}{2}$, $x = \operatorname{arctg} \frac{1}{2} + \pi n$.

335. а) $\operatorname{ctg} x = 1$, $x = \frac{\pi}{4} + \pi n$;

б) $\operatorname{ctg} x = \sqrt{3}$, $x = \frac{\pi}{6} + \pi n$;

в) $\operatorname{ctg} x = 0$, $x = \frac{\pi}{2} + \pi n$;

г) $\operatorname{ctg} x = \frac{\sqrt{3}}{2}$, $x = \frac{\pi}{3} + \pi n$.

336. а) $\operatorname{ctg} x = -\sqrt{3}$, $x = -\frac{\pi}{6} + \pi n$;

б) $\operatorname{ctg} x = -1$, $x = -\frac{\pi}{4} + \pi n$;

в) $\operatorname{ctg} x = -\frac{\sqrt{3}}{3}$, $x = -\frac{\pi}{3} + \pi n$;

г) $\operatorname{ctg} x = -5$, $x = -\operatorname{arctg} 5 + \pi n$.

337. а) $2 \arcsin\left(-\frac{\sqrt{3}}{2}\right) + \operatorname{arctg}(-1) + \arccos \frac{\sqrt{2}}{2} = -\frac{2\pi}{3} - \frac{\pi}{4} + \frac{\pi}{4} = -\frac{2\pi}{3}$;

б) $3 \arcsin \frac{1}{2} + 4 \arccos\left(-\frac{\sqrt{2}}{2}\right) - \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = \frac{\pi}{2} + 3\pi + \frac{\pi}{3} = \frac{11\pi}{3}$;

в) $\operatorname{arctg}(-\sqrt{3}) + \arccos\left(-\frac{\sqrt{3}}{2}\right) + \arcsin 1 = -\frac{\pi}{3} + \frac{5\pi}{6} + \frac{\pi}{2} = \pi$;

г) $\arcsin(-1) - \frac{3}{2} \arccos \frac{1}{2} + 3 \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{2} + \frac{\pi}{2} + \pi = \pi$.

338. а) $\sin(\operatorname{arctg}(-\sqrt{3})) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$;

б) $\operatorname{tg}\left(\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)\right) = \operatorname{tg}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$;

в) $\cos(\operatorname{arctg} 0) = \cos 0 = 1$;

г) $\operatorname{ctg}(\operatorname{arctg}(-1)) = \operatorname{ctg}\left(-\frac{\pi}{4}\right) = -1$.

339. а) $\operatorname{tg}(\operatorname{arccctg} 1) = \operatorname{tg} \frac{\pi}{4} = 1$;

б) $\sin(\operatorname{arccctg} \sqrt{3}) = \sin \frac{\pi}{6} = \frac{1}{2}$;

в) $\cos(\operatorname{arccctg}(-1)) = \cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$;

г) $\operatorname{ctg}\left(2 \operatorname{arccctg}\left(-\frac{\sqrt{3}}{3}\right)\right) = \operatorname{ctg}\left(\frac{2\pi}{3}\right) = \sqrt{3}$.

340. а) $\operatorname{tg}^2 x - 3 = 0$, $\operatorname{tg} x = \pm\sqrt{3}$, $x = \pm\frac{\pi}{3} + \pi n$;

б) $2 \operatorname{tg}^2 x + 3 \operatorname{tg} x = 0$, $\operatorname{tg} x = 0$, $\operatorname{tg} x = -\frac{3}{2}$, $x = \pi n$, $x = -\operatorname{arccctg} \frac{3}{2} + \pi n$;

в) $4 \operatorname{tg}^2 x - 9 = 0$, $\operatorname{tg} x = \pm\frac{3}{2}$, $\operatorname{arccctg} \frac{3}{2} + \pi n$;

г) $3 \operatorname{tg}^2 x - 2 \operatorname{tg} x = 0$, $\operatorname{tg} x = 0$, $\operatorname{tg} x = \frac{2}{3}$, $x = \pi n$, $x = \operatorname{arccctg} \frac{2}{3} + \pi n$.

341. а) $\operatorname{tg}^2 x - 6 \operatorname{tg} x + 5 = 0$, $\operatorname{tg} x = 5$, $\operatorname{tg} x = 1$, $x = \operatorname{arctg} 5 + \pi n$, $x = \frac{\pi}{4} + \pi n$;

б) $\operatorname{tg}^2 x - 2 \operatorname{tg} x - 3 = 0$, $\operatorname{tg} x = 3$, $\operatorname{tg} x = -1$, $x = \operatorname{arctg} 3 + \pi n$, $x = -\frac{\pi}{4} + \pi n$.

342. а) $\operatorname{tg}(\pi + x) = \sqrt{3}$, $\operatorname{tg} x = \sqrt{3}$, $x = \frac{\pi}{3} + \pi n$;

б) $2 \operatorname{ctg}(2\pi + x) - \operatorname{tg}\left(\frac{\pi}{2} + x\right) = \sqrt{3}$, $2 \operatorname{ctg} x + \operatorname{ctg} x = \sqrt{3}$, $\operatorname{ctg} x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{3} + \pi n$;

в) $-\sqrt{3} \operatorname{tg}(\pi - x) = 1$, $\operatorname{tg} x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{6} + \pi n$;

г) $\operatorname{ctg}(2\pi - x) + \operatorname{ctg}(\pi - x) = 2$, $\operatorname{ctg} x = -1$, $x = \frac{\pi}{4} + \pi n$.

343. а) $\operatorname{tg} x < \sqrt{3}$, $x \in \left(-\frac{\pi}{2} + \pi k; \frac{\pi}{3} + \pi k\right)$; б) $\operatorname{ctg} x > 0$, $x \in \left(\pi k; \frac{\pi}{2} + \pi k\right)$;

в) $\operatorname{tg} x < 0$, $x \in \left(-\frac{\pi}{2} + \pi k; \pi k\right)$; г) $\operatorname{ctg} x > -1$, $x \in \left(\pi k; \frac{3\pi}{4} + \pi k\right)$.

344. а) $\operatorname{tg} x < 3$, $x \in \left(-\frac{\pi}{2} + \pi k; \operatorname{arctg} 3 + \pi k\right)$;

б) $3 \operatorname{ctg} x - 1 > 0$, $\operatorname{ctg} x > \frac{1}{3}$, $x \in \left(\pi k; \operatorname{arctg} \frac{1}{3} + \pi k\right)$;

в) $\operatorname{ctg} x \leq 2$, $x \in (\operatorname{arctg} 2 + \pi k; \pi + \pi k)$;

г) $2 \operatorname{tg} x + 1 \geq 0$, $\operatorname{tg} x \geq -\frac{1}{2}$, $x \in \left(-\operatorname{arctg} \frac{1}{2} + \pi k; \frac{\pi}{2} + \pi k\right)$.

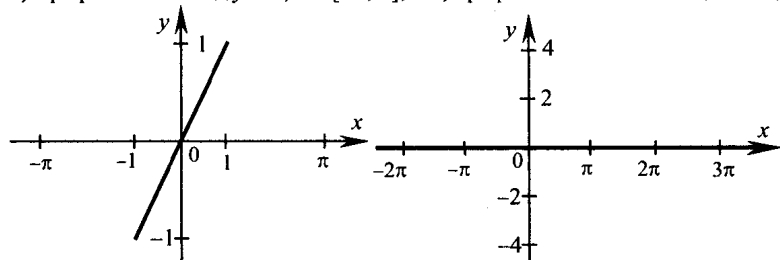
345. а) $\operatorname{tg}^2 x > 9$, $\begin{cases} \operatorname{tg} x < -3 \\ \operatorname{tg} x > 3 \end{cases}$, $x \in \left(-\frac{\pi}{2} + \pi k; \operatorname{arctg} 3 + \pi k\right) \cup \left(\operatorname{arctg} 3 + \pi k; \frac{\pi}{2} + \pi k\right)$;

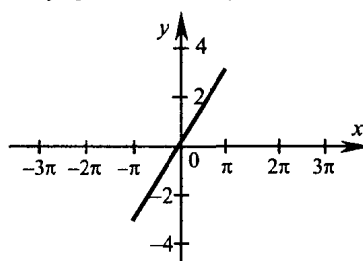
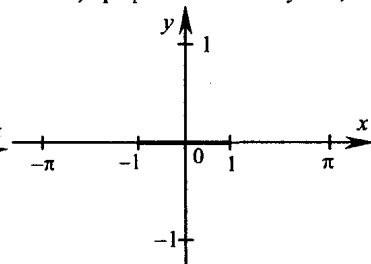
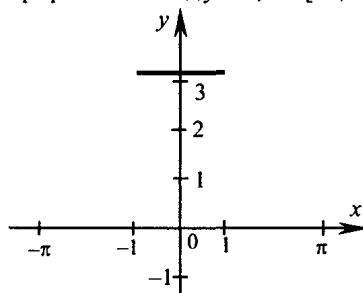
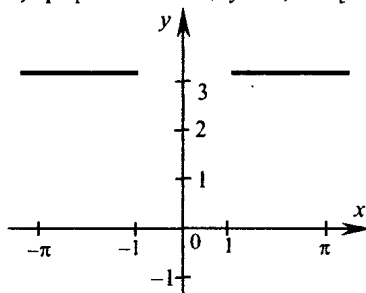
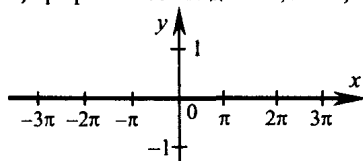
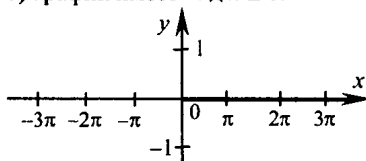
б) $\operatorname{tg}^2 x > \operatorname{tg} x$, $\operatorname{tg} x(\operatorname{tg} x - 1) > 0$, $\begin{cases} \operatorname{tg} x < 0 \\ \operatorname{tg} x > 1 \end{cases}$, $x \in \left(-\frac{\pi}{2} + \pi k; \pi k\right) \cup \left(\frac{\pi}{4} + \pi k; \frac{\pi}{2} + \pi k\right)$

в) $\operatorname{tg}^2 x < 9$, $\operatorname{tg} x \in (-3; 3)$, $x \in (-\operatorname{arctg} 3 + \pi k; \operatorname{arctg} 3 + \pi k)$;

г) $\operatorname{tg}^2 x < 2$, $\operatorname{tg} x(\operatorname{tg} x - 2) < 0$, $\operatorname{tg} x \in (0; 2)$, $x \in (\pi k; \operatorname{arctg} 2 + \pi k)$.

346. а) График имеет вид $y = x$, $x \in [-1; 1]$; б) график имеет вид $x = 0$, $x \in R$;



в) график имеет вид $y = x, x \in R$;г) график имеет вид $y = 0, x \in [-1; 1]$.347. График имеет вид $y = \pi, x \in [-1; 1]$;б) график имеет вид $y = \pi, x \notin [-1; 1]$;в) график имеет вид $x = 0, x \in R$;г) график имеет вид $x \geq 0$.348. а) $\sin\left(\operatorname{arctg} \frac{3}{4}\right), \operatorname{arctg} \frac{3}{4} = x, x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right), \operatorname{tg} x = \frac{3}{4},$

$$\sin x = \frac{3}{4} \sqrt{1 - \sin^2 x}, 16 \sin^2 x = 9 - 9 \sin^2 x, \sin^2 x = \frac{9}{25},$$

$$\sin x = \pm \frac{3}{5} \Rightarrow \sin\left(\operatorname{arctg} \frac{3}{4}\right) = \frac{3}{5};$$

б) $\cos\left(\operatorname{arctg} \frac{12}{5}\right), \operatorname{arctg} \frac{12}{5} = x, x \in (0; \pi), \operatorname{ctg} x = \frac{12}{5},$

$$\frac{12}{5} = \frac{\cos x}{\sqrt{1 - \cos^2 x}}, 144 - 144 \cos^2 x = 25 \cos^2 x, \cos^2 x = \frac{144}{169},$$

$$\cos x = \pm \frac{12}{13} \Rightarrow \left(\operatorname{arctg} \frac{12}{5}\right) = \frac{12}{13};$$

в) $\sin\left(\operatorname{arctg}\left(-\frac{4}{3}\right)\right), \operatorname{arctg}\left(-\frac{4}{3}\right) = x, x \in (0; \pi), -\frac{4}{3} = \operatorname{ctg} x, -\frac{4}{3} = \frac{\cos x}{\sin x},$

$$16 \sin^2 x = 9 - 9 \sin^2 x, \sin^2 x = \frac{9}{25}, \sin x = \pm \frac{3}{5} \Rightarrow \sin\left(\operatorname{arctg}\left(-\frac{4}{3}\right)\right) = \frac{3}{5};$$

$$r) \cos\left(\arctg\left(-\frac{5}{12}\right)\right), \arctg\left(-\frac{5}{12}\right) = x, x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right), -\frac{5}{12} = \operatorname{tg} x,$$

$$25 \cos^2 x = \frac{144}{160}, \cos x = \pm \frac{12}{13} \Rightarrow \cos\left(\arctg\left(-\frac{5}{12}\right)\right) = \frac{12}{13}.$$

§20. Тригонометрические уравнения

$$349. a) 2 \cos x + \sqrt{3} = 0, \quad \cos x = -\frac{\sqrt{3}}{2}, \quad x = \pm \frac{5\pi}{6} + 2\pi n;$$

$$б) 2 \sin x - 1 = 0, \quad \sin x = \frac{1}{2}, \quad x = (-1)^n \frac{\pi}{6} + \pi k;$$

$$в) 2 \cos x - 1 = 0, \quad \cos x = \frac{1}{2}, \quad x = \pm \frac{\pi}{3} + 2\pi n;$$

$$г) 2 \sin x + \sqrt{2} = 0, \quad \sin x = -\frac{\sqrt{2}}{2}, \quad x = (-1)^{k+1} \frac{\pi}{6} + \pi k.$$

$$350. a) \operatorname{tg} x + \sqrt{3} = 0, \quad \operatorname{tg} x = -\sqrt{3}, \quad x = -\frac{\pi}{3} + \pi n;$$

$$б) \sqrt{3} \operatorname{tg} x - 1 = 0, \quad \operatorname{tg} x = \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{6} + \pi n;$$

$$в) \operatorname{ctg} x + 1 = 0, \quad \operatorname{ctg} x = -1, \quad x = -\frac{\pi}{4} + \pi n;$$

$$г) \sqrt{3} \operatorname{ctg} x - 1 = 0, \quad \operatorname{ctg} x = \frac{\sqrt{3}}{3}, \quad x = \frac{\pi}{3} + \pi n.$$

$$351. a) \sin 2x = \frac{\sqrt{2}}{2}, \quad 2x = (-1)^k \frac{\pi}{4} + \pi k, \quad x = (-1)^k \frac{\pi}{8} + \frac{\pi k}{2};$$

$$б) \cos \frac{x}{3} = -\frac{1}{2}, \quad \frac{x}{3} = \pm \frac{2\pi}{3} + 2\pi n, \quad x = \pm 2\pi + 6\pi n;$$

$$в) \sin \frac{x}{4} = \frac{1}{2}, \quad \frac{x}{4} = (-1)^k \frac{\pi}{6} + 2\pi k, \quad x = (-1)^k \frac{2\pi}{3} + 4\pi k;$$

$$г) \cos 4x = 0, \quad 4x = \frac{\pi}{2} + \pi n, \quad x = \frac{\pi}{8} + \frac{\pi n}{4}.$$

$$352. a) \sin\left(-\frac{x}{3}\right) = \frac{\sqrt{2}}{2}, \quad \sin \frac{x}{3} = -\frac{\sqrt{2}}{2}, \quad \frac{x}{3} = (-1)^{k+1} \frac{\pi}{4} + \pi k, \quad x = (-1)^{k+1} \frac{3\pi}{4} + 3\pi k;$$

$$б) \cos(-2x) = -\frac{\sqrt{3}}{2}, \quad 2x = \pm \frac{5\pi}{6} + 2\pi n, \quad x = \pm \frac{5\pi}{12} + \pi n;$$

$$в) \operatorname{tg}(-4x) = \frac{\sqrt{3}}{3}, \quad \operatorname{tg} 4x = -\frac{\sqrt{3}}{3}, \quad 4x = -\frac{\pi}{6} + \pi n, \quad x = -\frac{\pi}{24} + \frac{\pi n}{4};$$

$$г) \operatorname{ctg}\left(-\frac{x}{2}\right) = 1, \quad \operatorname{ctg} \frac{x}{2} = -1, \quad \frac{x}{2} = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{2} + 2\pi n.$$

$$353. \text{ а) } 2 \cos\left(\frac{x}{2} - \frac{\pi}{6}\right) = \sqrt{3}, \quad \frac{x}{2} - \frac{\pi}{6} = \pm \frac{\pi}{6} + 2\pi n, \quad x = \pm \frac{\pi}{3} + \frac{\pi}{3} + 4\pi n;$$

$$\text{ б) } \sqrt{3} \operatorname{tg}\left(\frac{x}{3} + \frac{\pi}{6}\right) = 3, \quad \operatorname{tg}\left(\frac{x}{3} + \frac{\pi}{6}\right) = \sqrt{3}, \quad \frac{x}{3} + \frac{\pi}{6} = \frac{\pi}{3} + \pi n, \quad x = \frac{\pi}{2} + 3\pi n;$$

$$\text{ в) } 2 \sin\left(3x - \frac{\pi}{4}\right) = -\sqrt{2}, \quad 3x - \frac{\pi}{4} = (-1)^{k+1} \frac{\pi}{4} + \pi k, \quad x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi k}{3};$$

$$\text{ г) } \sin\left(\frac{x}{2} - \frac{\pi}{6}\right) + 1 = 0, \quad \frac{x}{2} - \frac{\pi}{6} = -\frac{\pi}{2} + 2\pi n, \quad x = -\frac{2\pi}{3} + 4\pi n.$$

$$354. \text{ а) } \cos\left(\frac{\pi}{6} - 2x\right) = -1, \quad 2x - \frac{\pi}{6} = \pi + 2\pi n, \quad x = \frac{\pi}{12} + \pi n;$$

$$\text{ б) } \operatorname{tg}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -1, \quad \frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{4} + \pi n, \quad x = \pi + 2\pi n;$$

$$\text{ в) } 2 \sin\left(\frac{\pi}{3} - \frac{x}{4}\right) = \sqrt{3}, \quad \frac{x}{4} - \frac{\pi}{3} = (-1)^{k+1} \frac{\pi}{3} + \pi n, \quad x = (-1)^{k+1} \frac{4\pi}{3} + \frac{4\pi}{3} + 4\pi k;$$

$$\text{ г) } 2 \cos\left(\frac{\pi}{4} - 3x\right) = \sqrt{2}, \quad 3x - \frac{\pi}{4} = \pm \frac{\pi}{4} + 2\pi n, \quad x = \pm \frac{\pi}{12} + \frac{\pi}{12} + \frac{2\pi n}{3}.$$

$$355. \text{ а) } 3 \sin^2 x - 5 \sin x - 2 = 0, \quad \sin x = \frac{5+7}{6} - \text{ не подходит,}$$

$$\sin x = -\frac{1}{3}, \quad x = (-1)^{k+1} \arcsin \frac{1}{3} + \pi n;$$

$$\text{ б) } 3 \sin^2 2x + 10 \sin 2x + 3 = 0, \quad \sin 2x = \frac{-10 \pm \sqrt{100 - 4 \cdot 3 \cdot 3}}{6} = \frac{-5 \pm 4}{3},$$

$$\sin 2x = \frac{-5-4}{3} - \text{ не подходит, } \sin 2x = -\frac{1}{3}, \quad x = (-1)^{k+1} \frac{\arcsin \frac{1}{3}}{2} + \frac{\pi k}{2};$$

$$\text{ г) } 2 \sin^2 \frac{x}{2} - 3 \sin \frac{x}{2} + 1 = 0, \quad \sin \frac{x}{2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot 1}}{4} = \frac{3 \pm 1}{4},$$

$$\sin \frac{x}{2} = \frac{3+1}{4} = 1, \quad \frac{x}{2} = \frac{\pi}{2} + 2\pi n, \quad x = \pi + 4\pi n, \quad \sin \frac{x}{2} = \frac{1}{2},$$

$$\frac{x}{2} = (-1)^k \frac{\pi}{6} + \pi k, \quad x = (-1)^k \frac{\pi}{3} + 2\pi k.$$

$$356. \text{ а) } 6 \cos^2 x + \cos x - 1 = 0, \quad \cos x = \frac{-1 \pm \sqrt{1 + 4 \cdot 6 \cdot 1}}{12} = \frac{-1 \pm 5}{12},$$

$$\cos x = -\frac{1}{2}, \quad x = \pm \frac{2\pi}{3} + 2\pi n, \quad \cos x = \frac{1}{3}, \quad x = \pm \arccos \frac{1}{3} + 2\pi n;$$

$$\text{ б) } 2 \cos^2 3x - 5 \cos 3x - 3 = 0, \quad \cos 3x = \frac{5+7}{4} - \text{ не подходит,}$$

$$\cos 3x = -\frac{1}{2}, \quad 3x = \pm \frac{2\pi}{3} + 2\pi n, \quad x = \pm \frac{2\pi}{9} + \frac{2\pi n}{3};$$

в) $2 \cos^2 x - \cos x - 3 = 0$, $\cos x = \frac{1+5}{4}$ – не подходит,

$\cos x = -1$, $x = \pi + 2\pi n$;

г) $2 \cos^2 \frac{x}{3} + 3 \cos \frac{x}{3} - 2 = 0$, $\cos \frac{x}{3} = \frac{-3-5}{4}$ – не подходит,

$\cos \frac{x}{3} = \frac{1}{2}$, $\frac{x}{3} = \pm \frac{\pi}{3} + 2\pi n$, $x = \pm \pi + 6\pi n$.

357. а) $2 \sin^2 x + 3 \cos x = 0$, $2 - 2 \cos^2 x - 3 \cos x - 2 = 0$,

$\cos x = \frac{3+5}{4}$ – не подходит, $\cos x = -\frac{1}{2}$, $x = \pm \frac{2\pi}{3} + 2\pi n$;

б) $8 \sin^2 2x + \cos 2x + 1 = 0$, $8 - 8 \cos^2 x + \cos 2x + 1 = 0$, $8 \cos^2 x - \cos 2x - 9 = 0$,

$\cos 2x = \frac{1+17}{16}$ – не подходит, $\cos 2x = -1$, $2x = \pi + 2\pi n$, $x = \frac{\pi}{2} + \pi n$;

в) $5 \cos^2 x + 6 \sin x - 6 = 0$, $5 - 5 \sin^2 x + 6 \sin x - 6 = 0$, $5 \sin^2 x - 6 \sin x + 1 = 0$,

$\sin x = \frac{6 \pm \sqrt{36 - 4 \cdot 5 \cdot 1}}{10} = \frac{3 \pm 2}{5}$, $\sin x = 1$, $x = \frac{\pi}{2} + 2\pi n$,

$\sin x = \frac{1}{5}$, $x = (-1)^n \arcsin \frac{1}{5} + \pi n$;

г) $4 \sin 3x + \cos^2 3x = 4$, $\sin^2 3x - 4 \sin 3x + 3 = 0$, $\sin 3x = 3$ – не подходит,

$\sin 3x = 1$, $3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$.

358. а) $3 \operatorname{tg}^2 x + 2 \operatorname{tg} x - 1 = 0$, $\operatorname{tg} x = \frac{-1+2}{3} = \frac{1}{3}$, $x = \operatorname{arctg} \frac{1}{3} + \pi n$, $\operatorname{tg} x = -1$, $x = -\frac{\pi}{4} + \pi n$;

б) $\operatorname{ctg}^2 2x - 6 \operatorname{ctg} 2x + 5 = 0$, $\operatorname{ctg} 2x = 5$, $2x = \operatorname{arctg} 5 + \pi n$,

$x = \frac{\operatorname{arctg} 5}{2} + \frac{\pi n}{2}$, $\operatorname{ctg} 2x = 1$, $2x = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{2}$;

в) $2 \operatorname{tg}^2 x + 3 \operatorname{tg} x - 2 = 0$, $\operatorname{tg} x = \frac{-3+5}{4} = \frac{1}{2}$, $x = -\operatorname{arctg} \frac{1}{2} + \pi n$,

$\operatorname{tg} x = -2$, $x = -\operatorname{arctg} 2 + \pi n$;

г) $7 \operatorname{ctg}^2 \frac{x}{2} + 2 \operatorname{ctg} \frac{x}{2} = 5$, $7 \operatorname{ctg}^2 \frac{x}{2} + 2 \operatorname{ctg} \frac{x}{2} - 5 = 0$,

$\operatorname{ctg} \frac{x}{2} = \frac{-1-6}{7} = -1$, $\frac{x}{2} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + 2\pi n$,

$\operatorname{ctg} \frac{x}{2} = \frac{-1+6}{7} = \frac{5}{7}$, $\frac{x}{2} = \operatorname{arctg} \frac{5}{7} + \pi n$, $x = 2 \operatorname{arctg} \frac{5}{7} + 2\pi n$.

359. а) $\left(\sin x - \frac{1}{2}\right)(\sin x + 1) = 0$, $\sin x = \frac{1}{2}$, $x = (-1)^k \frac{\pi}{6} + \pi k$, $\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$;

б) $\left(\cos x + \frac{1}{2}\right)(\cos x - 1) = 0$, $\cos x = -\frac{1}{2}$, $x = \pm \frac{2\pi}{3} + 2\pi n$, $\cos x = 1$, $x = 2\pi n$;

$$\text{в)} \left(\cos x - \frac{\sqrt{2}}{2} \right) \left(\sin x + \frac{\sqrt{2}}{2} \right) = 0, \cos x = \frac{\sqrt{2}}{2}, x = \pm \frac{\pi}{4} + 2\pi n,$$

$$\sin x = -\frac{\sqrt{2}}{2}, x = (-1)^{n+1} \frac{\pi}{4} + \pi n;$$

$$\text{г)} (1 + \cos x)(\sqrt{2} \sin x - 1) = 0, \cos x = -1, x = \pi + 2\pi n,$$

$$\sin x = \frac{\sqrt{2}}{2}, x = (-1)^n \frac{\pi}{4} + \pi n.$$

$$360. \text{ а)} \sin x + \sqrt{3} \cos x = 0, \operatorname{tg} x = -\sqrt{3}, \cos x \neq 0, x = -\frac{\pi}{3} + \pi n;$$

$$\text{б)} \sin x + \cos x = 0, \operatorname{tg} x = -1, \cos x \neq 0, x = -\frac{\pi}{4} + \pi n;$$

$$\text{в)} \sin x - 3 \cos x = 0, \operatorname{tg} x = 3, \cos x \neq 0, x = \operatorname{arctg} 3 + \pi n;$$

$$\text{г)} \sqrt{3} \sin x + \cos x = 0, \operatorname{tg} x = -\frac{\sqrt{3}}{3}, \cos x \neq 0, x = -\frac{\pi}{6} + \pi n.$$

$$361. \text{ а)} \sin^2 x + \sin x \cos x = 0, \sin x(\sin x + \cos x) = 0, \sin x = 0, x = \pi n,$$

$$\sin x + \cos x = 0, x = -\frac{\pi}{4} + \pi n;$$

$$\text{б)} \sqrt{3} \sin x \cos x + \cos^2 x = 0, \cos x(\sqrt{3} \sin x + \cos x) = 0, \cos x = 0,$$

$$x = \frac{\pi}{2} + \pi n, \sqrt{3} \sin x + \cos x = 0, x = -\frac{\pi}{6} + \pi n;$$

$$\text{в)} \sin^2 x = 3 \sin x \cos x, \sin x(\sin x - 3 \cos x) = 0, \sin x = 0, x = \pi n,$$

$$\sin x - 3 \cos x = 0, x = \operatorname{arctg} 3 + \pi n;$$

$$\text{г)} \sqrt{3} \cos^2 x = \sin x \cos x, \cos x(\sqrt{3} \cos x - \sin x) = 0, \cos x = 0,$$

$$x = \frac{\pi}{2} + \pi n, \sqrt{3} \cos x - \sin x = 0, \operatorname{tg} x = \sqrt{3}, x = \frac{\pi}{3} + \pi n.$$

$$362. \text{ а)} \sin^2 x + \sin x \cos x - 3 \cos^2 x = 0, \operatorname{tg}^2 x + 2 \operatorname{tg} x - 3 = 0, \operatorname{tg} x = -3,$$

$$x = -\operatorname{arctg} 3 + \pi n, \operatorname{tg} x = 1, x = \frac{\pi}{4} + \pi n;$$

$$\text{б)} \sin^2 x - 4 \sin x \cos x + 3 \cos^2 x = 0, \operatorname{tg}^2 x - 4 \operatorname{tg} x + 3 = 0, \operatorname{tg} x = 3,$$

$$x = \operatorname{arctg} 3 + \pi n, \operatorname{tg} x = 1, x = \frac{\pi}{4} + \pi n;$$

$$\text{в)} \sin^2 x + \sin x \cos x - 2 \cos^2 x = 0, \operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0, \operatorname{tg} x = \frac{-1 \pm 3}{2},$$

$$\operatorname{tg} x = 1, x = \frac{\pi}{4} + \pi n, \operatorname{tg} x = -2, x = \operatorname{arctg} 2 + \pi n;$$

$$\text{г)} 3 \sin^2 x + \sin x \cos x - 2 \cos^2 x = 0, 3 \operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0, \cos x \neq 0,$$

$$\operatorname{tg} x = \frac{-1 \pm \sqrt{1 + 4 \cdot 3 \cdot 2}}{6} = \frac{-1 \pm 5}{6}, \operatorname{tg} x = -1, x = -\frac{\pi}{4} + \pi n, \operatorname{tg} x = \frac{2}{3}, x = \operatorname{arctg} \frac{2}{3} + \pi n.$$

$$363. \text{ а) } \sin^2 \frac{3x}{4} - \frac{\sqrt{2}}{2} = \sin x - \cos^2 \frac{3x}{4} + 1, \quad 1 - \frac{\sqrt{2}}{2} - 1 = \sin x,$$

$$\sin x = -\frac{\sqrt{2}}{2}, \quad x = (-1)^{k+1} \frac{\pi}{4} + \pi k;$$

$$\text{б) } \cos^2 2x - 1 - \cos x = \frac{\sqrt{3}}{2} - \sin^2 2x, \quad \cos x = -\frac{\sqrt{3}}{2}, \quad x = \pm \frac{5\pi}{6} + 2\pi n.$$

$$364. \text{ а) } \sin x = \frac{1}{2}, \quad x \in [0; \pi]; \quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6};$$

$$\text{б) } \cos x = -\frac{1}{2}, \quad x \in [-2\pi; 3\pi]; \quad x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}.$$

$$365. \text{ а) } \sin 3x = \frac{\sqrt{2}}{2}, \quad x \in [0; 2\pi], \quad 3x = (-1)^k \frac{\pi}{4} + \pi k,$$

$$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}; \quad x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12};$$

$$\text{б) } \cos 3x = \frac{\sqrt{3}}{2}, \quad x \in [-\pi; \pi], \quad 3x = \pm \frac{\pi}{6} + 2\pi n,$$

$$3x = -\frac{13\pi}{6}, -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}; \quad x = -\frac{13\pi}{18}, -\frac{11\pi}{18}, -\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18};$$

$$\text{в) } \operatorname{tg} \frac{x}{2} = \frac{\sqrt{3}}{3}, \quad x \in [-3\pi; 3\pi], \quad \frac{x}{2} = \frac{\pi}{6} + \pi n, \quad x = \frac{\pi}{3} + 2\pi n, \quad x = -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3};$$

$$\text{г) } \operatorname{ctg} 4x = -1, \quad x \in [0; \pi], \quad 4x = -\frac{\pi}{4} + \pi n,$$

$$4x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}; \quad x = \frac{3\pi}{16}, \frac{7\pi}{16}, \frac{11\pi}{16}, \frac{15\pi}{16}.$$

$$366. \text{ а) } \sin 3x = -\frac{1}{2}, \quad x \in [-4; 4], \quad 3x = (-1)^{k+1} \frac{\pi}{6} + \pi k,$$

$$x = (-1)^{k+1} \frac{\pi}{18} + \frac{\pi k}{3}, \quad x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{7\pi}{6};$$

$$\text{б) } \cos x = 1, \quad x \in [-6; 16], \quad x = 2\pi n, \quad x = 0; 2\pi; 4\pi.$$

$$367. \text{ а) } \sin x = \frac{x}{2}, \quad x \in [-12; 18], \quad \frac{x}{2} = \pi n, \quad x = -2\pi; 0; 2\pi; 4\pi;$$

$$\text{б) } \cos x = -\frac{\sqrt{2}}{2}, \quad x \in [1; 7], \quad 3x = \pm \frac{3\pi}{4} + 2\pi n, \quad x = \pm \frac{\pi}{4} + \frac{2\pi n}{3}, \quad x = \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}$$

$$368. \sin\left(2x - \frac{\pi}{4}\right) = -1, \quad 2x - \frac{\pi}{4} = -\frac{\pi}{2} + 2\pi n, \quad x = -\frac{\pi}{8} + \pi n,$$

$$\text{а) } x = \frac{7\pi}{8}; \quad \text{б) } -\frac{\pi}{8}; \frac{7\pi}{8}; \quad \text{в) } -\frac{\pi}{8}; \quad \text{г) } -\frac{\pi}{8}.$$

$$369. \cos\left(\frac{\pi}{3} - 2x\right) = \frac{1}{2}, \quad 2x - \frac{\pi}{3} = \pm \frac{\pi}{3} + 2\pi n, \quad x = \frac{\pi}{3} + \pi n, \quad x = \pi n,$$

$$a) \frac{\pi}{3}; \quad б) 0; \frac{\pi}{3}; \pi; \frac{4\pi}{3}; \quad в) -\frac{2\pi}{3}; \quad г) -\frac{2\pi}{3}; 0; \frac{\pi}{3}.$$

$$370. a) \sqrt{16 - x^2} \sin x = 0, \quad |x| \leq 4, \quad x = 4, \quad x = -4, \quad \sin x = 0, \quad x = \pi n, \quad n = 0, \pm 1.$$

$$\text{Ответ: } x = \pm 4; x = \pi n, n = 0, \pm 1 \dots$$

$$б) \sqrt{7x - x^2} (2 \cos x - 1) = 0, \quad 0 \leq x \leq 7, \quad 7x - x^2 = 0, \quad x = 0, \quad x = 7,$$

$$2 \cos x - 1 = 0, \quad \cos x = \frac{1}{2}, \quad x = \frac{\pi}{3}, \quad x = \frac{5\pi}{3}.$$

$$\text{Ответ: } x = 0; \frac{\pi}{3}; \frac{5\pi}{3}; 7.$$

$$371. a) (\sqrt{2} \cos x - 1) \sqrt{4x^2 - 7x + 3} = 0, \quad 4x^2 - 7x + 3 \geq 0,$$

$$x \geq \frac{7+1}{8} = 1, \quad x \leq \frac{3}{4}, \quad \sqrt{2} \cos x - 1 = 0, \quad x = \pm \frac{\pi}{4} + 2\pi n.$$

$$\text{Ответ: } x = 1, x = \frac{3}{4}, x = -\frac{\pi}{4}, x = \pm \frac{\pi}{4} + 2\pi n; n = \pm 1; \pm 2; \pm 3 \dots$$

$$б) (2 \sin x - \sqrt{3}) \sqrt{3x^2 - 7x + 4} = 0, \quad 3x^2 - 7x + 4 \geq 0; \quad x \leq 1; \quad x \geq \frac{4}{3},$$

$$2 \sin x - \sqrt{3} = 0, \quad \sin x = \frac{\sqrt{3}}{2}, \quad x = \frac{2\pi}{3} + 2\pi k; \quad k = \pm 1; \pm 2 \dots$$

$$3x^2 - 7x + 4 = 0, \quad x = 1; \quad x = \frac{4}{3}.$$

$$\text{Ответ: } 1; \frac{4}{3}; \frac{2\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k; \quad k = \pm 1; \pm 2 \dots$$

$$372. a) \operatorname{tg} x - 2 \operatorname{ctg} x + 1 = 0, \quad \operatorname{tg}^2 x - \operatorname{tg} x - 2 = 0,$$

$$\operatorname{tg} x = -2, \quad x = -\operatorname{arctg} 2 + \pi n, \quad \operatorname{tg} x = 1, \quad x = \frac{\pi}{4} + \pi n;$$

$$б) \frac{\operatorname{tg} x + 5}{2} = \frac{1}{\cos^2 x}, \quad 2 \operatorname{tg}^2 x - \operatorname{tg} x - 3 = 0,$$

$$\operatorname{tg} x = \frac{1+5}{4} = \frac{3}{2}, \quad x = \operatorname{arctg} \frac{3}{2} + \pi k, \quad \operatorname{tg} x = -1, \quad x = -\frac{\pi}{4} + \pi k;$$

$$в) 2 \operatorname{ctg} x - 3 \operatorname{tg} x + 5 = 0, \quad 2 \operatorname{ctg}^2 x + 5 \operatorname{ctg} x - 3 = 0, \quad \operatorname{ctg} x = \frac{-5+7}{4} = \frac{1}{2},$$

$$x = \operatorname{arccotg} \frac{1}{2} + \pi n, \quad \operatorname{ctg} x = -3, \quad x = -\operatorname{arccotg} 3 + \pi n;$$

$$г) \frac{7 - \operatorname{ctg} x}{4} = \frac{1}{\sin^2 x}, \quad 7 - \operatorname{ctg} x = 4 \operatorname{ctg}^2 x + 4, \quad 4 \operatorname{ctg}^2 x + \operatorname{ctg} x - 3 = 0,$$

$$\operatorname{ctg} x = \frac{-1+7}{8} = \frac{3}{4}, \quad x = \operatorname{arctg} \frac{3}{4} + \pi n, \quad \operatorname{ctg} x = -1, \quad x = -\frac{\pi}{4} + \pi n.$$

$$373. \text{ а) } 2 \cos^2 \frac{x}{2} + \sqrt{3} \cos \frac{x}{2} = 0, \cos \frac{x}{2} \left(2 \cos \frac{x}{2} + \sqrt{3} \right) = 0,$$

$$\cos \frac{x}{2} = 0, \frac{x}{2} = \frac{\pi}{2} + \pi k, x = \pi + 2\pi k,$$

$$\cos \frac{\sqrt{3}}{2}, \frac{x}{2} = \pm \frac{5\pi}{6} + 2\pi n, x = \pm \frac{5\pi}{3} + 4\pi n;$$

$$\text{ б) } 4 \cos^2 \left(x - \frac{\pi}{6} \right) - 3 = 0, \cos \left(x - \frac{\pi}{6} \right) = \pm \frac{\sqrt{3}}{2},$$

$$x - \frac{\pi}{6} = \pm \frac{\pi}{6} + 2\pi n, x = \frac{\pi}{3} + 2\pi n, x = 2\pi n,$$

$$x - \frac{\pi}{6} = \pm \frac{5\pi}{6} + 2\pi n, x = \pi + 2\pi n, x = -\frac{2\pi}{3} + 2\pi n, x = \frac{\pi}{3} + \pi n, x = \pi n;$$

$$\text{ в) } \sqrt{3} \operatorname{tg}^2 3x - 3 \operatorname{tg} 3x = 0, \operatorname{tg} 3x (\sqrt{3} \operatorname{tg} 3x - 3) = 0, \operatorname{tg} 3x = 0,$$

$$3x = \pi n, x = \frac{\pi n}{3}, \operatorname{tg} 3x = \sqrt{3}, 3x = \frac{\pi}{3} + \pi n, x = \frac{\pi}{9} + \frac{\pi n}{3};$$

$$\text{ г) } 4 \sin^2 \left(2x + \frac{\pi}{3} \right) - 1 = 0, \sin \left(2x + \frac{\pi}{3} \right) = \pm \frac{1}{2},$$

$$\left(2x + \frac{\pi}{3} \right) = (-1)^n \frac{\pi}{6} + \pi n, x = (-1)^n \frac{\pi}{12} - \frac{\pi}{6} + \frac{\pi n}{2},$$

$$2x + \frac{\pi}{3} = (-1)^{n+1} \frac{\pi}{6} + \pi n, x = (-1)^n \frac{\pi}{12} - \frac{\pi}{6} + \frac{\pi n}{2}.$$

$$374. \text{ а) } \sin^2 x - \frac{12 - \sqrt{2}}{2} \cdot \sin x - 3\sqrt{2} = 0, \sin x = 6, \text{ решений нет,}$$

$$\sin x = -\frac{\sqrt{2}}{2}, x = (-1)^{k+1} \frac{\pi}{4} + \pi k;$$

$$\text{ б) } \cos^2 x - \frac{8 - \sqrt{3}}{2} \cos x - 2\sqrt{3} = 0, \cos x = 4, \text{ не подходит,}$$

$$\cos x = -\frac{\sqrt{2}}{2}, x = \pm \frac{5\pi}{6} + 2\pi n.$$

$$375. \text{ а) } \sin 2x = \cos 2x, \operatorname{tg} 2x = 1, \cos 2x \neq 0, 2x = \frac{\pi}{4} + \pi n, x = \frac{\pi}{8} + \frac{\pi n}{2};$$

$$\text{ б) } \sqrt{3} \sin 3x = \cos 3x, \operatorname{ctg} 3x = \sqrt{3}, \sin 3x \neq 0, 3x = \frac{\pi}{6} + \pi n, x = \frac{\pi}{18} + \frac{\pi n}{3};$$

$$\text{ в) } \sin \frac{x}{2} = \sqrt{3} \cos \frac{x}{2}, \operatorname{tg} \frac{x}{2} = \sqrt{3}, \cos \frac{x}{2} \neq 0, \frac{x}{2} = \frac{\pi}{3} + \pi n, x = \frac{2\pi}{3} + 2\pi n;$$

$$\text{ г) } \sqrt{3} \sin 17x = \sqrt{6} \cos 17x, \operatorname{tg} 17x = \sqrt{3}, \cos 17x \neq 0, 17x = \frac{\pi}{3} + \pi n, x = \frac{\pi}{51} + \frac{\pi n}{17}.$$

$$376. \text{ а) } 2 \sin^2 2x - 5 \sin 2x \cos 2x + \cos^2 2x = 0, 2 \operatorname{tg}^2 2x - 5 \operatorname{tg} 2x + 1 = 0, \cos 2x \neq 0,$$

$$\operatorname{tg} 2x = \frac{5 - \sqrt{17}}{4}, \quad 2x = \operatorname{arctg} \frac{5 - \sqrt{17}}{4} + \pi n, \quad x = \frac{1}{2} \operatorname{arctg} \frac{5 - \sqrt{17}}{4} + \frac{\pi n}{2};$$

$$6) 3 \sin^2 3x + 10 \sin 3x \cos 3x + 3 \cos^2 3x = 0, \quad 2 \sin 3x \cos 3x = -\frac{3}{5},$$

$$3 \operatorname{tg}^2 3x + 10 \operatorname{tg} 3x + 3 = 0, \quad \operatorname{tg} 3x = \frac{-10 \pm \sqrt{100 - 4 \cdot 3 \cdot 3}}{6} = \frac{-10 \pm 8}{6}, \quad \operatorname{tg} 3x = -3,$$

$$x = \frac{1}{3} \operatorname{arctg}(-3) + \frac{\pi n}{3}, \quad \operatorname{tg} 3x = -\frac{1}{3}, \quad x = \frac{1}{3} \operatorname{arctg}\left(-\frac{1}{3}\right) + \frac{\pi n}{3}, \quad 6x = (-1)^{k+1} \arcsin \frac{3}{5} + \frac{\pi k}{6}.$$

$$377. \text{ а) } \sin^2 \frac{x}{2} = 3 \cos^2 \frac{x}{2}, \quad \cos^2 \frac{x}{2} = \frac{1}{4}, \quad \cos \frac{x}{2} = \pm \frac{1}{2}, \quad \frac{x}{2} = \pm \frac{2\pi}{3} + 2\pi n, \quad x = \pm \frac{4\pi}{3} + 4\pi n;$$

$$6) \sin^2 4x = \cos^2 4x, \quad \operatorname{tg}^2 4x = 1, \quad \cos 4x \neq 0, \quad \operatorname{tg} 4x = \pm 1,$$

$$4x = \frac{\pi}{4} + \pi n, \quad x = \frac{\pi}{16} + \frac{\pi n}{4}, \quad 4x = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{16} + \frac{\pi n}{4}.$$

$$378. \text{ а) } 5 \sin^2 x - 14 \sin x \cos x - 3 \cos^2 x = 2,$$

$$3 \sin^2 x - 14 \sin x \cos x - 5 \cos^2 x = 0, \quad 3 \operatorname{tg}^2 x - 14 \operatorname{tg} x - 5 = 0, \quad \cos x \neq 0,$$

$$\operatorname{tg}^2 x = \frac{7+8}{3} = 5, \quad x = \operatorname{arctg} 5 + \pi k, \quad \operatorname{tg} x = -\frac{1}{3}, \quad x = -\operatorname{arctg} \frac{1}{3} + \pi k;$$

$$6) 3 \sin^2 x - \sin x \cos x = 2, \quad \sin^2 x - \sin x \cos x - 2 \cos^2 x = 0, \quad \operatorname{tg}^2 x - \operatorname{tg} x - 2 = 0,$$

$$\cos x \neq 0, \quad \operatorname{tg} x = 2, \quad x = \operatorname{arctg} 2 + \pi n, \quad \operatorname{tg} x = -1, \quad x = -\frac{\pi}{4} + \pi n;$$

$$\text{в) } 2 \cos^2 x - \sin x \cos x + 5 \sin^2 x = 3, \quad 2 \sin^2 x - \sin x \cos x - \cos^2 x = 0,$$

$$2 \operatorname{tg}^2 x - \operatorname{tg} x - 1 = 0, \quad \cos x \neq 0, \quad \operatorname{tg} x = 1, \quad x = \frac{\pi}{4} + \pi k, \quad \operatorname{tg} x = -\frac{1}{2}, \quad x = -\operatorname{arctg} \frac{1}{2} + \pi k;$$

$$\text{р) } 4 \sin^2 x - 2 \sin x \cos x = 3, \quad \sin^2 x - 2 \sin x \cos x - 3 \cos^2 x = 0,$$

$$\operatorname{tg}^2 x - 2 \operatorname{tg} x - 3 = 0, \quad \cos x \neq 0, \quad \operatorname{tg} x = -1, \quad x = -\frac{\pi}{4} + \pi k, \quad \operatorname{tg} x = 3, \quad x = \operatorname{arctg} 3 + \pi k.$$

$$379. \text{ а) } \sqrt{3} \sin x \cos x + \cos^2 x = 0, \quad \cos x (\sqrt{3} \sin x + \cos x) = 0, \quad \cos x = 0,$$

$$x = \frac{\pi}{2} + \pi n, \quad \sqrt{3} \sin x + \cos x = 0, \quad \sqrt{3} \operatorname{tg} x = -1, \quad \operatorname{tg} x = \frac{\sqrt{3}}{3}, \quad x = -\frac{\pi}{6} + \pi n;$$

$$6) 2 \sin^2 x - 3 \sin x \cos x + 4 \cos^2 x = 4, \quad 3 \sin x \cos x = 2 - 2 \cos^2 x,$$

$$3 \sin x \cos x = 2 \sin^2 x, \quad \sin x (3 \cos x - 2 \sin x) = 0, \quad \sin x = 0, \quad x = \pi n,$$

$$3 \cos x - 2 \sin x = 0, \quad \operatorname{tg} x = \frac{3}{2}, \quad \cos x \neq 0, \quad x = \operatorname{arctg} \frac{3}{2} + \pi n.$$

$$380. \text{ а) } 3 \sin^2 2x - 2 = \sin 2x \cos 2x, \quad \sin^2 2x - \sin 2x \cos 2x - 2 \cos^2 2x = 0,$$

$$\operatorname{tg}^2 2x - \operatorname{tg} 2x - 2 = 0, \quad \cos^2 2x \neq 0, \quad \operatorname{tg} 2x = 2, \quad 2x = \operatorname{arctg} 2 + \pi n,$$

$$x = \frac{1}{2} \operatorname{arctg} 2 + \frac{\pi n}{2}, \quad \operatorname{tg} 2x = -1, \quad 2x = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{8} + \frac{\pi n}{2};$$

$$\begin{aligned} \text{б)} \quad & 2 \sin^2 4x - 4 - 3 \sin 4x \cos 4x - 4 \cos^2 4x = 0, \quad 2 + 2 \cos^2 4x - 4 = 3 \sin 4x \cos 4x, \\ & 2 \sin^2 4x + 3 \sin 4x \cos 4x = 0, \quad \sin 4x(2 \sin 4x + 3 \cos 4x) = 0, \quad \sin 4x = 0, \\ & 4x = \pi n, \quad x = \frac{\pi n}{4}, \quad 2 \sin 4x + 3 \cos 4x = 0, \quad 2 \operatorname{tg} 4x = -3, \quad \cos 4x \neq 0, \\ & x = -\frac{1}{4} \operatorname{arctg} \frac{3}{2} + \frac{\pi n}{4}. \end{aligned}$$

$$381. \text{ а)} \quad \sin^2 \frac{x}{2} - 3 = 2 \sin \frac{x}{2} \cos \frac{x}{2}, \quad 2 \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2} + 3 \cos^2 \frac{x}{2} = 0,$$

$$2 \operatorname{tg}^2 \frac{x}{2} + 2 \operatorname{tg} \frac{x}{2} + 3 = 0, \quad \cos \frac{x}{2} \neq 0, \text{ т.е. решений нет;}$$

$$\text{б)} \quad 3 \sin^2 \frac{x}{3} + 4 \cos^2 \frac{x}{3} = 3 + \sqrt{3} \sin \frac{x}{3} \cos \frac{x}{3}, \quad \cos^2 \frac{x}{3} - 3\sqrt{3} \sin \frac{x}{3} \cos \frac{x}{3} = 0,$$

$$\cos \frac{x}{3} \left(\cos \frac{x}{3} - \sqrt{3} \sin \frac{x}{3} \right) = 0, \quad \cos \frac{x}{3} = 0, \quad \frac{x}{3} = \frac{\pi}{2} + \pi n, \quad x = \frac{3\pi}{2} + 3\pi n,$$

$$\cos \frac{x}{3} - \sqrt{3} \sin \frac{x}{3} = 0, \quad \operatorname{ctg} \frac{x}{3} = \sqrt{3}, \quad \sin \frac{x}{3} \neq 0, \quad \frac{x}{3} = \frac{\pi}{6} + \pi n, \quad x = \frac{\pi}{2} + 3\pi n.$$

$$382. \text{ а)} \quad \sin \left(\frac{\pi}{2} + 2x \right) + \cos \left(\frac{\pi}{2} - 2x \right) = 0, \quad \cos 2x + \sin 2x = 0,$$

$$\operatorname{tg} 2x = -1, \quad \cos 2x \neq 0, \quad 2x = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{8} + \frac{\pi n}{2};$$

$$\text{б)} \quad 2 \sin(\pi - 3x) + \cos(2\pi - 3x) = 0, \quad 2 \sin 3x + \cos 3x = 0,$$

$$\operatorname{tg} 3x = -\frac{1}{2}, \quad \cos 3x \neq 0, \quad 3x = -\operatorname{arctg} \frac{1}{2} + \pi n, \quad x = -\frac{1}{3} \operatorname{arctg} \frac{1}{2} + \pi n.$$

$$383. \text{ а)} \quad \cos \left(\frac{\pi}{2} - \frac{x}{2} \right) - 3 \cos \left(\pi - \frac{x}{2} \right) = 0, \quad \sin \frac{x}{2} + 3 \cos \frac{x}{2} = 0,$$

$$\operatorname{tg} \frac{x}{2} = -3, \quad \cos \frac{x}{2} \neq 0, \quad \frac{x}{2} = -\operatorname{arctg} 3 + \pi n, \quad x = -2 \operatorname{arctg} 3 + 2\pi n;$$

$$\text{б)} \quad \sqrt{3} \sin \left(\pi - \frac{x}{3} \right) + \sin \left(\frac{\pi}{2} - \frac{x}{3} \right) = 0, \quad \sqrt{3} \sin \frac{x}{3} + 3 \cos \frac{x}{3} = 0,$$

$$\operatorname{tg} \frac{x}{3} = -\sqrt{3}, \quad \cos \frac{x}{3} \neq 0, \quad x = -\pi + 3\pi n.$$

$$384. \text{ а)} \quad |\sin x| = |\cos x|, \quad \sin x = \pm \cos x, \quad \operatorname{tg} x = \pm 1, \quad \cos x \neq 0, \quad x = \pm \frac{\pi}{4} + \pi n;$$

$$\text{б)} \quad |\sin 2x| = |\sqrt{3} \cos 2x|, \quad \sin 2x = \pm \sqrt{3} \cos 2x, \quad \operatorname{tg} 2x = \pm \sqrt{3}, \quad \cos 2x \neq 0,$$

$$2x = \pm \frac{\pi}{3} + \pi n, \quad x = \pm \frac{\pi}{6} + \frac{\pi n}{2}.$$

$$385. \text{ а)} \quad \sin \left(2x - \frac{\pi}{6} \right) + \cos \left(\frac{13\pi}{6} - 2x \right) = 0, \quad \sin \left(2x - \frac{\pi}{6} \right) + \cos \left(2x - \frac{\pi}{6} \right) = 0,$$

$$\operatorname{tg}\left(2x - \frac{\pi}{6}\right) = -1, \cos\left(2x - \frac{\pi}{6}\right) \neq 0, 2x - \frac{\pi}{6} = -\frac{\pi}{4} + \pi n, x = -\frac{\pi}{24} + \frac{\pi n}{2};$$

$$\text{б) } \sin\left(\frac{x}{2} + \frac{\pi}{3}\right) = \sqrt{3} \cos\left(\frac{47\pi}{3} - \frac{x}{2}\right), \sin\left(\frac{x}{2} + \frac{\pi}{3}\right) = \sqrt{3} \cos\left(\frac{x}{2} + \frac{\pi}{3}\right),$$

$$\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{3}\right) = \sqrt{3}, \cos\left(\frac{x}{2} + \frac{\pi}{3}\right) \neq 0, \frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{3} + \pi n, x = 2\pi n.$$

$$386. \text{ а) } \sin^2 x - 5 \cos x = \sin x \cos x - 5 \sin x, \sin x(\sin x + 5) - \cos x(\sin x + 5) = 0,$$

$$(\sin x + 5)(\sin x - \cos x) = 0, \sin x - \cos x = 0, x = \frac{\pi}{4} + \pi n;$$

$$\text{б) } \cos^2 x - 7 \sin x + \sin x \cos x = 7 \cos x, \cos x(\cos x - 7) + \sin x(\cos x - 7) = 0,$$

$$(\cos x - 7)(\cos x + \sin x) = 0, \cos x + \sin x = 0, x = -\frac{\pi}{4} + \pi n.$$

$$387. \text{ а) } \sin^2 x + \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right) - 2 \cos^2 x, \operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0, \cos x \neq 0,$$

$$\operatorname{tg} x = -2, x = -\operatorname{arctg} 2 + \pi n, \operatorname{tg} x = 1, x = \frac{\pi}{4} + \pi n;$$

$$\text{б) } \sin^2 3x + 3 \cos^2 3x - 4 \sin\left(\frac{\pi}{2} + 3x\right)\cos\left(\frac{\pi}{2} + 3x\right) = 0,$$

$$\sin^2 3x + 3 \cos^2 3x + 4 \sin 3x \cos 3x = 0, \operatorname{tg}^2 3x + 4 \operatorname{tg} 3x + 3 = 0, \cos 3x \neq 0,$$

$$\operatorname{tg} 3x = -1, x = -\frac{\pi}{12} + \frac{\pi k}{3}, \operatorname{tg} 3x = -3, x = -\frac{1}{3} \operatorname{arctg} 3 + \frac{\pi k}{3};$$

$$\text{в) } \sin^2 x + 2 \sin(\pi - x) \cos x - 3 \cos^2(2\pi - x) = 0, \operatorname{tg}^2 x + 2 \operatorname{tg} x - 3 = 0,$$

$$\cos x \neq 0, \operatorname{tg} x = -3, x = -\operatorname{arctg} 3 + \pi n, \operatorname{tg} x = 1, x = \frac{\pi}{4} + \pi n;$$

$$\text{г) } \sin^2(2\pi - 3x) + 5 \sin(\pi - 3x) \cos 3x + 4 \sin^2\left(\frac{3\pi}{2} - 3x\right) = 0,$$

$$\operatorname{tg}^2 3x + 5 \operatorname{tg} 3x + 4 = 0, \operatorname{tg} 3x = -4, 3x = -\operatorname{arctg} 4 + \pi n, x = -\frac{1}{3} \operatorname{arctg} 4 + \frac{\pi n}{3},$$

$$\operatorname{tg} 3x = -1, 3x = -\frac{\pi}{4} + \pi n, x = -\frac{\pi}{12} + \frac{\pi n}{3}.$$

$$388. \text{ а) } 3 \sin^2 \frac{x}{2} + \sin \frac{x}{2} \sin\left(\frac{\pi}{2} - \frac{x}{2}\right) = 2, \sin^2 \frac{x}{2} + \sin \frac{x}{2} \cos \frac{x}{2} - 2 \cos^2 x = 0,$$

$$\operatorname{tg}^2 \frac{x}{2} + \operatorname{tg} \frac{x}{2} - 2 = 0, \cos \frac{x}{2} \neq 0, \operatorname{tg} \frac{x}{2} = -2, \frac{x}{2} = -\operatorname{arctg} 2 + \pi n, x = -2 \operatorname{arctg} 2 + 2\pi n,$$

$$\operatorname{tg} \frac{x}{2} = 1; \frac{x}{2} = \frac{\pi}{4} + \pi n, x = \frac{\pi}{2} + 2\pi n;$$

$$\text{б) } 2 \cos^2 \frac{x}{2} - 3 \sin\left(\pi - \frac{x}{2}\right) \cos\left(2\pi - \frac{x}{2}\right) + 7 \sin^2 \frac{x}{2} = 3,$$

$$4 \sin^2 \frac{x}{2} - 3 \sin \frac{x}{2} \cos \frac{x}{2} - \cos^2 \frac{x}{2} = 0, \quad 4 \operatorname{tg}^2 \frac{x}{2} - 3 \operatorname{tg} \frac{x}{2} - 1 = 0,$$

$$\cos \frac{x}{2} \neq 0, \quad \operatorname{tg} \frac{x}{2} = \frac{3+5}{8} = 1, \quad \frac{x}{2} = \frac{\pi}{4} + \pi n, \quad x = \frac{\pi}{2} + 2\pi n, \quad \operatorname{tg} \frac{x}{2} = -\frac{1}{4},$$

$$\frac{x}{2} = -\operatorname{arctg} \frac{1}{4} + \pi n, \quad x = -2 \operatorname{arctg} \frac{1}{4} + 2\pi n;$$

$$\text{в)} \quad 4 \cos^2 \left(\frac{\pi}{2} + x \right) + \sqrt{3} \sin \left(\frac{3\pi}{2} - x \right) \sin(\pi + x) + 3 \cos^2(\pi + x) = 3,$$

$$\sin^2 x + \sqrt{3} \sin x \cos x = 0, \quad \sin x (\sin x + \sqrt{3} \cos x) = 0, \quad \sin x = 0, \quad x = \pi n,$$

$$\sin x + \sqrt{3} \cos x = 0, \quad \operatorname{tg} x = -\sqrt{3}, \quad \cos x \neq 0, \quad x = -\frac{\pi}{3} + \pi n;$$

$$\text{г)} \quad 3 \sin^2 \left(x - \frac{3\pi}{2} \right) - 2 \cos \left(\frac{3\pi}{2} + x \right) \cos(\pi + x) + 2 \sin^2(x - \pi) = 2,$$

$$\cos^2 x + 2 \sin x \cos x = 0, \quad \cos x (\cos x + 2 \sin x) = 0, \quad \cos x = 0,$$

$$x = \frac{\pi}{2} + \pi n, \quad \cos x + 2 \sin x = 0, \quad \operatorname{tg} x = -\frac{1}{2}, \quad \cos x \neq 0, \quad x = -\operatorname{arctg} \frac{1}{2} + \pi n.$$

$$389. \text{ а)} \quad 2 \sin^2(\pi + x) - 5 \cos \left(\frac{\pi}{2} + x \right) + 2 = 0, \quad 2 \sin^2 x + 5 \sin x + 2 = 0,$$

$$\sin x = \frac{-5-3}{4} \text{ — не подходит, } \sin x = -\frac{1}{2}, \quad x = (-1)^{k+1} \frac{\pi}{6} + \pi k;$$

$$\text{б)} \quad 2 \cos^2 x + 5 \cos \left(\frac{\pi}{2} - x \right) - 4 = 0, \quad 2 \sin^2 x - 5 \sin x + 2 = 0,$$

$$\sin x = \frac{5+3}{4} \text{ — не подходит, } \sin x = \frac{1}{2}, \quad x = (-1)^k \frac{\pi}{6} + \pi k;$$

$$\text{в)} \quad 2 \cos^2 x + \sin \left(\frac{\pi}{2} - x \right) - 1 = 0, \quad 2 \cos^2 x + \cos x - 1 = 0,$$

$$\cos x = \frac{-1-3}{4} = -1, \quad x = \pi + 2\pi k, \quad \cos x = \frac{1}{2}, \quad x = \pm \frac{\pi}{3} + 2\pi n;$$

$$\text{г)} \quad 5 - 5 \sin(3(\pi - x)) = \cos^2(\pi - 3x), \quad 5 - 5 \sin(3\pi - 3x) = \cos^2 3x,$$

$$\sin^2 3x - 5 \sin 3x + 4 = 0, \quad \sin 3x = 4 \text{ — не подходит, } \sin 3x = 1,$$

$$3x = \frac{\pi}{2} + 2\pi n, \quad x = \frac{\pi}{6} + \frac{2\pi n}{3}.$$

$$390. \text{ а)} \quad 2 \operatorname{tg}^2 2x + 3 \operatorname{tg}(\pi + 2x) = 0, \quad 2 \operatorname{tg}^2 2x + 3 \operatorname{tg} 2x = 0,$$

$$\operatorname{tg} 2x (2 \operatorname{tg} 2x + 3) = 0, \quad \operatorname{tg} 2x = 0, \quad 2x = \pi n, \quad x = \frac{\pi n}{2}, \quad \operatorname{tg} 2x = -\frac{3}{2},$$

$$2x = -\operatorname{arctg} \frac{3}{2} + \pi n, \quad x = -\frac{1}{2} \operatorname{arctg} \frac{3}{2} + \frac{\pi n}{2};$$

$$\text{б)} \quad \operatorname{tg}^2 3x - 6 \operatorname{ctg} \left(\frac{\pi}{2} - 3x \right) = 0, \quad \operatorname{tg}^2 3x - 6 \operatorname{tg} 3x = 0, \quad \operatorname{tg} 3x (\operatorname{tg} 3x - 6) = 0,$$

$$\operatorname{tg} 3x = 0, \quad x = \frac{\pi n}{3}, \quad \operatorname{tg} 3x = 6, \quad x = \frac{1}{3} \operatorname{arctg} 6 + \frac{\pi n}{3}.$$

$$391. \text{ а) } 3 \operatorname{tg}^2 \frac{x}{2} - 2 \operatorname{ctg} \left(\frac{3\pi}{2} + \frac{x}{2} \right) - 1 = 0, \quad 3 \operatorname{tg}^2 \frac{x}{2} + 2 \operatorname{tg} \frac{x}{2} - 1 = 0,$$

$$\operatorname{tg} \frac{x}{2} = \frac{-1-2}{3} = -1, \quad \frac{x}{2} = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{2} + 2\pi n, \quad \operatorname{tg} \frac{x}{2} = \frac{1}{3}, \quad x = 2 \operatorname{arctg} \frac{1}{3} + 2\pi n;$$

$$\text{ б) } 3 \operatorname{tg}^2 4x - 2 \operatorname{ctg} \left(\frac{\pi}{2} - 4x \right) = 1, \quad 3 \operatorname{tg}^2 4x - 2 \operatorname{tg} 4x - 1 = 0,$$

$$\operatorname{tg} 4x = 1, \quad x = \frac{\pi}{16} + \frac{\pi n}{4}, \quad \operatorname{tg} 4x = -\frac{1}{3}, \quad x = -\frac{1}{4} \operatorname{arctg} \frac{1}{3} + \frac{\pi n}{4}.$$

$$392. \text{ а) } \operatorname{tg}(\pi + x) + 2 \operatorname{tg} \left(\frac{\pi}{2} + x \right) + 1 = 0, \quad \operatorname{tg} x - 2 \operatorname{ctg} x + 1 = 0,$$

$$\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0, \quad \operatorname{tg} x = -2, \quad x = -\operatorname{arctg} 2 + \pi n, \quad \operatorname{tg} x = 1, \quad x = \frac{\pi}{4} + \pi n;$$

$$\text{ б) } 2 \operatorname{ctg} x - 3 \operatorname{ctg} \left(\frac{\pi}{2} - x \right) + 5 = 0, \quad 2 \operatorname{ctg} x - 3 \operatorname{tg} x + 5 = 0, \quad 3 \operatorname{tg}^2 x - 5 \operatorname{tg} x - 2 = 0,$$

$$\operatorname{tg} x = \frac{5 \pm \sqrt{25 + 4 \cdot 3 \cdot 2}}{6} = \frac{5 \pm 7}{6}, \quad \operatorname{tg} x = 2, \quad x = \operatorname{arctg} 2 + \pi n, \quad \operatorname{tg} x = -\frac{1}{3},$$

$$x = -\operatorname{arctg} \frac{1}{3} + \pi n.$$

$$393. \text{ а) } \sin^2 x + \cos^2 2x + \cos^2 \left(\frac{3\pi}{2} + 2x \right) + 2 \cos x \operatorname{tg} x = 1,$$

$$\sin^2 x + \cos^2 2x + \sin^2 2x + 2 \sin x - 1 = 0, \quad \sin^2 x + 2 \sin x = 0, \quad \sin x(\sin x + 2) = 0, \\ \sin x = 0, \quad x = \pi n;$$

$$\text{ б) } 2 \cos^2 x - \sin \left(x - \frac{\pi}{2} \right) + \operatorname{tg} x \operatorname{tg} \left(x + \frac{\pi}{2} \right) = 0, \quad 2 \cos^2 x + \cos x - 1 = 0,$$

$$\cos x = \frac{-1-3}{4} = -1, \quad x = \pi + \pi n, \quad \cos x = \frac{1}{2}, \quad x = \pm \frac{\pi}{3} + 2\pi n.$$

$$394. \text{ а) } \sin 2x < \frac{1}{2}, \quad 2x \in \left(-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n \right), \quad x \in \left(-\frac{7\pi}{12} + \pi n; \frac{\pi}{12} + \pi n \right);$$

$$\text{ б) } 3 \cos 4x < 1, \quad \cos 4x < \frac{1}{3}, \quad 4x \in \left(\arccos \frac{1}{3} + 2\pi n; 2\pi - \arccos \frac{1}{3} + 2\pi n \right),$$

$$x \in \left(\frac{1}{4} \arccos \frac{1}{3} + \frac{\pi n}{2}; \frac{\pi}{2} - \frac{1}{4} \arccos \frac{1}{3} + \frac{\pi n}{2} \right);$$

$$\text{ в) } \cos 3x > \frac{\sqrt{3}}{2}, \quad 3x \in \left(-\frac{\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n \right), \quad x \in \left(-\frac{\pi}{18} + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{2\pi n}{3} \right);$$

$$\text{г) } 7 \sin \frac{x}{2} > -1, \sin \frac{x}{2} > -\frac{1}{7}, \frac{x}{2} \in \left(-\arcsin \frac{1}{7} + 2\pi n; \arcsin \frac{1}{7} + \pi + 2\pi n \right), \\ x \in \left(-2 \arcsin \frac{1}{7} + 4\pi n; 2 \arcsin \frac{1}{7} + 2\pi + 4\pi n \right).$$

$$395. \text{ а) } \sin \left(2x - \frac{\pi}{3} \right) > \frac{1}{3}, 2x - \frac{\pi}{3} \in \left(\arcsin \frac{1}{3} + 2\pi n; \pi - \arcsin \frac{1}{3} + 2\pi n \right),$$

$$x \in \left(\frac{1}{2} \arcsin \frac{1}{3} + \frac{\pi}{6} + \pi n; \frac{2\pi}{3} - \frac{1}{2} \arcsin \frac{1}{3} + \pi n \right);$$

$$\text{б) } \cos \left(\frac{\pi}{4} - x \right) < \frac{\sqrt{2}}{2}, x - \frac{\pi}{4} \in \left(\frac{\pi}{4} + 2\pi n; \frac{7\pi}{4} + 2\pi n \right), x \in \left(\frac{\pi}{2} + 2\pi n; 2\pi + 2\pi n \right);$$

$$\text{в) } \cos \left(3x - \frac{\pi}{6} \right) > -\frac{1}{4}, 3x - \frac{\pi}{6} \in \left(-\arccos \left(-\frac{1}{4} \right) + 2\pi n; \arccos \left(-\frac{1}{4} \right) + 2\pi n \right),$$

$$x \in \left(\frac{\pi}{18} - \frac{1}{3} \arccos \left(-\frac{1}{4} \right) + \frac{2\pi n}{3}; \frac{\pi}{18} + \frac{1}{3} \arccos \left(-\frac{1}{4} \right) + \frac{2\pi n}{3} \right);$$

$$\text{г) } \sin \left(\frac{3\pi}{4} - x \right) < \frac{\sqrt{3}}{2}, \sin \left(x - \frac{3\pi}{4} \right) > -\frac{\sqrt{3}}{2},$$

$$x - \frac{3\pi}{4} \in \left(-\frac{\pi}{3} + 2\pi n; \frac{4\pi}{3} + 2\pi n \right), x \in \left(-\frac{\pi}{3} + \frac{3\pi}{4} + 2\pi n; \frac{4\pi}{3} + \frac{3\pi}{4} + 2\pi n \right).$$

$$396. \text{ а) } \sin^2 x - 6 \sin x \cos x + 5 \cos^2 x > 0, \operatorname{tg}^2 x - 6 \operatorname{tg} x + 5 > 0, \cos x \neq 0,$$

$$\operatorname{tg} x < 1, \operatorname{tg} x > 5, x \in \left(-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n \right), \left(\operatorname{arctg} 5 + \pi n; \frac{\pi}{2} + \pi n \right);$$

$$\text{б) } \sin^2 x - 6 \sin x \cos x + 5 \cos^2 x < 0, \operatorname{tg}^2 x - 6 \operatorname{tg} x + 5 < 0, \operatorname{tg} x \in (1; 5),$$

$$x \in \left(\frac{\pi}{4} + \pi n; \operatorname{arctg} 5 + \pi n \right).$$

$$397. \text{ а) } y = \sin x + \sqrt{-\cos^2 x}, \cos^2 x \geq 0, x = \frac{\pi}{2} + \pi n,$$

Область значений функции: $\{-1; 1\}$.

$$\text{б) } y = \cos x + \sqrt{-\sin^2 x}, \sin^2 x \geq 0, \sin x = 0, x = \pi n,$$

Область значений функции: $\{-1; 1\}$.

$$398. \text{ а) } y = \cos 3x + \sqrt{\cos^2 3x - 1} = \cos 3x + \sqrt{-\sin^2 3x}, \sin^2 3x \geq 0, \sin 3x = 0, x = \frac{\pi n}{3},$$

Область значений функции: $\{-1; 1\}$.

$$\text{б) } y = \sin 2x + \sqrt{\sin^2 4x - 1} = \sin 2x + \sqrt{-\cos^2 4x}, \cos^2 4x \geq 0, \cos 4x = 0,$$

$$2x = \frac{\pi}{4} + \frac{\pi n}{2}, x = \frac{\pi}{8} + \frac{\pi n}{4}. \text{ Область определения функции: } \left\{ \frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2} \right\}.$$

Глава 3. Преобразование тригонометрических выражений

§21. Синус и косинус суммы аргументов

399. а) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$;

б) $\cos 105^\circ = \cos(60^\circ + 45^\circ) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$.

400. а) $\sin(\alpha + \beta) - \sin \alpha \cos \beta = \sin \alpha \cos \beta + \sin \beta \cos \alpha - \sin \alpha \cos \beta = \sin \beta \cos \alpha$;

б) $\sin\left(\frac{\pi}{3} + \alpha\right) - \frac{1}{2} \sin \alpha = \sin \frac{\pi}{3} \cos \alpha + \cos \frac{\pi}{3} \sin \alpha - \frac{1}{2} \sin \alpha = \frac{\sqrt{3}}{2} \cos \alpha$;

в) $\sin \alpha \sin \beta + \cos(\alpha + \beta) = \sin \alpha \sin \beta + \cos \alpha \cos \beta - \sin \alpha \sin \beta = \cos \alpha \cos \beta$;

г) $\cos\left(\alpha + \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2} \sin \alpha = \frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \sin \alpha + \frac{\sqrt{2}}{2} \sin \alpha = \frac{\sqrt{2}}{2} \cos \alpha$.

401. а) $\sin(\alpha + \beta) + \sin(-\alpha) \cos(-\beta) = \sin \alpha \cos \beta$,

$\sin \alpha \cos \beta + \cos \alpha \sin \beta = \sin \beta \cos \alpha$ тождество неверно;

б) $\cos(\alpha + \beta) + \sin(-\alpha) \sin(-\beta) = \cos \alpha \cos \beta$,

$\cos(\alpha + \beta) + \sin(-\alpha) \sin(-\beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta + \sin \alpha \sin \beta = \cos \alpha \cos \beta$.

402. а) $\sin 74^\circ \cos 16^\circ + \cos 74^\circ \sin 16^\circ = \sin(74^\circ + 16^\circ) = \sin 90^\circ = 1$;

б) $\cos 23^\circ \cos 22^\circ - \sin 23^\circ \sin 22^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$;

в) $\sin 89^\circ \cos 1^\circ + \cos 89^\circ \sin 1^\circ = \sin(89^\circ + 1^\circ) = \sin 90^\circ = 1$;

г) $\cos 178^\circ \cos 2^\circ - \sin 178^\circ \sin 2^\circ = \cos(178^\circ + 2^\circ) = -1$.

403. а) $\sin \frac{\pi}{5} \cos \frac{\pi}{20} + \cos \frac{\pi}{5} \sin \frac{\pi}{20} = \sin\left(\frac{\pi}{5} + \frac{\pi}{20}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$;

б) $\cos \frac{2\pi}{7} \cos \frac{5\pi}{7} - \sin \frac{2\pi}{7} \sin \frac{5\pi}{7} = \cos\left(\frac{2\pi}{7} + \frac{5\pi}{7}\right) = \cos \pi = -1$;

в) $\sin \frac{\pi}{12} \cos \frac{11\pi}{12} + \cos \frac{\pi}{12} \sin \frac{11\pi}{12} = \sin\left(\frac{\pi}{12} + \frac{11\pi}{12}\right) = \sin \pi = 0$;

г) $\cos \frac{2\pi}{15} \cos \frac{\pi}{5} - \sin \frac{2\pi}{15} \sin \frac{\pi}{5} = \cos\left(\frac{2\pi}{15} + \frac{\pi}{5}\right) = \cos \frac{\pi}{3} = \frac{1}{2}$.

404. а) $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \sin\left(\frac{\pi}{3} + x\right)$, $\sin \frac{\pi}{3} \cos x + \cos \frac{\pi}{3} \sin x = \sin\left(\frac{\pi}{3} + x\right)$;

б) $\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \cos\left(x + \frac{\pi}{3}\right)$.

405. а) $\sin 5x \cos 3x + \cos 5x \sin 3x = \sin 8x$, $\sin(5x + 3x) = \sin 8x$;

б) $\cos 5x \cos 3x - \sin 5x \sin 3x = \cos 8x$, $\cos(5x + 3x) = \cos 8x$;

$$\text{в)} \sin 7x \cos 4x + \cos 7x \sin 4x = \sin 11x, \quad \sin(7x + 4x) = \sin 11x;$$

$$\text{г)} \cos 2x \cos 12x - \sin 2x \sin 12x = \cos 14x.$$

$$406. \text{ а)} \sin 2x \cos x + \cos 2x \sin x = 1, \quad \sin 3x = 1, \quad 3x = \frac{\pi}{2} + 2\pi n, \quad x = \frac{\pi}{6} + \frac{2\pi n}{3};$$

$$\text{б)} \cos 3x \cos 5x = \sin 3x \sin 5x, \quad \cos 3x \cos 5x - \sin 3x \sin 5x = 0,$$

$$\cos 8x = 0, \quad 8x = \frac{\pi}{2} + \pi n, \quad x = \frac{\pi}{16} + \frac{\pi n}{8}.$$

$$407. \text{ а)} \sin 6x \cos x + \cos 6x \sin x = \frac{1}{2}, \quad \sin 7x = \frac{1}{2},$$

$$7x = (-1)^k \frac{\pi}{6} + \pi k, \quad x = (-1)^k \frac{\pi}{42} + \frac{\pi k}{7};$$

$$\text{б)} \cos 5x \cos 7x - \sin 5x \sin 7x = -\frac{\sqrt{3}}{2}, \quad \cos 12x = -\frac{\sqrt{3}}{2},$$

$$12x = \pm \frac{5\pi}{6} + 2\pi n, \quad x = \pm \frac{5\pi}{72} + \frac{\pi n}{6}.$$

$$408. \sin t = \frac{3}{5}, \quad 0 < t < \frac{\pi}{2}, \quad \cos t = \frac{4}{5}.$$

$$\text{а)} \sin\left(\frac{\pi}{3} + t\right) = \sin \frac{\pi}{3} \cos t + \sin t \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{3}{5} = \frac{2\sqrt{3}}{5} + \frac{3}{10} = \frac{4\sqrt{3} + 3}{10};$$

$$\text{б)} \cos\left(\frac{\pi}{2} + t\right) = -\sin t = -\frac{3}{5}; \quad \text{в)} \sin\left(\frac{\pi}{2} + t\right) = \cos t = \frac{4}{5};$$

$$\text{г)} \cos\left(\frac{\pi}{3} + t\right) = \cos \frac{\pi}{3} \cos t - \sin \frac{\pi}{3} \sin t = \frac{1}{2} \cdot \frac{4}{5} - \frac{\sqrt{3}}{2} \cdot \frac{3}{5} = \frac{4 - 3\sqrt{3}}{10}.$$

$$409. \cos t = -\frac{5}{13}, \quad \frac{\pi}{2} < t < \pi, \quad \sin t = \frac{12}{13}.$$

$$\text{а)} \sin\left(t + \frac{\pi}{6}\right) = \sin t \frac{\sqrt{3}}{2} + \cos t \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{12}{13} - \frac{5}{13} \cdot \frac{1}{2} = \frac{12\sqrt{3} - 5}{26};$$

$$\text{б)} \cos\left(t + \frac{3\pi}{2}\right) = \sin t = \frac{12}{13};$$

$$\text{в)} \cos\left(t + \frac{\pi}{6}\right) = \cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6} = -\frac{5}{13} \cdot \frac{\sqrt{3}}{2} - \frac{12}{13} \cdot \frac{1}{2} = \frac{-5\sqrt{3} - 12}{26};$$

$$\text{г)} \sin\left(t + \frac{3\pi}{2}\right) = -\cos t = \frac{5}{13}.$$

$$410. \cos \alpha = \frac{15}{17}, \quad \cos \beta = \frac{4}{5}, \quad 0 < \alpha < \frac{\pi}{2},$$

$$0 < \beta < \frac{\pi}{2}, \quad \sin \alpha = \frac{8}{17}, \quad \sin \beta = \frac{3}{5}.$$

$$a) \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha = \frac{8}{17} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{15}{17} = \frac{77}{85};$$

$$b) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{5}{17} \cdot \frac{4}{5} - \frac{8}{17} \cdot \frac{3}{5} = \frac{60 - 24}{85} = \frac{36}{85}.$$

$$411. \sin \alpha = \frac{4}{5}, \cos \beta = -\frac{15}{17}, \frac{\pi}{2} < \alpha < \pi, \frac{\pi}{2} < \beta < \pi, \cos \alpha = -\frac{3}{5}, \sin \beta = \frac{8}{17}.$$

$$a) \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha = \frac{4}{5} \left(-\frac{15}{17}\right) + \frac{8}{17} \left(-\frac{3}{5}\right) = -\frac{84}{85};$$

$$b) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{13}{85}.$$

$$412. \sin \alpha = \frac{9}{41}, \sin \beta = -\frac{40}{41}, 0 < \alpha < \frac{\pi}{2}, \frac{3\pi}{2} < \beta < 2\pi, \cos \alpha = \frac{40}{41}, \cos \beta = \frac{9}{41}.$$

$$a) \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha = \left(\frac{9}{41}\right)^2 - \left(\frac{40}{41}\right)^2 = \frac{9 - 40}{41} \cdot \frac{9 + 40}{41} = \frac{1519}{1681};$$

$$b) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{40}{41} \cdot \frac{9}{41} + \frac{9}{41} \cdot \frac{40}{41} = \frac{720}{1681}.$$

$$413. a) \sin 75^\circ \cos 75^\circ = \frac{1}{4},$$

$$\sin 75^\circ \cos 75^\circ = \frac{1}{2} (\sin 75^\circ \cos 75^\circ + \sin 75^\circ \cos 75^\circ) = \frac{1}{2} \sin 150^\circ = \frac{1}{2} \sin 30^\circ = \frac{1}{4};$$

$$b) \cos^2 75^\circ - \sin^2 75^\circ - \sin 75^\circ \sin 75^\circ = \cos(75^\circ + 75^\circ) = \cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2};$$

$$b) \sin 105^\circ \cos 105^\circ = -\frac{1}{4}, \quad \sin 105^\circ \cos 105^\circ = \frac{1}{2} (\sin 105^\circ \cos 105^\circ + \sin 105^\circ \cos 105^\circ) = \sin 210^\circ = -\frac{1}{2} \sin 30^\circ = -\frac{1}{4};$$

$$r) \cos^2 75^\circ + \sin^2 75^\circ = 1.$$

$$414. a) \sin 2x = 2 \sin x \cos x, \sin 2x = \sin(x + x) = \sin x \cos x + \sin x \cos x = 2 \sin x \cos x;$$

$$b) \cos 2x = \cos^2 x - \sin^2 x, \cos 2x = \cos(x + x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x.$$

$$415. a) \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta,$$

$$\sin(\alpha - \beta) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha;$$

$$b) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta,$$

$$\cos(\alpha + (-\beta)) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$$

$$416. a) \frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x = 1, \sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = 1,$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1, x - \frac{\pi}{4} = 2\pi n, x = \frac{\pi}{4} + 2\pi n;$$

$$б) \sin x + \cos x = 1, \quad \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}},$$

$$x + \frac{\pi}{4} = (-1)^k \frac{\pi}{4} + \pi n, \quad x = (-1)^k \frac{\pi}{4} - \frac{\pi}{4} + \pi n;$$

$$в) \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = 1, \quad \cos \frac{\pi}{6} \cos x - \sin \frac{\pi}{6} \sin x = 1,$$

$$x + \frac{\pi}{6} = 2\pi n, \quad x = -\frac{\pi}{6} + 2\pi n;$$

$$г) \sqrt{3} \cos x - \sin x = 1, \quad \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{1}{2},$$

$$x + \frac{\pi}{6} = \pm \frac{\pi}{3} + 2\pi n, \quad x = \pm \frac{\pi}{3} - \frac{\pi}{6} + 2\pi n.$$

$$417. а) \sin x \cos 3x + \cos x \sin 3x > \frac{1}{2}, \quad \sin 4x > \frac{1}{2},$$

$$4x \in \left(\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n\right), \quad x \in \left(\frac{\pi}{24} + \frac{\pi n}{2}; \frac{5\pi}{24} + \frac{\pi n}{2}\right);$$

$$б) \cos 2x \cos 5x - \sin 2x \sin 5x < -\frac{1}{3}, \quad \cos 7x < -\frac{1}{3},$$

$$7x \in \left(\pi - \arccos \frac{1}{3} + 2\pi n; \pi + \arccos \frac{1}{3} + 2\pi n\right),$$

$$x \in \left(\frac{\pi}{7} - \frac{1}{7} \arccos \frac{1}{3} + \frac{2\pi n}{7}; \frac{\pi}{7} + \frac{1}{7} \arccos \frac{1}{3} + \frac{2\pi n}{7}\right);$$

$$в) \sin x \cos \frac{x}{2} + \cos x \sin \frac{x}{2} \leq -\frac{2}{7}, \quad \sin \frac{3x}{2} \leq -\frac{2}{7},$$

$$\frac{3x}{2} \in \left[-\pi + \arcsin \frac{2}{7} + 2\pi n; -\arcsin \frac{2}{7} + 2\pi n\right],$$

$$x \in \left[-\frac{2\pi}{3} + \frac{2}{3} \arcsin \frac{2}{7} + \frac{4\pi n}{3}; -\frac{2}{3} \arcsin \frac{2}{7} + \frac{4\pi n}{3}\right];$$

$$г) \cos \frac{x}{2} \cos \frac{x}{4} - \sin \frac{x}{2} \sin \frac{x}{4} > \frac{\sqrt{2}}{2}, \quad \cos \frac{3x}{4} > \frac{\sqrt{2}}{2},$$

$$\frac{3x}{4} \in \left(-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right), \quad x \in \left(-\frac{\pi}{3} + \frac{8\pi n}{3}; \frac{\pi}{3} + \frac{8\pi n}{3}\right).$$

§22. Синус и косинус разности аргументов

$$418. а) \sin(60^\circ - \beta) = \sin 60^\circ \cos \beta - \cos 60^\circ \sin \beta = \frac{\sqrt{3}}{2} \cos \beta - \frac{1}{2} \sin \beta;$$

$$б) \cos(\beta - 30^\circ) = \cos \beta \cos 30^\circ + \sin \beta \sin 30^\circ = \frac{\sqrt{3}}{2} \cos \beta + \frac{1}{2} \sin \beta;$$

$$\text{в)} \sin(\alpha - 30^\circ) = \sin \alpha \cos 30^\circ - \sin 30^\circ \cos \alpha = -\frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha;$$

$$\text{г)} \cos(60^\circ - \alpha) = \cos 60^\circ \cos \alpha + \sin 60^\circ \sin \alpha = \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha.$$

$$\begin{aligned} 419. \text{ а)} \sin\left(\frac{5\pi}{6} - \alpha\right) - \frac{1}{2} \cos \alpha &= \sin \frac{5\pi}{6} \cos \alpha - \sin \alpha \cos \frac{5\pi}{6} - \frac{1}{2} \cos \alpha = \\ &= \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha - \frac{1}{2} \cos \alpha = \frac{\sqrt{3}}{2} \sin \alpha; \end{aligned}$$

$$\text{б)} \sqrt{3} \cos \alpha - 2 \cos\left(\alpha - \frac{\pi}{6}\right) = \sqrt{3} \cos \alpha - 2 \cos \alpha \cos \frac{\pi}{6} - 2 \sin \alpha \sin \frac{\pi}{6} = -\sin \alpha;$$

$$\text{в)} \frac{\sqrt{3}}{2} \sin \alpha + \cos\left(\alpha - \frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2} \sin \alpha + \cos \alpha \cos \frac{5\pi}{3} + \sin \alpha \sin \frac{5\pi}{3} = \frac{1}{2} \cos \alpha;$$

$$\text{г)} \sqrt{2} \sin\left(\alpha - \frac{\pi}{4}\right) - \sin \alpha = \sqrt{2} \sin \alpha \cos \frac{\pi}{4} - \sqrt{2} \cos \alpha \sin \frac{\pi}{4} - \sin \alpha = -\cos \alpha.$$

$$420. \text{ а)} \cos(\alpha - \beta) - \cos \alpha \cos \beta = \cos \alpha \cos \beta - \sin \alpha \sin \beta - \cos \alpha \cos \beta = -\sin \alpha \sin \beta;$$

$$\text{б)} \sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha + \sin \alpha \cos \beta - \sin \beta \cos \alpha = 2 \sin \alpha \cos \beta;$$

$$\text{в)} \sin \alpha \cos \beta - \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \alpha \cos \beta + \sin \beta \cos \alpha = \sin \beta \cos \alpha;$$

$$\text{г)} \cos(\alpha - \beta) - \cos(\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta - \cos \alpha \cos \beta + \sin \alpha \sin \beta = 2 \sin \alpha \sin \beta.$$

$$421. \text{ а)} \cos 36^\circ \cos 24^\circ - \sin 36^\circ \sin 24^\circ = \cos(36^\circ + 24^\circ) = \cos 60^\circ = \frac{1}{2};$$

$$\text{б)} \cos 36^\circ \cos 24^\circ - \sin 36^\circ \sin 24^\circ = \cos(36^\circ + 24^\circ) = \cos 60^\circ = \frac{1}{2};$$

$$\text{в)} \sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = \sin(63^\circ + 27^\circ) = \sin 90^\circ = 1;$$

$$\text{г)} \sin 51^\circ \cos 21^\circ - \cos 51^\circ \sin 21^\circ = \sin(51^\circ - 21^\circ) = \frac{1}{2}.$$

$$422. \text{ а)} \cos \frac{5\pi}{8} \cos \frac{3\pi}{8} + \sin \frac{5\pi}{8} \sin \frac{3\pi}{8} = \cos\left(\frac{5\pi}{8} - \frac{3\pi}{8}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2};$$

$$\text{б)} \sin \frac{2\pi}{15} \cos \frac{\pi}{5} + \cos \frac{2\pi}{15} \sin \frac{\pi}{5} = \sin\left(\frac{2\pi}{15} + \frac{\pi}{5}\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2};$$

$$\text{в)} \cos \frac{\pi}{12} \cos \frac{\pi}{4} - \sin \frac{\pi}{12} \sin \frac{\pi}{4} = \sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = \cos \frac{\pi}{3} = \frac{1}{2};$$

$$\text{г)} \sin \frac{\pi}{12} \cos \frac{\pi}{4} - \cos \frac{\pi}{12} \sin \frac{\pi}{4} = \sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = -\sin \frac{\pi}{6} = -\frac{1}{2}.$$

$$423. \text{ а)} \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \sin\left(\frac{\pi}{3} - x\right), \quad \sin \frac{\pi}{3} \cos x - \cos x \frac{\pi}{3} \sin x = \sin\left(\frac{\pi}{3} - x\right);$$

$$\text{б)} \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = \cos\left(\frac{\pi}{3} - x\right), \quad \cos \frac{\pi}{3} \cos x + \sin \frac{\pi}{3} \sin x = \cos\left(\frac{\pi}{3} - x\right).$$

$$424. \text{ а) } \cos(\alpha - \beta) + \sin(-\alpha) \sin \beta = \cos \alpha \cos \beta,$$

$$\cos \alpha \cos \beta + \sin \alpha \sin \beta - \sin \alpha \sin \beta = \cos \alpha \cos \beta;$$

$$\text{б) } \sin(30^\circ - \alpha) - \cos(60^\circ - \alpha) = -\sqrt{3} \sin \alpha,$$

$$\sin 30^\circ \cos \alpha - \sin \alpha \cos 30^\circ - \cos 60^\circ \cos \alpha - \sin 60^\circ \sin \alpha =$$

$$= \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha - \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha = -\sqrt{3} \sin \alpha;$$

$$\text{в) } \sin(\alpha - \beta) - \cos \alpha \sin(-\beta) = \sin \alpha \cos \beta,$$

$$\sin \alpha \cos \beta - \sin \beta \cos \alpha + \sin \beta \cos \alpha = \sin \alpha \cos \beta;$$

$$\text{г) } \sin(30^\circ - \alpha) + \sin(30^\circ + \alpha) = \cos \alpha,$$

$$\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \cos \alpha.$$

$$425. \text{ а) } \sin(\alpha - \beta) - \sin(\alpha + \beta) = -2 \cos \alpha \sin \beta,$$

$$\sin \alpha \cos \beta - \sin \beta \cos \alpha - \sin \alpha \cos \beta - \sin \beta \cos \alpha = -2 \cos \alpha \sin \beta;$$

$$\text{б) } \cos(\alpha - \beta) + \cos(\alpha + \beta) = 2 \cos \alpha \cos \beta,$$

$$\cos \alpha \cos \beta + \sin \alpha \sin \beta + \cos \alpha \cos \beta - \sin \alpha \sin \beta = 2 \cos \alpha \cos \beta.$$

$$426. \text{ а) } \cos 6x \cos 5x + \sin 6x \sin 5x = -1, \cos(6x - 5x) = -1, \cos x = -1, x = \pi + 2\pi n;$$

$$\text{б) } \sin 3x \cos 5x - \sin 5x \cos 3x = \frac{1}{2}, \sin(3x - 5x) = \frac{1}{2}, \sin 2x = -\frac{1}{2},$$

$$2x = (-1)^{k+1} \frac{\pi}{6} + \pi k, x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi k}{2}.$$

$$427. \text{ а) } \sin 15^\circ = \sin(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{2};$$

$$\text{б) } \cos 15^\circ = \cos(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{2};$$

$$\text{в) } \sin 15^\circ \cos 15^\circ = \frac{1}{2} (\sin(15^\circ + 15^\circ)) = \frac{1}{2} \sin 30^\circ = \frac{1}{4};$$

$$\text{г) } \cos^2 15^\circ - \sin^2 15^\circ = \cos(15^\circ + 15^\circ) = \frac{\sqrt{3}}{2}.$$

$$428. \text{ а) Опечатка в условии:}$$

$$\sin 77^\circ \cos 17^\circ - \sin 13^\circ \cos 73^\circ = \sin 77^\circ \cos 17^\circ - \cos 77^\circ \sin 17^\circ =$$

$$= \sin(77^\circ - 17^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2};$$

$$\text{б) } \cos 125^\circ \cos 5^\circ + \sin 55^\circ \cos 85^\circ = -\cos 55^\circ \cos 5^\circ + \sin 55^\circ \cos 5^\circ = -\cos 60^\circ = -\frac{1}{2}.$$

$$429. \text{ а) } \sin\left(\frac{\pi}{6} + t\right) \cos\left(\frac{\pi}{3} - t\right) + \sin\left(\frac{2\pi}{3} + t\right) \sin\left(\frac{\pi}{3} - t\right) =$$

$$\sin\left(\frac{\pi}{6} + t\right) \cos\left(\frac{\pi}{3} - t\right) + \sin\left(\frac{\pi}{3} - t\right) \sin\left(\frac{\pi}{3} - t\right) = \cos^2 t \left(\frac{\pi}{3} - t\right) + \sin^2 \left(\frac{\pi}{3} - t\right) = 1;$$

$$\begin{aligned} \text{б)} \quad & \cos\left(\frac{\pi}{4} + t\right) \cos\left(\frac{\pi}{12} - t\right) - \cos\left(\frac{\pi}{4} - t\right) \cos\left(\frac{5\pi}{12} + t\right) = \\ & = \cos\left(\frac{\pi}{12} - t\right) \sin\left(\frac{\pi}{4} - t\right) - \sin\left(\frac{\pi}{12} - t\right) \cos\left(\frac{\pi}{4} - t\right) = \sin\left(\frac{\pi}{4} - t - \frac{\pi}{12} + t\right) = \sin \frac{\pi}{6} = \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} 430. \text{ а)} \quad & \frac{\cos 105^\circ \cos 55^\circ + \sin 105^\circ \cos 85^\circ}{\sin 95^\circ \cos 5^\circ - \cos 95^\circ \sin 185^\circ} = \frac{\sin 15^\circ \cos 5^\circ + \cos 15^\circ \sin 15^\circ}{\cos^2 5^\circ - \sin^2 5^\circ} = \\ & = \frac{-\sin 10^\circ}{\cos 10^\circ} = -\operatorname{tg} 10^\circ; \end{aligned}$$

$$\text{б)} \quad \frac{\sin 25^\circ \cos 5^\circ - \cos 25^\circ \cos 85^\circ}{\cos 375^\circ \cos 5^\circ - \sin 15^\circ \sin 365^\circ} = \frac{\sin(25^\circ - 5^\circ)}{\cos 15^\circ \cos 5^\circ - \sin 15^\circ \sin 5^\circ} = \frac{\sin 20^\circ}{\cos 20^\circ} = \operatorname{tg} 20^\circ.$$

$$431. \text{ а)} \quad \frac{\sin(\alpha + \beta) - \cos \alpha \sin \beta}{\sin(\alpha - \beta) + \cos \alpha \sin \beta} = \frac{\sin \alpha \cos \beta}{\sin \alpha \cos \beta} = 1;$$

$$\text{б)} \quad \frac{\sin(\alpha - \beta) + 2 \cos \alpha \sin \beta}{2 \cos \alpha \cos \beta - \cos(\alpha - \beta)} = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \operatorname{tg}(\alpha + \beta);$$

$$\text{в)} \quad \frac{\cos(\alpha + \beta) + \sin \alpha \sin \beta}{\cos(\alpha - \beta) - \sin \alpha \sin \beta} = \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} = 1;$$

$$\text{г)} \quad \frac{\cos(\alpha - \beta) + 2 \sin \alpha \sin \beta}{2 \sin \alpha \cos \beta - \sin(\alpha - \beta)} = \operatorname{ctg}(\alpha + \beta).$$

$$432. \sin t = \frac{5}{13}, \quad \frac{\pi}{2} < t < \pi, \quad \cos t = -\frac{12}{13},$$

$$\text{а)} \quad \sin\left(\frac{\pi}{3} - t\right) = \sin \frac{\pi}{3} \cos t - \cos \frac{\pi}{3} \sin t = -\frac{\sqrt{3}}{2} \cdot \frac{12}{13} - \frac{1}{2} \cdot \frac{5}{13} = \frac{-12\sqrt{3} - 5}{26};$$

$$\text{б)} \quad \cos\left(t - \frac{\pi}{2}\right) = \sin t = \frac{5}{13}; \quad \text{в)} \quad \sin\left(\frac{\pi}{2} - t\right) = \sin \frac{\pi}{2} \cos t - \cos \frac{\pi}{2} \sin t = \cos t = -\frac{12}{13};$$

$$\text{г)} \quad \cos\left(\frac{\pi}{3} - t\right) = \cos \frac{\pi}{3} \cos t + \sin \frac{\pi}{3} \sin t = -\frac{1}{2} \cdot \frac{12}{13} + \frac{\sqrt{3}}{2} \cdot \frac{5}{13} = \frac{5\sqrt{3} - 12}{26}.$$

$$433. \cos t = \frac{3}{5}, \quad \frac{3\pi}{2} < t < 2\pi$$

$$\text{а)} \quad \sin\left(t - \frac{\pi}{5}\right) = \sin t \cos \frac{\pi}{6} - \sin \frac{\pi}{6} \cos t = -\frac{4}{5} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{3}{5} = -\frac{4\sqrt{3} + 3}{10};$$

$$\text{б)} \quad \sin\left(t - \frac{3\pi}{2}\right) = \sin t \cos \frac{3\pi}{2} - \sin \frac{3\pi}{2} \cos t = \cos t = \frac{3}{5};$$

$$\text{в)} \quad \cos\left(t - \frac{3\pi}{2}\right) = \cos t \cos \frac{3\pi}{2} + \sin t \sin \frac{3\pi}{2} = -\sin t = \frac{4}{5};$$

$$\text{г)} \quad \cos\left(t - \frac{\pi}{5}\right) = \cos t \cos \frac{\pi}{6} + \sin t \sin \frac{\pi}{6} = \cos t \cdot \frac{\sqrt{3}}{2} + \sin t \cdot \frac{1}{2} = \frac{3}{5} \cdot \frac{\sqrt{3}}{2} - \frac{4}{5} \cdot \frac{1}{2} = \frac{3\sqrt{3} - 4}{10}.$$

$$434. \sin \alpha = \frac{4}{5}, \cos \beta = -\frac{15}{17}, \frac{\pi}{2} < \alpha < \pi, \frac{\pi}{2} < \beta < \pi$$

$$a) \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha = -\frac{4}{5} \cdot \frac{15}{17} + \frac{8}{17} \cdot \frac{3}{5} = \frac{-60 + 24}{85} = -\frac{36}{85};$$

$$b) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \frac{3}{5} \cdot \frac{15}{17} + \frac{4}{5} \cdot \frac{8}{17} = \frac{77}{85}.$$

$$435. \sin \beta = -\frac{12}{13}, \cos \alpha = -\frac{4}{5}, \pi < \beta < \frac{3\pi}{2}, \cos \beta = -\frac{5}{13}, \sin \alpha = \frac{3}{5}, \frac{\pi}{2} < \alpha < \pi$$

$$a) \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha = -\frac{3}{5} \cdot \frac{5}{13} - \frac{12}{13} \cdot \frac{4}{5} = \frac{-15 - 48}{65} = -\frac{63}{65};$$

$$b) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}.$$

$$436. a) \frac{\sqrt{2} \cos \alpha - 2 \cos\left(\frac{\pi}{4} - \alpha\right)}{2 \sin\left(\frac{\pi}{6} + \alpha\right) - \sqrt{3} \sin \alpha} = -\sqrt{2} \operatorname{tg} \alpha,$$

$$\frac{\sqrt{2} \cos \alpha - \sqrt{2} \cos \alpha - \sqrt{2} \sin \alpha}{\cos \alpha + \sqrt{3} \sin \alpha - \sqrt{3} \sin \alpha} = \frac{-\sqrt{2} \sin \alpha}{\cos \alpha} = -\sqrt{2} \operatorname{tg} \alpha;$$

$$b) \frac{\cos \alpha - 2 \cos\left(\frac{\pi}{3} + \alpha\right)}{2 \sin\left(-\frac{\pi}{6} + \alpha\right) - \sqrt{3} \sin \alpha} = -\sqrt{3} \operatorname{tg} \alpha,$$

$$\frac{\cos \alpha - \cos \alpha + \sqrt{3} \sin \alpha}{2 \sin \alpha \cos \frac{\pi}{6} - 2 \sin \frac{\pi}{6} \cos \alpha - \sqrt{3} \sin \alpha} = \frac{\sqrt{3} \sin \alpha}{-\cos \alpha} = -\sqrt{3} \operatorname{tg} \alpha.$$

$$437. a) \sqrt{2} \cos\left(\frac{\pi}{4} - x\right) - \cos x = \frac{1}{2}, \cos x + \sin x - \cos x = \frac{1}{2},$$

$$\sin x = \frac{1}{2}, x = (-1)^k \frac{\pi}{6} + \pi k;$$

$$b) \sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sin \frac{x}{2} = \frac{\sqrt{3}}{2}, \cos \frac{x}{2} - \sin \frac{x}{2} + \sin \frac{x}{2} = \frac{\sqrt{3}}{2},$$

$$\cos \frac{x}{2} = \frac{\sqrt{3}}{2}, \frac{x}{2} = \pm \frac{\pi}{6} + 2\pi n, x = \pm \frac{\pi}{3} + 4\pi n.$$

$$438. a) \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x = 1, \sin\left(x - \frac{\pi}{4}\right) = 1, x - \frac{\pi}{4} = \frac{\pi}{2} + 2\pi n, x = \frac{3\pi}{4} + 2\pi n;$$

$$b) \sin x - \cos x = 1, \sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},$$

$$x - \frac{\pi}{4} = (-1)^n \frac{\pi}{4} + 2\pi n, \quad x = (-1)^n \frac{\pi}{4} + \frac{\pi}{4} + 2\pi n;$$

$$\text{в)} \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 1, \quad \cos\left(x - \frac{\pi}{6}\right) = 1, \quad x - \frac{\pi}{6} = 2\pi n, \quad x = \frac{\pi}{6} + 2\pi n;$$

$$\text{г)} \sqrt{3} \cos x + \sin x = 1, \quad \cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}, \quad x - \frac{\pi}{6} = \pm \frac{\pi}{3} + 2\pi n, \quad x = \pm \frac{\pi}{3} + \frac{\pi}{6} + 2\pi n.$$

$$439. \text{ а)} \sin 5x \cos 3x - \cos 5x \sin 3x > \frac{1}{2}, \quad \sin 2x > \frac{1}{2},$$

$$2x \in \left(\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n\right), \quad x \in \left(\frac{\pi}{12} + \pi n; \frac{5\pi}{12} + \pi n\right);$$

$$\text{б)} \cos x \cos \frac{x}{2} + \sin x \sin \frac{x}{2} < -\frac{2}{7}, \quad \cos \frac{x}{2} < -\frac{2}{7},$$

$$\frac{x}{2} \in \left(\pi - \arccos \frac{2}{7} + 2\pi n; \pi - \arccos \frac{2}{7} + 2\pi n\right),$$

$$x \in \left(2\pi - 2 \arccos \frac{2}{7} + 4\pi n; 2\pi - 2 \arccos \frac{2}{7} + 4\pi n\right);$$

$$\text{в)} \sin \frac{x}{4} \cos \frac{x}{2} - \cos \frac{x}{4} \sin \frac{x}{2} < \frac{1}{3}, \quad \sin\left(\frac{x}{4} - \frac{x}{2}\right) < \frac{1}{3}, \quad \sin \frac{x}{4} > -\frac{1}{3},$$

$$\frac{x}{4} \in \left(-\arcsin \frac{1}{3} + 2\pi n; \pi + \arcsin \frac{1}{3} + 2\pi n\right),$$

$$x \in \left(-4 \arcsin \frac{1}{3} + 8\pi n; 4\pi + 4 \arcsin \frac{1}{3} + 8\pi n\right);$$

$$\text{г)} \sin 2x \sin 5x + \cos 2x \cos 5x > -\frac{\sqrt{3}}{2}, \quad \cos 3x > -\frac{\sqrt{3}}{2},$$

$$3x \in \left(-\frac{5\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n\right), \quad x \in \left(-\frac{5\pi}{18} + \frac{2\pi n}{3}; \frac{5\pi}{18} + \frac{2\pi n}{3}\right).$$

§23. Тангенс суммы и разности аргументов

$$440. \text{ а)} \operatorname{tg} 15^\circ = \operatorname{tg}(45^\circ - 30^\circ) = \frac{\operatorname{tg} 45^\circ - \operatorname{tg} 30^\circ}{1 + \operatorname{tg} 45^\circ \operatorname{tg} 30^\circ} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}};$$

$$\text{б)} \operatorname{tg} 75^\circ = \operatorname{tg}(45^\circ + 30^\circ) = \frac{\operatorname{tg} 45^\circ + \operatorname{tg} 30^\circ}{1 - \operatorname{tg} 45^\circ \operatorname{tg} 30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}};$$

$$\text{в)} \operatorname{tg} 105^\circ = \operatorname{tg}(-60^\circ + 45^\circ) = \frac{\operatorname{tg} 60^\circ + \operatorname{tg} 45^\circ}{1 - \operatorname{tg} 60^\circ \operatorname{tg} 45^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}};$$

$$\text{r) } \operatorname{tg} 165^\circ = \operatorname{tg} -15^\circ = -\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}.$$

$$441. \text{ a) } \frac{\operatorname{tg} 25^\circ + \operatorname{tg} 20^\circ}{1 - \operatorname{tg} 25^\circ \operatorname{tg} 20^\circ} = \operatorname{tg} 45^\circ = -1;$$

$$\text{б) } \frac{1 - \operatorname{tg} 70^\circ \operatorname{tg} 65^\circ}{\operatorname{tg} 70^\circ + \operatorname{tg} 65^\circ} = \operatorname{ctg} 135^\circ = -1;$$

$$\text{в) } \frac{\operatorname{tg} 9^\circ + \operatorname{tg} 51^\circ}{1 - \operatorname{tg} 9^\circ \operatorname{tg} 51^\circ} = \operatorname{tg} 60^\circ = \sqrt{3};$$

$$\text{г) } \frac{1 + \operatorname{tg} 54^\circ \operatorname{tg} 9^\circ}{\operatorname{tg} 54^\circ - \operatorname{tg} 9^\circ} = \operatorname{ctg} 45^\circ = 1.$$

$$442. \text{ a) } \operatorname{tg}\left(\frac{\pi}{4} - \alpha\right), \operatorname{tg} \alpha = \frac{2}{3}, \operatorname{tg}\left(\frac{\pi}{4} - \alpha\right) = \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha} = \frac{1 - \frac{2}{3}}{1 + \frac{2}{3}} = \frac{1}{5};$$

$$\text{б) } \operatorname{tg} \alpha = \frac{4}{5}, \operatorname{tg}\left(\alpha + \frac{\pi}{4}\right) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \frac{\pi}{4}}{1 - \operatorname{tg} \alpha \operatorname{tg} \frac{\pi}{4}} = \frac{\frac{4}{5} + \frac{\sqrt{3}}{3}}{1 - \frac{4}{5} \cdot \frac{\sqrt{3}}{3}} = \frac{12 + 5\sqrt{3}}{15 - 4\sqrt{3}};$$

$$\text{в) } \operatorname{ctg} \alpha = \frac{4}{3}, \operatorname{tg}\left(\frac{\pi}{2} + \alpha\right) = -\operatorname{ctg} \alpha = -\frac{4}{3};$$

$$\text{г) } \operatorname{ctg} \alpha = \frac{8}{5}, \operatorname{tg}\left(\alpha - \frac{\pi}{6}\right) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \frac{\pi}{6}}{1 + \operatorname{tg} \alpha \operatorname{tg} \frac{\pi}{6}} = \frac{\frac{8}{5} - \frac{\sqrt{3}}{3}}{1 + \frac{8}{5} \cdot \frac{\sqrt{3}}{3}} = \frac{24 - 5\sqrt{3}}{15 + 8\sqrt{3}}.$$

$$443. \operatorname{tg} \alpha = \frac{1}{2}, \quad \operatorname{tg} \beta = \frac{1}{3};$$

$$\text{a) } \operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = \frac{5}{6} \cdot \frac{6}{5} = 1;$$

$$\text{б) } \operatorname{tg}(\alpha - \beta) = \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{6}} = \frac{1}{6} \cdot \frac{6}{7} = \frac{1}{7}.$$

$$444. \operatorname{tg} \alpha = \frac{2}{5}, \quad \operatorname{tg}\left(\frac{\pi}{2} + \beta\right) = -3, \quad -\operatorname{ctg} \beta = -3, \quad \operatorname{tg} \beta = \frac{1}{3};$$

$$\text{a) } \operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{\frac{2}{5} + \frac{1}{3}}{1 - \frac{2}{15}} = \frac{11}{15} \cdot \frac{15}{13} = \frac{11}{13};$$

$$\text{б) } \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{\frac{2}{5} - \frac{1}{3}}{1 + \frac{2}{15}} = \frac{1}{15} \cdot \frac{15}{17} = \frac{1}{17}.$$

$$445. \text{ а) } \frac{\operatorname{tg} 2,22 + \operatorname{tg} 0,92}{1 - \operatorname{tg} 2,22 \cdot \operatorname{tg} 0,92} = \operatorname{tg}(2,22 + 0,92) = \operatorname{tg} 3,14;$$

$$\text{ б) } \frac{\operatorname{tg} 1,47 - \operatorname{tg} 0,69}{1 + \operatorname{tg} 1,47 \cdot \operatorname{tg} 0,69} = \operatorname{tg}(1,47 - 0,69) = \operatorname{tg} 0,78.$$

$$446. \text{ а) } \frac{\operatorname{tg} x + \operatorname{tg} 3x}{1 - \operatorname{tg} x \operatorname{tg} 3x} = 1, \operatorname{tg} 4x = 1, 4x = \frac{\pi}{4} + \pi n, x = \frac{\pi}{16} + \frac{\pi n}{4};$$

$$\text{ б) } \frac{\operatorname{tg} 5x - \operatorname{tg} 3x}{1 + \operatorname{tg} 5x \operatorname{tg} 3x} = \sqrt{3}, 2x = \frac{\pi}{3} + \pi n, x = \frac{\pi}{6} + \frac{\pi n}{2}.$$

$$447. \text{ а) } \operatorname{tg}\left(\alpha - \frac{\pi}{4}\right) = 3, \frac{\operatorname{tg} \alpha - 1}{1 + \operatorname{tg} \alpha} = 3, \operatorname{tg} \alpha - 1 = 3 + 3 \operatorname{tg} \alpha, 2 \operatorname{tg} \alpha = -4, \operatorname{tg} \alpha = -2;$$

$$\text{ б) } \operatorname{tg}\left(\alpha + \frac{\pi}{4}\right) = \frac{1}{5}, \frac{\operatorname{tg} \alpha + 1}{1 - \operatorname{tg} \alpha} = \frac{1}{5}, 5 \operatorname{tg} \alpha + 5 = 1 - \operatorname{tg} \alpha, \operatorname{tg} \alpha = -\frac{2}{3}, \operatorname{ctg} \alpha = -\frac{3}{2}.$$

$$448. \text{ а) } \operatorname{tg} \alpha = 3, \operatorname{tg}(\alpha + \beta) = 1, \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = 1, 3 + \operatorname{tg} \beta = 1 - 3 \operatorname{tg} \beta, \operatorname{tg} \beta = -\frac{1}{2};$$

$$\text{ б) } \operatorname{tg} \alpha = \frac{1}{4}, \operatorname{tg}(\alpha - \beta) = 2, \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} = 2, \frac{1}{4} - \operatorname{tg} \beta = 2 + \frac{1}{2} \operatorname{tg} \beta, \frac{3}{2} \operatorname{tg} \beta = -\frac{7}{4}, \operatorname{tg} \beta = -\frac{7}{6}.$$

$$449. \sin \alpha = -\frac{12}{13}, \quad \pi < \alpha < \frac{3\pi}{2}, \quad \cos \alpha = -\frac{5}{13}, \quad \operatorname{tg} \alpha = \frac{12}{5};$$

$$\text{ а) } \operatorname{tg}\left(\alpha + \frac{\pi}{4}\right) = \frac{\operatorname{tg} \alpha + 1}{1 - \operatorname{tg} \alpha} = \frac{\frac{12}{5} + 1}{1 - \frac{12}{5}} = -\frac{17}{5} \cdot \frac{5}{7} = -\frac{17}{7},$$

$$\text{ б) } \operatorname{tg}\left(\alpha - \frac{\pi}{4}\right) = \frac{\operatorname{tg} \alpha - 1}{1 + \operatorname{tg} \alpha} = \frac{\frac{12}{5} - 1}{1 + \frac{12}{5}} = \frac{17}{5} \cdot \frac{5}{7} = \frac{17}{7}.$$

$$450. \cos \alpha = \frac{3}{5}, \quad 0 < \alpha < \frac{\pi}{2}, \quad \sin \alpha = \frac{4}{5}, \quad \operatorname{tg} \alpha = \frac{4}{3};$$

$$\text{ а) } \operatorname{tg}\left(\alpha + \frac{\pi}{3}\right) = \frac{\operatorname{tg} \alpha + \sqrt{3}}{1 - \sqrt{3} \operatorname{tg} \alpha} = \frac{\frac{4}{3} + \sqrt{3}}{1 - \frac{4}{\sqrt{3}}} = \frac{4 + 3\sqrt{3}}{3}.$$

$$\frac{\sqrt{3}}{\sqrt{3} - 4} = \frac{4\sqrt{3} + 9}{3\sqrt{3} - 12} = -\frac{48 + 25\sqrt{3}}{39};$$

$$\text{ б) } \operatorname{tg}\left(\alpha - \frac{\pi}{3}\right) = \frac{\operatorname{tg} \alpha - \sqrt{3}}{1 + \sqrt{3} \operatorname{tg} \alpha} = \frac{\frac{4}{3} - \sqrt{3}}{3} \cdot \frac{3}{3 + 4\sqrt{3}} = \frac{4 - 3\sqrt{3}}{3 + 4\sqrt{3}}.$$

$$451. \text{ а) } \frac{\operatorname{tg}\left(\frac{\pi}{8} + \alpha\right) + \operatorname{tg}\left(\frac{\pi}{8} - \alpha\right)}{1 - \operatorname{tg}\left(\frac{\pi}{8} + \alpha\right) \operatorname{tg}\left(\frac{\pi}{8} - \alpha\right)} = \operatorname{tg}\left(\frac{\pi}{8} + \alpha + \frac{\pi}{8} - \alpha\right) = \operatorname{tg} \frac{\pi}{4} = 1;$$

$$\text{ б) } \frac{\operatorname{tg}(45^\circ + \alpha) - \operatorname{tg} \alpha}{1 + \operatorname{tg}(45^\circ + \alpha) \operatorname{tg} \alpha} = \operatorname{tg}(45^\circ + \alpha - \alpha) = \operatorname{tg} 45^\circ = 1.$$

$$452. \text{ а) } \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha} = \operatorname{tg}(45^\circ - \alpha), \quad \operatorname{tg}(45^\circ - \alpha) = \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha};$$

$$\text{ б) } \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg}(\alpha + \beta)} + \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta)} = 2,$$

$$\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg}(\alpha + \beta)} + \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta)} = 1 - \operatorname{tg} \alpha \operatorname{tg} \beta + 1 + \operatorname{tg} \alpha \operatorname{tg} \beta = 2.$$

$$453. \text{ а) } \operatorname{tg}\left(\frac{3\pi}{4} - x\right) + \operatorname{tg} x = \operatorname{tg}\left(\frac{3\pi}{4} - x\right) \operatorname{tg} x - 1,$$

$$\frac{\operatorname{tg} x + 1}{\operatorname{tg} x - 1} + \operatorname{tg} x = \frac{\operatorname{tg} x + 1}{\operatorname{tg} x - 1} \operatorname{tg} x - 1, \quad \frac{\operatorname{tg}^2 x + 1}{\operatorname{tg} x - 1} = \frac{\operatorname{tg}^2 x + \operatorname{tg} x - \operatorname{tg} x + 1}{\operatorname{tg} x - 1};$$

$$\text{ б) } \operatorname{tg}\left(\alpha + \frac{\pi}{4}\right) - \operatorname{tg} \alpha = 1 + \operatorname{tg}\left(\frac{\pi}{4} + \alpha\right) \operatorname{tg} \alpha,$$

$$\frac{\operatorname{tg} \alpha + 1}{1 - \operatorname{tg} \alpha} - \operatorname{tg} \alpha = \frac{-\operatorname{tg} \alpha + \operatorname{tg}^2 \alpha + \operatorname{tg} \alpha + 1}{1 - \operatorname{tg} \alpha} = \frac{1 - \operatorname{tg} \alpha + \operatorname{tg}^2 \alpha + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha}.$$

$$\frac{\operatorname{tg}^2 \alpha + 1}{1 - \operatorname{tg} \alpha} = \frac{\operatorname{tg}^2 \alpha + 1}{1 - \operatorname{tg} \alpha}.$$

$$454. \text{ а) } \operatorname{tg}(\alpha + \beta) - (\operatorname{tg} \alpha + \operatorname{tg} \beta) = \operatorname{tg}(\alpha + \beta) \operatorname{tg} \alpha \operatorname{tg} \beta,$$

$$\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg}^2 \alpha \operatorname{tg} \beta - \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg}^2 \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{\operatorname{tg}^2 \alpha \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg}^2 \beta}{1 - \operatorname{tg} \alpha} = \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg}(\alpha + \beta);$$

$$\text{ б) } \operatorname{tg}(\alpha - \beta) - (\operatorname{tg} \alpha - \operatorname{tg} \beta) = \operatorname{tg}(\beta + \alpha) - \operatorname{tg} \alpha \operatorname{tg} \beta,$$

$$\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta - \operatorname{tg} \alpha - \operatorname{tg}^2 \alpha \operatorname{tg} \beta + \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg}^2 \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} = \operatorname{tg}(\beta - \alpha) \operatorname{tg} \alpha \operatorname{tg} \beta.$$

$$455. \text{ а) } \frac{\sqrt{3} - \operatorname{tg} x}{1 + \sqrt{3} \operatorname{tg} x} = 1, \quad \operatorname{tg}\left(\frac{\pi}{3} - x\right) = 1, \quad x - \frac{\pi}{3} = -\frac{\pi}{4} + \pi n, \quad x = \frac{\pi}{12} + \pi n;$$

$$\text{ б) } \frac{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} 2x}{\operatorname{tg} \frac{\pi}{4} \operatorname{tg} 2x + 1} = \sqrt{3}, \quad \operatorname{tg}\left(2x - \frac{\pi}{5}\right) = -\sqrt{3}, \quad 2x - \frac{\pi}{5} = -\frac{\pi}{3} + \pi n, \quad x = -\frac{\pi}{15} + \frac{\pi n}{2}.$$

$$456. \operatorname{tg} 2x = \operatorname{tg}(x + x) = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}.$$

$$457. \alpha - \beta = \frac{\pi}{4}; \quad \alpha = \frac{\pi}{4} + \beta; \quad \beta = \alpha - \frac{\pi}{4};$$

$$а) \frac{1 + \operatorname{tg} \beta}{1 - \operatorname{tg} \beta} = \operatorname{tg} \alpha, \operatorname{tg} \alpha = \operatorname{tg} \left(\beta + \frac{\pi}{4} \right) = \frac{\operatorname{tg} \beta + 1}{1 - \operatorname{tg} \beta};$$

$$б) \frac{\operatorname{tg} \alpha - 1}{\operatorname{tg} \alpha + 1} = \operatorname{tg} \beta, \operatorname{tg} \beta = \operatorname{tg} \left(\alpha - \frac{\pi}{4} \right) = \frac{\operatorname{tg} \alpha - 1}{1 + \operatorname{tg} \alpha}.$$

$$458. \frac{\operatorname{tg}(\alpha - \beta) - \operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta) \operatorname{tg} \beta} = \frac{\operatorname{tg}(\beta - \alpha) \operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta) \operatorname{tg} \beta} = -\operatorname{tg} \alpha.$$

$$459. \frac{\operatorname{tg} \frac{\pi}{5} + \operatorname{tg} x}{1 - \operatorname{tg} \frac{\pi}{5} \operatorname{tg} x} < 1$$

$$а) \operatorname{tg} \left(x + \frac{\pi}{5} \right) < 1, x + \frac{\pi}{5} \in \left(-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n \right), x \in \left(-\frac{7\pi}{10} + \pi n; \frac{\pi}{20} + \pi n \right);$$

$$б) \frac{\operatorname{tg} 3x - 1}{\operatorname{tg} 3x + 1} > 1, \operatorname{tg} \left(3x - \frac{\pi}{4} \right) > 1, 3x - \frac{\pi}{4} \in \left(\frac{\pi}{4} + \pi n; \frac{\pi}{2} + \pi n \right)$$

$$x \in \left(\frac{\pi}{6} + \frac{\pi n}{3}; \frac{\pi}{4} + \frac{\pi n}{3} \right).$$

$$560. y_1 = 3x + 1; \quad y_2 = 6 - 2x$$

$\operatorname{tg} y_1 = 3$ – тангенс угла наклона 1-ой прямой;

$\operatorname{tg} y_2 = -2$ – тангенс угла наклона 2-ой прямой;

$$y_1 = \operatorname{arctg} 3$$

$$y_2 = -\operatorname{arctg} 2$$

$$y_1 - y_2 = \operatorname{arctg} 3 + \operatorname{arctg} 2$$

$$\operatorname{tg}(y_1 - y_2) = \frac{3 + 2}{1 - 6} = -1$$

$$y_1 - y_2 = \frac{3\pi}{4}.$$

$$461. \operatorname{tg} \angle KBC = \frac{1}{2}, \angle KBC = \operatorname{arctg} \frac{1}{2} - \text{см. рис. 144}$$

$$\angle BKC = 90^\circ - \angle KBC$$

$$\angle KOC = 180^\circ - 45^\circ - 90^\circ + \angle KBC = 45^\circ + \angle KBC.$$

$$\operatorname{tg} \angle KOC = \operatorname{tg} \left(45^\circ + \operatorname{arctg} \frac{1}{2} \right) = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = \frac{3}{2} \cdot 2 = 3$$

$$\operatorname{tg} \angle KOC = \operatorname{arctg} 3.$$

§24. Формулы двойного аргумента

$$462. а) \frac{\sin 2t}{\cos t} - \sin t = 2 \sin t - \sin t = \sin t; \quad б) \frac{\sin 6t}{\cos^2 3t} = 2 \operatorname{tg} 3t;$$

$$в) \cos^2 t - \cos 2t = \cos^2 t - \cos^2 t + \sin^2 t = \sin^2 t;$$

$$r) \frac{\cos 2t}{\cos t - \sin t} - \sin t = \frac{(\cos t - \sin t)(\cos t + \sin t)}{\cos t - \sin t} - \sin t = \cos t.$$

$$463. a) \frac{\sin 40^\circ}{\sin 20^\circ} = 2 \cos 20^\circ;$$

$$б) \frac{\cos 80^\circ}{\cos 40^\circ + \sin 40^\circ} = \cos 40^\circ - \sin 40^\circ;$$

$$в) \frac{\sin 100^\circ}{2 \cos 50^\circ} = \sin 50^\circ;$$

$$r) \frac{\cos 36^\circ + \sin^2 18^\circ}{\cos 18^\circ} = \frac{\cos^2 18^\circ}{\cos 18^\circ} = \cos 18^\circ.$$

$$464. a) 2 \sin 15^\circ \cos 15^\circ = \sin 30^\circ = \frac{1}{2};$$

$$б) (\cos 75^\circ - \sin 75^\circ)^2 = 1 - 2 \sin 75^\circ \cos 75^\circ = 1 - \sin 150^\circ = \frac{1}{2};$$

$$в) \cos^2 15^\circ - \sin^2 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2};$$

$$r) (\cos 15^\circ + \sin 15^\circ)^2 = 1 + 2 \sin 15^\circ \cos 15^\circ = \frac{3}{2}.$$

$$465. a) 2 \sin \frac{\pi}{8} \cos \frac{\pi}{8} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}; \quad б) \sin \frac{\pi}{8} \cos \frac{\pi}{8} + \frac{1}{4} = \frac{\sqrt{2}}{2} + \frac{1}{4} = \frac{\sqrt{2} + 1}{4};$$

$$в) \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}; \quad r) \frac{\sqrt{2}}{2} - \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right)^2 = \frac{\sqrt{2}}{2} - 1 - \sin \frac{\pi}{4} = -1.$$

$$466. a) \frac{2 \operatorname{tg} 15^\circ}{1 - \operatorname{tg}^2 15^\circ} = \operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3}; \quad б) \frac{\operatorname{tg} \frac{\pi}{8}}{1 - \operatorname{tg}^2 \frac{\pi}{8}} = \frac{1}{2} \operatorname{tg} \frac{\pi}{8} = \frac{1}{2};$$

$$в) \frac{\operatorname{tg} 75^\circ}{1 - \operatorname{tg}^2 75^\circ} = -\operatorname{tg} \frac{\pi}{3} = -\sqrt{3}; \quad r) \frac{2 \operatorname{tg} \frac{\pi}{6}}{\operatorname{tg}^2 \frac{\pi}{6} - 1} = -\operatorname{tg} 150^\circ = -\frac{\sqrt{3}}{3}.$$

$$467. a) \sin \frac{x}{2} \cos \frac{x}{2} = \frac{1}{2} \sin x, \quad \frac{1}{2} \sin x = \sin \frac{x}{2} \cos \frac{x}{2};$$

$$б) \cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} = \cos \frac{x}{2}, \quad \cos \frac{x}{2} = \cos^2 \frac{x}{4} - \sin^2 \frac{x}{4};$$

$$в) \sin 2x \cos 2x = \frac{1}{2} \sin 4x, \quad \frac{1}{2} \sin 4x = \sin 2x \cos 2x;$$

$$r) \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x, \quad \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}.$$

$$468. a) \cos(2\alpha + 2\beta) = \cos^2(\alpha + \beta) - \sin^2(\alpha + \beta),$$

$$\cos(2\alpha + 2\beta) = \cos 2(\alpha + \beta) = \cos^2(\alpha + \beta) - \sin^2(\alpha + \beta);$$

$$б) \sin(2\alpha + 2\beta) = 2 \sin(\alpha + \beta) \cos(\alpha + \beta),$$

$$\sin(2\alpha + 2\beta) = \sin 2(\alpha + \beta) = 2 \sin(\alpha + \beta) \cos(\alpha + \beta).$$

$$469. a) \operatorname{tg}(2\alpha + 2\beta) = \frac{2 \operatorname{tg}(\alpha + \beta)}{1 - \operatorname{tg}^2(\alpha + \beta)}, \quad \operatorname{tg}(2\alpha + 2\beta) = \operatorname{tg} 2(\alpha + \beta) = \frac{2 \operatorname{tg}(\alpha + \beta)}{1 - \operatorname{tg}^2(\alpha + \beta)};$$

$$6) \operatorname{tg}(\alpha + \beta) = \frac{2 \operatorname{tg}\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)}{1 - \operatorname{tg}^2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)}, \quad \operatorname{tg}(\alpha + \beta) = \operatorname{tg} 2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \frac{2 \operatorname{tg}\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)}{1 - \operatorname{tg}^2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)}.$$

$$470. \sin t = \frac{5}{13}, \quad \frac{\pi}{2} < t < \pi, \quad \cos t = -\frac{12}{13}, \quad \operatorname{tg} t = \frac{5}{13} \left(-\frac{13}{12}\right) = -\frac{5}{12};$$

$$a) \sin 2t = 2 \sin t \cos t = 2 \cdot \frac{5}{13} \cdot \left(-\frac{12}{13}\right) = -\frac{120}{169};$$

$$6) \cos 2t = \cos^2 t - \sin^2 t = \frac{144}{169} - \frac{25}{169} = -\frac{119}{169};$$

$$b) \operatorname{tg} 2t = \frac{2 \operatorname{tg} t}{1 - \operatorname{tg}^2 t} = \frac{2 \cdot \left(-\frac{5}{12}\right)}{1 - \frac{25}{144}} = -\frac{5}{6} \cdot \frac{144}{119} = -\frac{120}{119};$$

$$r) \operatorname{ctg} 2t = \frac{1}{\operatorname{tg} 2t} = -\frac{119}{120}.$$

$$471. \cos x = \frac{4}{5}, \quad 0 < x < \frac{\pi}{2}, \quad \sin x = \frac{3}{5}, \quad \operatorname{tg} x = \frac{3}{4}, \quad \operatorname{ctg} x = \frac{4}{3};$$

$$a) \sin 2x = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25};$$

$$6) \cos 2x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25};$$

$$b) \operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}; \quad r) \operatorname{ctg} 2x = \frac{7}{24}.$$

$$472. a) \frac{\sin t}{2 \cos^2 \frac{t}{2}} = \frac{2 \sin \frac{t}{2} \left(\cos \frac{t}{2}\right)}{2 \cos^2 \frac{t}{2}} = \operatorname{tg} \frac{t}{2};$$

$$6) \frac{\cos t}{\cos \frac{t}{2} + \sin \frac{t}{2}} = \frac{\cos^2 \frac{t}{2} - \sin^2 \frac{t}{2}}{\cos \frac{t}{2} + \sin \frac{t}{2}} = \cos \frac{t}{2} - \sin \frac{t}{2};$$

$$b) \frac{\sin 4t}{\cos 2t} = \frac{2 \sin 2t \cos 2t}{\cos 2t} = 2 \sin 2t;$$

$$r) \frac{\cos 2t - \sin 2t}{\cos 4t} = \frac{\cos 2t - \sin 2t}{(\sin 2t - \cos 2t)(\cos 2t + \sin 2t)} = \frac{1}{\cos 2t + \sin 2t}.$$

$$473. a) \frac{\sin 2t - 2 \sin t}{\cos t - 1} = \frac{2 \sin t (\cos t - 1)}{\cos t - 1} = 2 \sin t;$$

$$6) \frac{\cos 2t - \cos^2 t}{1 - \cos^2 t} = \frac{\cos^2 t - \sin^2 t - \cos^2 t}{\sin^2 t} = -1;$$

$$\text{в) } \sin 2t \operatorname{ctg} t - 1 = 2 \sin t \cos t \frac{\cos t}{\sin t} - 1 = 2 \cos^2 t - 1 = \cos 2t ;$$

$$\text{г) } (\operatorname{tg} t + \operatorname{ctg} t) \sin 2t = \frac{1}{\sin t \cos t} 2 \sin t \cos t = 2 .$$

$$474. \text{ а) } \frac{2}{\operatorname{tg} t + \operatorname{ctg} t} = \frac{2}{\frac{1}{\sin t \cos t}} = \sin 2t ; \text{ б) } \frac{2}{\operatorname{tg} t - \operatorname{ctg} t} = \frac{2}{\frac{-\cos 2t}{\sin t \cos t}} = \frac{\sin 2t}{-\cos 2t} = -\operatorname{tg} 2t .$$

$$475. \text{ а) } (1 - \operatorname{tg}^2 t) \cos^2 t = \cos^2 t - \sin^2 t = \cos 2t ;$$

$$\text{б) } 2 \cos^2 \frac{\pi + t}{4} - 2 \sin^2 \frac{\pi + t}{4} = 2 \cos \left(\frac{\pi}{2} + \frac{t}{2} \right) = -2 \sin \frac{t}{2} .$$

$$476. \text{ а) } (\sin t - \cos t)^2 = 1 - \sin 2t, \sin^2 t - 2 \sin t \cos t + \cos^2 t = 1 - \sin 2t ;$$

$$\text{б) } 2 \cos^2 t = 1 + \cos 2t, 1 + \cos 2t = \sin^2 t + \cos^2 t - \sin^2 t + \cos^2 t = 2 \cos^2 t ;$$

$$\text{в) } (\sin t + \cos t)^2 = 1 + \sin 2t, \sin^2 t + 2 \sin t \cos t + \cos^2 t = 1 + \sin 2t ;$$

$$\text{г) } 2 \sin^2 t = 1 - \cos 2t, 1 - \cos 2t = \sin^2 t + \cos^2 t - \cos^2 t + \sin^2 t = 2 \sin^2 t .$$

$$477. \text{ а) } \cos^4 t - \sin^4 t = \cos 2t ,$$

$$\cos 2t = (\cos^2 t - \sin^2 t) \cdot 1 = (\cos^2 t - \sin^2 t) (\cos^2 t + \sin^2 t) = \cos^4 t - \sin^4 t ;$$

$$\text{б) } \cos^4 t + \sin^4 t = 1 - \frac{1}{2} \sin^2 2t ,$$

$$\cos^4 t + \sin^4 t + 2 \sin^2 t \cos^2 t - 2 \sin^2 t \cos^2 t = (\sin^2 t + \cos^2 t)^2 - \frac{1}{2} \sin^2 2t = 1 - \frac{1}{2} \sin^2 2t .$$

$$478. \text{ а) } \operatorname{ctg} t - \sin 2t = \operatorname{ctg} t \cos 2t ,$$

$$\frac{\cos t - 2 \sin^2 t \cos t}{\sin t} = \frac{\cos t (\cos^2 t - \sin^2 t)}{\sin t} = \frac{\cos t \cos 2t}{\sin t} = \operatorname{ctg} t \cos 2t ;$$

$$\text{б) } \sin 2t - \operatorname{tg} t = \cos 2t \operatorname{tg} t ,$$

$$\frac{2 \sin t \cos^2 t - \sin t}{\cos t} = \frac{(\cos^2 t - \sin^2 t) \sin t}{\cos t} = \frac{\cos 2t \sin t}{\cos t} = \cos 2t \operatorname{tg} t .$$

$$479. \text{ а) } \sin 2x - 2 \cos x = 0$$

$$2 \cos x = (\sin x - 1) = 0, \cos x = 0, x = \frac{\pi}{2} + \pi n, \sin x = 1, x = \frac{\pi}{2} + 2\pi n ;$$

$$\text{б) } 2 \sin x = \sin 2x, 2 \sin (\cos x - 1) = 0,$$

$$\sin x = 0, \cos x = 1, x = \pi n, x = 2\pi n ;$$

$$\text{в) } \sin 2x - \sin x = 0, \sin x (2 \cos x - 1) = 0, \sin x (2 \cos x - 1) = 0,$$

$$\sin x = 0, x = \pi n, \cos x = \frac{1}{2}, x = \pm \frac{\pi}{3} + 2\pi n ;$$

$$\text{г) } \sin 2x - \cos x = 0, \cos x (2 \sin x - 1) = 0, \cos x = 0,$$

$$x = \frac{\pi}{2} + \pi n, \sin x = \frac{1}{2}, x = (-1)^k \frac{\pi}{6} + \pi k .$$

$$480. \text{ а) } \sin x \cos x = 1,$$

$$\frac{1}{2} \sin x \cos x = \frac{1}{2}, \sin 2x = \frac{1}{2}, 2x = (-1)^n \frac{\pi}{6} + 2\pi n, x = (-1)^n \frac{\pi}{12} + \pi n ,$$

$$\sin 2x = 2, \text{ решений нет;}$$

$$б) \sin 4x \cos 4x = \frac{1}{2},$$

$$\sin 8x = 1, 8x = \frac{\pi}{2} + \pi n, x = \frac{\pi}{16} + \frac{\pi n}{8};$$

$$в) \cos^2 \frac{x}{3} - \sin^2 \frac{x}{3} = \frac{1}{2},$$

$$\cos \frac{2x}{3} = \frac{1}{2}, \frac{2x}{3} = \pm \frac{\pi}{3} + 2\pi n, x = \pm \frac{\pi}{2} + 3\pi n;$$

$$г) \sin^2 x - \cos^2 x = \frac{1}{2},$$

$$\cos 2x = -\frac{1}{2}, 2x = \pm \frac{2\pi}{3} + 2\pi n, x = \pm \frac{\pi}{3} + \pi n.$$

$$481. а) \cos 2x + 3 \sin x = 1,$$

$$1 - 2 \sin^2 x + 3 \sin x = 1, \sin x (2 \sin x - 3) = 0, \sin x = 0, x = \pi n;$$

$$б) \sin^2 x = -\cos 2x,$$

$$\sin^2 x + 1 - 2 \sin^2 x = 0, \sin^2 x = 1, \sin x = \pm 1, x = \frac{\pi}{2} + \pi n;$$

$$в) \cos 2x = \cos^2 2x, 2 \cos^2 2x - 1 - \cos^2 x = 0, \cos^2 2x = 1, \cos x = 1, x = \pi n;$$

$$г) 2 \cos 2x = 2 \sin^2 x, 1 - 2 \sin^2 x = 2 \sin^2 x, \sin x = \pm \frac{1}{2}, x = (-1)^k \frac{\pi}{6} + \pi n;$$

$$482. а) \sin 11^\circ 15' \cos 11^\circ 15' \cos 22^\circ 30' \cos 45^\circ =$$

$$= \frac{1}{2} \sin 22^\circ 30' \cos 22^\circ 30' \cos 45^\circ = \frac{1}{8} \sin 90^\circ = \frac{1}{8};$$

$$б) \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} = \frac{1}{8} \sin \frac{\pi}{6} = \frac{1}{16}.$$

$$483. а) \frac{1 + \cos 40^\circ + \cos 80^\circ}{\sin 80^\circ + \sin 40^\circ} \operatorname{tg} 40^\circ = \frac{2 \cos^2 40^\circ + \cos 40^\circ}{\sin 40^\circ (2 \cos 40^\circ + 1)} \operatorname{tg} 40^\circ = \operatorname{tg} 40^\circ \operatorname{ctg} 40^\circ = 1;$$

$$б) \frac{1 - \cos 25^\circ + \cos 50^\circ}{\sin 50^\circ - \sin 25^\circ} - \operatorname{tg} 65^\circ = \frac{2 \cos^2 25^\circ - \cos 25^\circ}{2 \sin 25^\circ \cos 25^\circ - \sin 25^\circ} - \operatorname{tg} 25^\circ =$$

$$= \frac{\cos 25^\circ \cdot 2 \cos 25^\circ - 1}{\sin 25^\circ \cdot 2 \cos 25^\circ - 1} - \operatorname{tg} 65^\circ = \operatorname{ctg} 25^\circ - \operatorname{tg} 25^\circ = \operatorname{tg} 65^\circ - \operatorname{tg} 65^\circ = 0.$$

$$484. \operatorname{tg} x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}, \cos x = -\frac{4}{5}, \sin x = -\frac{3}{5};$$

$$а) \sin 2x = 2 \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{24}{25};$$

$$б) \cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25};$$

$$в) \operatorname{tg} 2x = \frac{24}{25} \cdot \frac{25}{7} = \frac{24}{7};$$

$$г) \operatorname{ctg} 2x = \frac{7}{24}.$$

$$485. \operatorname{ctg} x = -\frac{4}{3}, \frac{3\pi}{2} < x < 2\pi, \operatorname{tg} x = -\frac{3}{45}, \cos x = \frac{4}{5}, \sin x = -\frac{3}{5};$$

$$а) \sin 2x = -2 \cdot \frac{4}{5} \cdot \frac{3}{5} = -\frac{24}{25};$$

$$б) \cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25};$$

$$\text{в) } \operatorname{tg} 2x = -\frac{24}{25} \cdot \frac{25}{7} = -\frac{24}{7}; \quad \text{г) } \operatorname{ctg} 2x = -\frac{7}{24}.$$

$$486. \text{ а) } \frac{\cos 2t}{\sin t \cos t + \sin^2 t} = \operatorname{ctg}(\pi + t) - 1,$$

$$\frac{\cos 2t}{\sin t(\cos t + \sin t)} = \frac{\cos t - \sin t}{\sin t} = \operatorname{ctg} t - 1 = \operatorname{ctg}(\pi + t) - 1;$$

$$\text{б) } (\operatorname{ctg} t - \operatorname{tg} t) \sin 2t = 2 \cos 2t,$$

$$\frac{\cos t}{\sin t} 2 \sin t \cos t - \frac{\sin t}{\cos t} 2 \sin t \cos t = 2 \cos^2 t - 2 \sin^2 t = 2 \cos 2t.$$

$$487. \text{ а) } \frac{\sin 2t - 2 \sin\left(\frac{\pi}{2} - t\right)}{\cos\left(\frac{\pi}{2} - t\right) - 2 \sin^2 t} = -2 \operatorname{ctg} t, \quad \frac{2 \cos t(\sin t - 1)}{-\sin t(\sin t - 1)} = -2 \operatorname{ctg} t;$$

$$\text{б) } \frac{1 - \cos 2t + \sin 2t}{1 + \cos 2t + \sin 2t} \operatorname{tg}\left(\frac{\pi}{2} - t\right) = 1,$$

$$\frac{2 \sin^2 t + \sin 2t}{2 \cos^2 t + \sin 2t} \operatorname{ctg} t = \frac{2 \sin t(\sin t + \cos t)}{2 \cos t(\sin t + \cos t)} \operatorname{ctg} t = 1.$$

$$488. \text{ а) } \sin 2\alpha = \frac{1}{3},$$

$$\sin^4 \alpha + \cos^4 \alpha = 1 - 2 \sin^2 \alpha \cos^2 \alpha = 1 - 2 \cdot \frac{1}{36} = \frac{17}{18};$$

$$\text{б) } \sin^4 \alpha + \cos^4 \alpha = \frac{49}{50}, \quad \frac{\pi}{2} < \alpha < \pi,$$

$$1 - 2 \sin^2 \alpha \cos^2 \alpha = \frac{49}{50}, \quad \sin^2 \alpha \cos^2 \alpha = \frac{1}{100}, \quad \sin \alpha \cos \alpha = -\frac{1}{10}, \quad \sin 2\alpha = -\frac{1}{5}.$$

$$489. \text{ а) } \sin 3x = 3 \sin x - 4 \sin^3 x,$$

$$\sin 3x = \sin(x + 2x) = \sin x \cos 2x + \sin 2x \cos x = \sin x - 2 \sin^3 x + 2 \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x.$$

$$\text{б) } \cos 3x = 4 \cos^3 x - 3 \cos x,$$

$$\cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x = 2 \cos^3 x - \cos x - 2 \cos x \sin x = 2 \cos^3 x - \cos x - 2 \cos x + 2 \cos^3 x = 4 \cos^3 x - 3 \cos x.$$

$$490. \text{ а) } \cos x \cos 2x = \frac{\sin 4x}{4 \sin x}; \quad \frac{\sin 4x}{4 \sin x} = \frac{2 \sin 2x \cos 2x}{4 \sin x} = \cos x \cos 2x;$$

$$\text{б) } \cos x \cos 2x \cos 4x = \frac{\sin 8x}{8 \sin x};$$

$$\frac{2 \sin 4x \cos 4x}{8 \sin x} = \frac{8 \sin x \cos x \cos 2x \cos 4x}{8 \sin x} = \cos x \cos 2x \cos 4x;$$

$$\text{в) } \sin x \cos 2x = \frac{\sin 4x}{4 \cos x}; \quad \frac{\sin 4x}{4 \cos x} = \frac{4 \sin x \cos x \cos 2x}{4 \cos x} = \sin x \cos 2x;$$

$$\text{г) } \sin x \cos 2x \cos 4x = \frac{\sin 8x}{8 \cos x};$$

$$\frac{\sin 8x}{8 \cos x} = \frac{8 \sin x \cos x \cos 2x \cos 4x}{8 \cos x} = \sin x \cos 2x \cos 4x.$$

$$491. \text{ а) } \sin 18^\circ \cos 18^\circ \cos 36^\circ = \frac{1}{4} \sin 72^\circ, \quad \frac{1}{2} \sin 36^\circ \cos 36^\circ = \frac{1}{4} \sin 72^\circ;$$

$$\text{б) } \sin 18^\circ \cos 36^\circ = \frac{1}{4}; \text{ т.к. } \sin 18^\circ \cos 18^\circ \cos 36^\circ = \frac{1}{4} \sin 72^\circ,$$

$$\sin 18^\circ \cos 18^\circ \cos 36^\circ = \frac{1}{4} \cos 18^\circ \Rightarrow \sin 18^\circ \cos 36^\circ = \frac{1}{4}.$$

$$492. \text{ а) } \cos \frac{\pi}{33} \cos \frac{2\pi}{33} \cos \frac{4\pi}{33} \cos \frac{8\pi}{33} \cos \frac{16\pi}{33} = \frac{\sin \frac{32\pi}{33}}{32 \sin \frac{\pi}{33}} = \frac{1}{32};$$

$$\text{б) } \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{1}{8}.$$

$$493. \text{ а) } 2 - \cos 2x + 3 \sin x = 0,$$

$$2 \sin^2 x - 1 + 2 + 3 \sin x = 0, \quad 2 \sin^2 x + 3 \sin x + 1 = 0,$$

$$\sin x = \frac{-3-1}{4} = -1, \quad x = -\frac{\pi}{2} + 2\pi n, \quad \sin x = -\frac{1}{2}, \quad x = (-1)^{k+1} \frac{\pi}{6} + \pi k;$$

$$\text{б) } \cos 6x - \cos 3x - 2 = 0, \quad 2 \cos^2 3x - \cos 3x - 3 = 0,$$

$$\cos 3x = \frac{1+5}{4} \text{ — решений нет;}$$

$$\cos 3x = -1, \quad x = \frac{\pi}{3} + \frac{2\pi n}{3}.$$

$$494. \text{ а) } 26 \sin x \cos x - \cos 4x + 7 = 0,$$

$$13 \sin 2x - 1 + 2 \sin^2 2x + 7 = 0, \quad 2 \sin^2 2x + 13 \sin 2x + 6 = 0,$$

$$\sin 2x = \frac{-13-11}{4} \text{ — решений нет; } \sin 2x = -\frac{1}{2},$$

$$2x = (-1)^{k+1} \frac{\pi}{6} + \frac{\pi k}{2};$$

$$\text{б) } \sin^4 x + \cos^4 x = \sin x \cos x$$

$$\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x = 2 \sin^2 x \cos^2 x + \frac{1}{2} \sin 2x,$$

$$\frac{1}{2} \sin^2 2x + \frac{1}{2} \sin 2x - 1 = 0,$$

$$1) \sin 2x = 1, \quad 2x = \frac{\pi}{2} + 2\pi n, \quad x = \frac{\pi}{4} + \pi n;$$

$$2) \sin 2x = -2 \text{ — решений нет.}$$

$$495. \text{ а) } 3 \sin 2x + \cos 2x = 1,$$

$$6 \sin x \cos x - 2 \sin^2 x = 0, \quad 2 \sin x (3 \cos x - \sin x) = 0,$$

$$\sin x = 3 \cos x, \quad \operatorname{tg} x = 3, \quad x = \operatorname{arctg} 3 + \pi n, \quad \sin x = 0, \quad x = \pi n;$$

$$6) \cos 4x + 2\sin 4x = 1,$$

$$-2\sin^2 2x + 4\sin 2x \cos 2x = 0, \sin 2x (2\cos 2x - \sin 2x) = 0,$$

$$\cos 2x = \frac{1}{2} \sin 2x, \operatorname{tg} 2x = 2, 2x = \arctg 2 + \pi n, x = \frac{1}{2} \arctg 2 + \frac{\pi n}{2},$$

$$\sin 2x = 0, 2x = \pi n, x = \frac{\pi n}{2}.$$

$$496. \text{ а) } 4\sin x + \sin 2x = 0, x \in [0; 2\pi],$$

$$2\sin x(2 + \cos x) = 0, \sin x = 0, \cos x = -2 \text{ — решений нет}$$

$$x = 0, x = \pi, x = 2\pi;$$

$$6) \cos^2\left(3x + \frac{\pi}{4}\right) - \sin^2\left(3x + \frac{\pi}{4}\right) + \frac{\sqrt{3}}{2} = 0, x \in \left[\frac{3\pi}{4}; \pi\right],$$

$$\cos\left(6x + \frac{2\pi}{4}\right) = -\frac{\sqrt{3}}{2}, \sin 6x = \frac{\sqrt{3}}{2},$$

$$6x = (-1)^k \frac{\pi}{3} + \pi k, x = (-1)^k \frac{\pi}{18} + \frac{\pi k}{6} \Rightarrow x = \frac{7\pi}{18}.$$

$$497. \text{ а) } (\cos x - \sin x)^2 = 1 - 2\sin x = 2x, x \in \left[\frac{20\pi}{9}; \frac{28\pi}{9}\right],$$

$$1 - \sin 2x = 1 - 2\sin 2x, \sin 2x = 2, x = \frac{\pi n}{2}, -2 \text{ корня};$$

$$6) 2\cos^2\left(2x - \frac{\pi}{4}\right) - 2\sin^2\left(2x - \frac{\pi}{4}\right) + 1 = 0, x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right],$$

$$\cos\left(4x - \frac{\pi}{2}\right) = -\frac{1}{2}, \sin 4x = -\frac{1}{2}, 4x = (-1)^{k+1} \frac{\pi}{6} + \pi k,$$

$$x = (-1)^{k+1} \frac{\pi}{24} + \frac{\pi k}{4} - 3 \text{ корня.}$$

$$498. \text{ а) } \sin x = \frac{2\operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} = 2\operatorname{tg} \frac{x}{2} \cos^2 \frac{x}{2} = 2\sin \frac{x}{2} \cos \frac{x}{2} = \sin x;$$

$$6) \cos x = \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} = \left(1 - \operatorname{tg}^2 \frac{x}{2}\right) \cos^2 \frac{x}{2} = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x.$$

$$499. \text{ а) } \sin x + 7\cos x = 5,$$

$$\frac{2\operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} + \frac{7 - 7\operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} = 2\operatorname{tg} \frac{x}{2} + 7 - 7\operatorname{tg}^2 \frac{x}{2} = 5 + 5\operatorname{tg}^2 \frac{x}{2},$$

$$12\operatorname{tg}^2 \frac{x}{2} - 2\operatorname{tg} \frac{x}{2} - 2 = 0, 6\operatorname{tg}^2 \frac{x}{2} - \operatorname{tg} \frac{x}{2} - 1 = 0,$$

$$\operatorname{tg} \frac{x}{2} = \frac{1+5}{12} = \frac{1}{2}, \quad x = \operatorname{arctg} \frac{1}{2} + 2\pi n,$$

$$\operatorname{tg} \frac{x}{2} = \frac{1-5}{12} = -\frac{1}{3}, \quad x = 2 \operatorname{arctg} \left(-\frac{1}{3} \right) + 2\pi n,$$

$$x = -2 \operatorname{arctg} \frac{1}{3} + 2\pi n;$$

$$6) 5 \sin x + 10 \cos x + 2 = 0, \quad \frac{10 \operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} + \frac{10 - 10 \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} + 2 = 0,$$

$$10 \operatorname{tg} \frac{x}{2} + 10 - 10 \operatorname{tg}^2 \frac{x}{2} + 2 + 2 \operatorname{tg}^2 \frac{x}{2} = 0,$$

$$4 \operatorname{tg}^2 \frac{x}{2} - 5 \operatorname{tg}^2 \frac{x}{2} - 6 = 0, \quad \operatorname{tg} \frac{x}{2} = \frac{5+11}{8} = 2, \quad x = 2 \operatorname{arctg} 2 + 2\pi n,$$

$$\operatorname{tg} \frac{x}{2} = -\frac{3}{4}, \quad x = -2 \operatorname{arctg} \frac{3}{4} + 2\pi n.$$

$$500. a) \cos \frac{1}{x^2 - \pi} \sin 2x = 8 \sin x \cos x,$$

$$\cos \frac{1}{x^2 - \pi} \sin 2x = 4 \sin 2x, \quad \sin 2x = 0,$$

$$x = \frac{\pi n}{2}, \quad n \neq \pm 2, \quad \cos \frac{1}{x^2 - \pi} = 4 - \text{решений нет};$$

$$6) 16 \sin x \cos x + \sin 2x \sin \frac{1}{x} = 0, \quad 8 \sin 2x + \sin 2x \sin \frac{1}{x} = 0,$$

$$\sin 2x = 0, \quad x = \frac{\pi n}{2}, \quad x \neq 0, \quad \sin \frac{1}{x} = -8 - \text{решений нет}.$$

$$501. a) \sin 2x + 2 \sin x = 2 - 2 \cos x, \quad \sin x \cos x + \sin x + \cos x = 1,$$

$$(\sin x + \cos x) = t, \quad \sin x \cos x = \frac{1}{2} t^2 - \frac{1}{2},$$

$$t^2 - 1 + 2t - 2 = 0, \quad t^2 + 2t - 3 = 0, \quad t = -3 - \text{решений нет},$$

$$t = 1, \quad \sin x + \cos x = 1, \quad \sin \left(x + \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2},$$

$$x + \frac{\pi}{4} = (-1)^k \frac{\pi}{4} + \pi k, \quad x = (-1)^k \frac{\pi}{4} - \frac{\pi}{4} + \pi k;$$

$$6) 4 \sin 2x + 8(\sin x - \cos x) = 7,$$

$$\sin x - \cos x = t, \quad 1 - \sin 2x = t^2, \quad \sin 2x = 1 - t^2,$$

$$4 - 4t^2 + 8t - 7 = 0, \quad t = \frac{4+2}{4} = \frac{3}{2},$$

$$\sin x + \cos x = \frac{3}{2} - \text{решений нет},$$

$$t = \frac{1}{2}, \quad \sin x + \cos x = \frac{1}{2}, \quad \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{4},$$

$$x + \frac{\pi}{4} = (-1)^k \arcsin \frac{\sqrt{2}}{4} + \pi k, \quad x = (-1)^k \arcsin \frac{\sqrt{2}}{4} + \pi k - \frac{\pi}{4}.$$

$$502. \text{ а) } \sin 2x \cos 2x < \frac{1}{4}, \quad \sin 4x < \frac{1}{2},$$

$$4x \in \left(-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n\right), \quad x \in \left(-\frac{7\pi}{24} + \frac{\pi n}{2}; \frac{\pi}{24} + \frac{\pi n}{2}\right);$$

$$\text{б) } \cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} > \frac{1}{2}, \quad \cos \frac{x}{2} > \frac{1}{2},$$

$$\frac{x}{2} \in \left(-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right), \quad x \in \left(-\frac{2\pi}{3} + 4\pi n; \frac{2\pi}{3} + 4\pi n\right).$$

$$503. \text{ а) } \cos^2 2x - \sin^2 2x \leq -1,$$

$$\cos 4x \leq -1, \quad 4x = \pi + 2\pi n, \quad x = \frac{\pi}{4} + \frac{\pi n}{2};$$

$$\text{б) } \sin 5x \cos 5x \geq \frac{1}{2}, \quad \sin 10x \geq \frac{1}{2}, \quad 10x = \frac{\pi}{2} + 2\pi n, \quad x = \frac{\pi}{20} + \frac{\pi n}{5};$$

$$\text{в) } \sin^2 3x - \cos^2 3x \leq -1, \quad \cos 6x \geq 1, \quad 6x = 2\pi n, \quad x = \frac{\pi n}{3};$$

$$\text{г) } \sin \frac{2x}{3} \cos \frac{2x}{3} \leq -\frac{1}{2}, \quad \sin \frac{4x}{3} \leq -\frac{1}{2}, \quad \frac{4x}{3} = -\frac{\pi}{2} + 2\pi n, \quad x = -\frac{3\pi}{8} + \frac{3\pi n}{2}.$$

§25. Формулы понижения степени

$$504. \text{ а) } \sin^2 2t = \frac{1 - \cos 4t}{2}; \quad \frac{1 - \cos 4t}{2} = \frac{1}{2}(1 - \cos^2 2t + \sin^2 2t) = \frac{1}{2}(2\sin^2 2t) = \sin^2 2t$$

$$\text{б) } 2\sin^2 \frac{t}{2} + \cos t = 1, \quad 2(1 - \cos t) \frac{1}{2} + \cos t = 1;$$

$$\text{в) } 2\sin^2 2t = 1 + \sin\left(\frac{3t}{2} - 4t\right), \quad 1 - \cos 4t = 1 + \sin\left(\frac{3t}{2} - 4t\right);$$

$$\text{г) } 2\cos^2 t - \cos 2t = 1, \quad 2\cos^2 t - \cos 2t = 2\cos^2 t - \cos^2 t + \sin^2 t = 1.$$

$$505. \text{ а) } \cos^2 3t = \frac{1 + \sin\left(\frac{\pi}{2} - 6t\right)}{2}, \quad \cos^2 3t = \frac{1 + \cos 6t}{2} = \frac{1 + \sin\left(\frac{\pi}{3} - 6t\right)}{2};$$

$$\text{б) } \frac{1 - \cos t}{1 + \cos t} = \operatorname{tg}^2 \frac{t}{2}, \quad \operatorname{tg}^2 \frac{t}{2} = \frac{\sin^2 \frac{t}{2}}{\cos^2 \frac{t}{2}} = \frac{1 - \cos t}{1 + \cos t};$$

$$\text{в) } \cos^2 3t = \frac{1 - \cos(6t + (-3\pi))}{2} = \cos^2 3t = \frac{1 + \cos 6t}{2} = \frac{1 - \cos(6t - 3\pi)}{2};$$

$$r) \frac{1 - \cos t}{\sin t} = \operatorname{tg} \frac{t}{2}, \quad \frac{1 - \cos t}{2} = \frac{2 \sin^2 \frac{t}{2}}{2 \sin \frac{t}{2} \cos \frac{t}{2}} = \operatorname{tg} \frac{t}{2}.$$

$$506. a) 1 + \sin \alpha = 2 \cos^2 \left(45^\circ - \frac{1}{2} \right); \quad 2 \cos^2 \left(45^\circ - \frac{\alpha}{2} \right) = 1 + \cos(90^\circ - \alpha) = 1 + \sin \alpha;$$

$$б) 2 \sin^2(45^\circ - \alpha) + \sin 2\alpha = 1; \quad 1 - \cos(90^\circ - 2\alpha) + \sin 2\alpha = 1, \quad 1 - \sin 2\alpha + \sin 2\alpha = 1;$$

$$в) 1 - \sin \alpha = 2 \sin^2 \left(45^\circ - \frac{\alpha}{2} \right); \quad 2 \sin^2 \left(45^\circ - \frac{\alpha}{2} \right) = 1 - \cos(90^\circ - \alpha) = 1 - \sin \alpha;$$

$$r) 2 \cos^2(45^\circ + \alpha) + \sin 2\alpha = 1; \quad 1 + \cos(90^\circ + 2\alpha) + \sin 2\alpha = 1 - \sin 2\alpha + \sin 2\alpha = 1.$$

$$507. a) \sin 22,5^\circ = \sqrt{\frac{1 - \cos 45^\circ}{2}} = \sqrt{\frac{2 - \sqrt{2}}{2}}; \quad б) \cos 22,5^\circ = \sqrt{\frac{1 + \cos 45^\circ}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}};$$

$$в) \sin \frac{3\pi}{8} = \sqrt{\frac{1 - \cos \frac{3\pi}{4}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{2}}; \quad r) \cos \frac{3\pi}{8} = \sqrt{\frac{1 + \cos \frac{3\pi}{4}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}}.$$

$$507. a) 1 - \cos x = 2 \sin^2 \frac{x}{2}, \quad 1 - \left(1 - 2 \sin^2 \frac{x}{2} \right) = 2 \sin^2 \frac{x}{2},$$

$$\sin^2 \frac{x}{2} = \sin^2 \frac{x}{2}, \quad \left(\sin \frac{x}{2} - 1 \right) \sin \frac{x}{2} = 0,$$

$$\begin{cases} \sin \frac{x}{2} = 0 \\ \sin \frac{x}{2} = 1 \end{cases}; \quad \begin{cases} x = 2\pi n \\ x = \pi + 4\pi k \end{cases};$$

$$б) 1 + \cos x = 2 \cos^2 \frac{x}{2},$$

$$1 + \left(2 \cos^2 \frac{x}{2} - 1 \right) = 2 \cos^2 \frac{x}{2}, \quad \cos^2 \frac{x}{2} = \cos^2 \frac{x}{2},$$

$$\left(\cos \frac{x}{2} - 1 \right) \cos \frac{x}{2} = 0, \quad \begin{cases} \cos \frac{x}{2} = 0 \\ \cos \frac{x}{2} = 1 \end{cases}; \quad \begin{cases} x = \pi + 2\pi n \\ x = 4\pi k \end{cases}.$$

$$508. a) 1 - \cos x = \sin x \sin \frac{x}{2}, \quad 1 - \left(1 - 2 \sin^2 \frac{x}{2} \right) = \sin x \sin \frac{x}{2}, \quad 2 \sin^2 \frac{x}{2} = 2 \sin^2 \frac{x}{2} \cos \frac{x}{2},$$

$$\sin^2 \frac{x}{2} \left(1 - \cos \frac{x}{2} \right) = 0, \quad \begin{cases} \sin \frac{x}{2} = 0 \\ \cos \frac{x}{2} = 1 \end{cases}; \quad \begin{cases} x = 2\pi n \\ x = 4\pi k \end{cases}; \quad x = 2\pi n;$$

$$б) \sin x = \operatorname{tg}^2 \frac{x}{2} (1 + \cos x), \sin x = \frac{1 - \cos x}{1 + \cos x} (1 + \cos x), \begin{cases} \sin x = 1 - \cos x \\ 1 + \cos x \neq 0 \end{cases},$$

$$\begin{cases} \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) = 1; \\ x \neq \pi + 2\pi n \end{cases}; \begin{cases} x + \frac{\pi}{4} = \frac{\pi}{4} + 2\pi k \\ x + \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k; \\ x \neq \pi + 2\pi n \end{cases}; \begin{cases} x = 2\pi k \\ x = \frac{\pi}{2} + 2\pi k; \\ x \neq \pi + 2\pi n \end{cases}; \begin{cases} x = 2\pi k \\ x = \frac{\pi}{2} + 2\pi k \end{cases}.$$

$$509. а) \sin^2 2x = 1; \begin{cases} \sin 2x = 1 \\ \sin 2x = -1 \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + 2\pi n \\ 2x = -\frac{\pi}{2} + 2\pi k \end{cases}; 2x = \frac{\pi}{2} + \pi l; x = \frac{\pi}{4} + \frac{\pi l}{2};$$

$$б) \cos^2 4x = \frac{1}{2}, 2\cos^2 4x - 1 = 0, \cos 8x = 0, 8x = \frac{\pi}{2} + \pi n, x = \frac{\pi}{16} + \frac{\pi n}{8};$$

$$в) \sin^2 \frac{x}{2} = \frac{3}{4},$$

$$\begin{cases} \sin \frac{x}{2} = \frac{\sqrt{3}}{2} \\ \sin \frac{x}{2} = -\frac{\sqrt{3}}{2} \end{cases}; \begin{cases} \frac{x}{2} = (-1)^n \frac{\pi}{3} + \pi n \\ \frac{x}{2} = (-1)^k \left(-\frac{\pi}{3} \right) + \pi k \end{cases}; \begin{cases} x = (-1)^n \frac{2\pi}{3} + 2\pi n \\ x = (-1)^k \left(-\frac{2\pi}{3} \right) + 2\pi k \end{cases}; x = \pm \frac{2\pi}{3} + 2\pi n;$$

$$г) \cos^2 \frac{x}{4} = \frac{1}{4},$$

$$\begin{cases} \cos \frac{x}{4} = \frac{1}{2} \\ \cos \frac{x}{4} = -\frac{1}{2} \end{cases}; \begin{cases} \frac{x}{4} = \pm \frac{\pi}{3} + 2\pi n \\ \frac{x}{4} = \pm \frac{2\pi}{3} + 2\pi k \end{cases}; \frac{x}{4} = \pm \frac{\pi}{3} + \pi n, x = \pm \frac{4\pi}{3} + 4\pi n.$$

$$510. 2\cos^2 \frac{x}{2} - \cos \frac{\pi}{9} = 1;$$

$$\left(2\cos^2 \frac{x}{2} - 1 \right) = \cos \frac{\pi}{9}, \cos x = \cos \frac{\pi}{9}, x = \pm \arccos \left(\cos \frac{\pi}{9} \right) + 2\pi n,$$

Отсюда имеем, что уравнение имеет отрезок $[-2\pi; 2\pi]$ 4 корня:

$$\frac{\pi}{9} - 2\pi; -\frac{\pi}{9}; \frac{\pi}{9}; 2\pi - \frac{\pi}{9}.$$

$$511. а) \text{Т.к. } 0 < t < \frac{\pi}{2}, \text{ то } \cos \frac{t}{2} = \sqrt{\frac{1}{2}(1 + \cos t)} = \sqrt{\frac{1}{2}\left(1 + \frac{3}{4}\right)} = \frac{\sqrt{7}}{2\sqrt{2}},$$

$$\sin \frac{t}{2} = \sqrt{\frac{1}{2}(1 - \cos t)} = \frac{1}{2\sqrt{2}}, \operatorname{tg} \frac{t}{2} = \frac{\sin \frac{t}{2}}{\cos \frac{t}{2}} = \frac{1}{\sqrt{7}}, \operatorname{ctg} \frac{t}{2} = \frac{1}{\operatorname{tg} \frac{t}{2}} = \sqrt{7};$$

б) Имеем: $\cos^2 t = \frac{\operatorname{ctg}^2 t}{1 + \operatorname{ctg}^2 t} = \frac{9/16}{1 + 9/16} = \frac{9}{25}$,

Т.к. $\pi < t < \frac{3\pi}{2}$ (т.е. $\cos t < 0$), получим $\cos t = -\frac{3}{5}$,

Поскольку $\frac{\pi}{2} < \frac{t}{2} < \frac{3\pi}{4}$, то $\sin \frac{t}{2} > 0$, $\cos \frac{t}{2} < 0$, т.е. имеем:

$$\sin \frac{t}{2} = \sqrt{\frac{1}{2}(1 - \cos t)} = \sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \frac{2}{\sqrt{5}},$$

$$\cos \frac{t}{2} = -\sqrt{\frac{1}{2}(1 + \cos t)} = -\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = -\frac{1}{\sqrt{5}},$$

$$\operatorname{tg} \frac{t}{2} = \frac{\sin \frac{t}{2}}{\cos \frac{t}{2}} = -2, \quad \operatorname{ctg} \frac{t}{2} = \frac{1}{\operatorname{tg} \frac{t}{2}} = -\frac{1}{2}.$$

512. а) Рассмотрим два случая:

1. $\frac{\pi}{2} < x < \frac{3\pi}{4}$. Тогда $\pi < 2x < \frac{3\pi}{2}$ и $\cos 2x = -\sqrt{1 - \sin^2 2x} = -\frac{4}{5}$,

$$\sin x = \sqrt{\frac{1}{2}(1 - \cos 2x)} = \sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)} = \frac{3}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1 + \cos 2x)} = -\sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)} = -\frac{1}{\sqrt{10}},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = -3, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = -\frac{1}{3};$$

2. $\frac{3\pi}{4} < x < \pi$. Тогда $\frac{3\pi}{2} < 2x < 2\pi$ и $\cos 2x = \sqrt{1 - \sin^2 2x} = \frac{4}{5}$,

$$\sin x = \sqrt{\frac{1}{2}(1 - \cos 2x)} = \sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)} = \frac{1}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1 + \cos 2x)} = -\sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)} = -\frac{3}{\sqrt{10}},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = -\frac{1}{3}, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = -3;$$

б) Т.к. $\pi < x < \frac{5\pi}{4}$, то $2\pi < 2x < \frac{5\pi}{2}$, т.е. $\cos 2x = \frac{1}{\sqrt{1 + \operatorname{tg}^2 2x}} = \frac{1}{\sqrt{1 + 9/16}} = \frac{4}{5}$,

$$\sin x = -\sqrt{\frac{1}{2}(1 - \cos 2x)} = -\sqrt{\frac{1}{2}\left(1 - \frac{4}{5}\right)} = -\frac{1}{\sqrt{10}},$$

$$\cos x = -\sqrt{\frac{1}{2}(1 + \cos 2x)} = -\sqrt{\frac{1}{2}\left(1 + \frac{4}{5}\right)} = -\frac{3}{\sqrt{10}},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{1}{3}, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = 3.$$

$$514. \text{ а) } \frac{\sin 2t}{1 + \cos 2t} \cdot \frac{\cos t}{1 + \cos t} = \operatorname{tg} \frac{t}{2},$$

$$\frac{\sin 2t}{1 + \cos 2t} \cdot \frac{\cos t}{1 + \cos t} = \frac{2 \sin t \cos^2 t}{2 \cos^2 t (1 + \cos t)} = \frac{\sin t}{1 + \cos t} = \operatorname{tg} \frac{t}{2};$$

$$\text{ б) } \frac{\sin 2t}{1 + \cos 2t} \cdot \frac{\cos t}{1 + \cos t} \cdot \frac{\cos \frac{t}{2}}{1 + \cos \frac{t}{2}} = \operatorname{tg} \frac{t}{4},$$

$$\frac{\sin 2t}{1 + \cos 2t} \cdot \frac{\cos t}{1 + \cos t} \cdot \frac{\cos \frac{t}{2}}{1 + \cos \frac{t}{2}} = \frac{2 \sin t \cos^2 t}{2 \cos^2 t (1 + \cos t)} \cdot \frac{\cos \frac{t}{2}}{1 + \cos \frac{t}{2}} =$$

$$= \frac{\sin t}{1 + \cos t} \cdot \frac{\cos \frac{t}{2}}{1 + \cos \frac{t}{2}} = \operatorname{tg} \frac{t}{2} \cdot \frac{\cos \frac{t}{2}}{1 + \cos \frac{t}{2}} = \frac{\sin \frac{t}{2}}{1 + \cos \frac{t}{2}} = \operatorname{tg} \frac{t}{4}.$$

$$515. \text{ а) } \frac{1 - \cos 2t + \sin 2t}{1 + \cos 2t + \sin 2t} = \operatorname{tg} t,$$

$$\frac{1 - (1 - 2 \sin^2 t) + 2 \sin t \cos t}{1 + 2 \sin t \cos t + 2 \cos^2 t - 1} = \frac{\sin t (\sin t + \cos t)}{\cos t (\sin t + \cos t)} = \frac{\sin t}{\cos t} = \operatorname{tg} t;$$

$$\text{ б) } \frac{1 + \cos 2t - \sin 2t}{1 + \sin 2t + \cos 2t} = \operatorname{tg} \left(\frac{\pi}{4} - t \right),$$

$$\frac{1 + \cos 2t - \sin 2t}{1 + \sin 2t + \cos 2t} = \frac{1 + 2 \cos^2 t - 1 - 2 \sin t \cos t}{1 + 2 \sin t \cos t + 2 \cos^2 t - 1} = \frac{\cos t - \sin t}{\cos t + \sin t} = \frac{\sqrt{2} \sin \left(\frac{\pi}{4} - t \right)}{\sqrt{2} \cos \left(\frac{\pi}{4} - t \right)} =$$

$$= \operatorname{tg} \left(\frac{\pi}{4} - t \right).$$

$$516. \text{ а) } \cos^2 t - \cos^2 \left(\frac{\pi}{4} - t \right) = \frac{1}{\sqrt{2}} \sin \left(\frac{\pi}{4} - 2t \right),$$

$$\frac{1}{\sqrt{2}} \sin \left(\frac{\pi}{4} - 2t \right) = \frac{1}{\sqrt{2}} \left(\sin \frac{\pi}{4} \cos 2t - \sin 2t \cos \frac{\pi}{4} \right) =$$

$$= \frac{1}{2} (\cos 2t - \sin 2t) = \frac{1}{2} \left(\cos 2t - \cos \left(\frac{\pi}{2} - 2t \right) \right) =$$

$$= \frac{1}{2} \left(2 \cos^2 t - 1 - 2 \cos^2 \left(\frac{\pi}{4} - t \right) + 1 \right) = \cos^2 t - \cos^2 \left(\frac{\pi}{4} - t \right);$$

$$6) \sin^2 t - \sin^2 \left(\frac{\pi}{4} - t \right) = \frac{1}{\sqrt{2}} \sin \left(2t - \frac{\pi}{4} \right),$$

$$\frac{1}{\sqrt{2}} \sin \left(2t - \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}} \left(\sin 2t \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos 2t \right) =$$

$$= \frac{1}{2} (\sin 2t - \cos 2t) = \frac{1}{2} \left(\cos \left(\frac{\pi}{2} - 2t \right) - \cos 2t \right) =$$

$$= \frac{1}{2} \left(1 - 2 \sin^2 \left(\frac{\pi}{4} - t \right) - 1 + 2 \sin^2 t \right) = \sin^2 t - \sin^2 \left(\frac{\pi}{4} - t \right).$$

$$517. a) f(x) = 2 \cos 2x + \sin^2 x = 2 \cos 2x + \frac{1}{2} (1 - \cos 2x) = \frac{3}{2} \cos 2x + \frac{1}{2}$$

Поскольку наибольшее значение функции $y = \cos 2x$ равно 1, а наименьшее -1 , то наибольшее значение функции $f(x)$ равно 2, а наименьшее -1 ;

$$6) f(x) = 2 \sin^2 3x - \cos 6x = (1 - \cos 6x) - \cos 6x = 1 - 2 \cos 6x$$

Поскольку наибольшее значение функции $y = \cos 6x$ равно 1, а наименьшее -1 , то наибольшее значение функции $f(x)$ равно 3, а наименьшее -1 .

$$518. a) t \in \left[\frac{\pi}{2}; \pi \right], \text{ т.е. } \sin t \geq 0, \cos t \leq 0,$$

$$\sqrt{1 - \cos 2t} + \sqrt{1 + \cos 2t} = \sqrt{2} (|\sin t| + |\cos t|) = \sqrt{2} (\sin t - \cos t) = 2 \sin \left(t - \frac{\pi}{4} \right);$$

$$6) t \in \left[\frac{3\pi}{2}; 2\pi \right], \text{ т.е. } \sin t \leq 0, \cos t \geq 0,$$

$$\sqrt{1 - \cos 2t} + \sqrt{1 + \cos 2t} = \sqrt{2} (|\sin t| + |\cos t|) = \sqrt{2} (-\sin t + \cos t) = 2 \sin \left(\frac{\pi}{4} - t \right);$$

$$в) t \in \left[0; \frac{\pi}{2} \right], \text{ т.е. } \sin t \geq 0, \cos t \geq 0,$$

$$\sqrt{1 - \cos 2t} + \sqrt{1 + \cos 2t} = \sqrt{2} (|\sin t| + |\cos t|) = \sqrt{2} (\sin t + \cos t) = 2 \sin \left(t + \frac{\pi}{4} \right);$$

$$г) t \in \left[\pi; \frac{3\pi}{2} \right], \text{ т.е. } \sin t \leq 0, \cos t \leq 0,$$

$$\sqrt{1 - \cos 2t} + \sqrt{1 + \cos 2t} = \sqrt{2} (|\sin t| + |\cos t|) = -\sqrt{2} (\sin t + \cos t) = -2 \sin \left(t + \frac{\pi}{4} \right).$$

$$519. \cos 2x = \frac{5}{13} = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x,$$

$$\text{откуда } \cos^2 x = \frac{1}{2} \left(1 + \frac{5}{13} \right) = \frac{9}{13}, \sin^2 x = \frac{1}{2} \left(1 - \frac{5}{13} \right) = \frac{4}{13},$$

$$\text{а) } \sin^4 x + \cos^4 x = (\sin^2 x)^2 + (\cos^2 x)^2 = \left(\frac{4}{13}\right)^2 + \left(\frac{9}{13}\right)^2 = \frac{97}{169};$$

$$\text{б) } \sin^8 x + \cos^8 x = (\sin^2 x)^4 + (\cos^2 x)^4 = \left(\frac{4}{13}\right)^4 + \left(\frac{9}{13}\right)^4 = \frac{6817}{28561}.$$

$$520. \text{ а) } \sin^2\left(2x - \frac{\pi}{6}\right) = \frac{3}{4},$$

$$\frac{1}{2}\left(1 - \cos\left(4x - \frac{\pi}{3}\right)\right) = \frac{3}{4}, \quad \cos\left(4x - \frac{\pi}{3}\right) = -\frac{1}{2},$$

$$4x - \frac{\pi}{3} = \pm \frac{2\pi}{3} + 2\pi n, \quad \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = -\frac{\pi}{12} + \frac{\pi k}{2} \end{cases};$$

$$\text{б) } \cos^2\left(x + \frac{\pi}{3}\right) = 1,$$

$$\frac{1}{2}\left(1 + \cos\left(2x - \frac{2\pi}{3}\right)\right) = 1, \quad \cos\left(2x + \frac{2\pi}{3}\right) = 1,$$

$$2x + \frac{2\pi}{3} = 2\pi n, \quad x = -\frac{\pi}{3} + \pi n;$$

$$\text{в) } \sin^2\left(x + \frac{\pi}{2}\right) = \frac{1}{2}, \quad \frac{1}{2}(1 - \cos(2x + \pi)) = \frac{1}{2}, \quad \frac{1}{2}(1 + \cos 2x) = \frac{1}{2},$$

$$\cos 2x = -1, \quad 2x = \pi + 2\pi n, \quad x = \frac{\pi}{2} + \pi n;$$

$$\text{г) } \cos^2\left(3x - \frac{\pi}{4}\right) = \frac{3}{4}, \quad \frac{1}{2}\left(1 + \cos\left(6x - \frac{\pi}{2}\right)\right) = \frac{3}{4}, \quad \frac{1}{2}(1 + \sin 6x) = \frac{3}{4},$$

$$\sin 6x = \frac{1}{2}, \quad 6x = (-1)^n \frac{\pi}{6} + \pi n, \quad x = (-1)^n \frac{\pi}{36} + \frac{\pi n}{6}.$$

$$521. \text{ а) } 4\sin^2 x + \sin^2 2x = 3,$$

$$2(1 - \cos 2x) + 1 - \cos^2 2x = 3, \quad \cos^2 2x + 2\cos 2x = 0,$$

$$\cos 2x (\cos 2x + 2) = 0, \quad \cos 2x = 0 \quad (\text{т.к. } \cos 2x + 2 > 0 \text{ для всех } x),$$

$$2x = \frac{\pi}{2} + \pi n, \quad x = \frac{\pi}{4} + \frac{\pi n}{2};$$

$$\text{б) } 4\cos^2 2x + 8\cos^2 x = 7, \quad 4\cos^2 2x + 4(1 + \cos 2x) = 7,$$

$$4\cos^2 2x + 4\cos 2x - 3 = 0,$$

$$\cos 2x = \frac{-2 \pm \sqrt{4+12}}{4} = \frac{-2 \pm 4}{4} = \begin{cases} \frac{1}{2} \\ -\frac{3}{2} \end{cases},$$

$$\cos 2x = \frac{1}{2} \quad (\text{т.к. } \cos 2x \neq -\frac{3}{2} \text{ при всех } x),$$

$$2x = \pm \frac{\pi}{3} + 2\pi n, \quad x = \pm \frac{\pi}{6} + \pi n.$$

$$522. \text{ а) } 4\sin^2 3x < 3,$$

$$2(1 - \cos 6x) < 3, \quad \cos 6x > -\frac{1}{2}, \quad -\frac{2\pi}{3} + 2\pi n < 6x < \frac{2\pi}{3} + 2\pi n,$$

$$-\frac{\pi}{9} + \frac{\pi n}{3} < x < \frac{\pi}{9} + \frac{\pi n}{3};$$

$$\text{б) } 4\cos^2 \frac{x}{4} > 1, \quad 2\left(1 + \cos \frac{x}{2}\right) > 1, \quad \cos \frac{x}{2} > -\frac{1}{2},$$

$$-\frac{2\pi}{3} + 2\pi n < \frac{x}{2} < \frac{2\pi}{3} + 2\pi n, \quad -\frac{4\pi}{3} + 4\pi n < x < \frac{4\pi}{3} + 4\pi n.$$

§26. Преобразование сумм тригонометрических функций в произведение

$$523. \text{ а) } \sin 40^\circ + \sin 16^\circ = 2 \sin \frac{40^\circ + 16^\circ}{2} \cos \frac{40^\circ - 16^\circ}{2} = 2 \sin 28^\circ \cos 12^\circ;$$

$$\text{б) } \sin 20^\circ - \sin 40^\circ = 2 \sin \frac{20^\circ - 40^\circ}{2} \cos \frac{20^\circ + 40^\circ}{2} = -2 \sin 10^\circ \cos 30^\circ = -\sqrt{3} \sin 10^\circ;$$

$$\text{в) } \sin 10^\circ + \sin 50^\circ = 2 \sin \frac{10^\circ + 50^\circ}{2} \cos \frac{50^\circ - 10^\circ}{2} = 2 \sin 30^\circ \cos 20^\circ = \cos 20^\circ;$$

$$\text{г) } \sin 52^\circ - \sin 36^\circ = 2 \sin \frac{52^\circ - 36^\circ}{2} \cos \frac{52^\circ + 36^\circ}{2} = 2 \sin 8^\circ \cos 44^\circ.$$

$$524. \text{ а) } \cos 15^\circ + \cos 45^\circ = 2 \cos \frac{15^\circ + 45^\circ}{2} \cos \frac{15^\circ - 45^\circ}{2} = 2 \cos 30^\circ \cos 15^\circ = \sqrt{3} \cos 15^\circ;$$

$$\text{б) } \cos 46^\circ - \cos 74^\circ = 2 \sin \frac{46^\circ + 74^\circ}{2} \sin \frac{74^\circ - 46^\circ}{2} = 2 \sin 60^\circ \sin 14^\circ = \sqrt{3} \cos 14^\circ;$$

$$\text{в) } \cos 20^\circ + \cos 40^\circ = 2 \cos \frac{20^\circ + 40^\circ}{2} \cos \frac{40^\circ - 20^\circ}{2} = 2 \cos 30^\circ \cos 10^\circ = \sqrt{3} \cos 10^\circ;$$

$$\text{г) } \cos 75^\circ - \cos 15^\circ = 2 \sin \frac{75^\circ + 15^\circ}{2} \sin \frac{15^\circ - 75^\circ}{2} = -2 \sin 45^\circ \sin 30^\circ = -\frac{\sqrt{2}}{2}.$$

$$525. \text{ а) } \sin \frac{\pi}{5} - \sin \frac{\pi}{10} = 2 \sin \frac{\frac{\pi}{5} - \frac{\pi}{10}}{2} \cos \frac{\frac{\pi}{5} + \frac{\pi}{10}}{2} = 2 \sin \frac{\pi}{20} \cos \frac{3\pi}{20};$$

$$\text{б) } \sin \frac{\pi}{3} + \sin \frac{\pi}{4} = 2 \sin \frac{\frac{\pi}{3} + \frac{\pi}{4}}{2} \cos \frac{\frac{\pi}{3} - \frac{\pi}{4}}{2} = 2 \sin \frac{7\pi}{24} \cos \frac{\pi}{24};$$

$$\text{в) } \sin \frac{\pi}{6} + \sin \frac{\pi}{7} = 2 \sin \frac{\frac{\pi}{6} + \frac{\pi}{7}}{2} \cos \frac{\frac{\pi}{6} - \frac{\pi}{7}}{2} = 2 \sin \frac{13\pi}{84} \cos \frac{\pi}{84};$$

$$r) \sin \frac{\pi}{3} - \sin \frac{\pi}{11} = 2 \sin \frac{\frac{\pi}{3} - \frac{\pi}{11}}{2} \cos \frac{\frac{\pi}{3} + \frac{\pi}{11}}{2} = 2 \sin \frac{4\pi}{33} \cos \frac{7\pi}{33}.$$

$$526. a) \cos \frac{\pi}{10} - \cos \frac{\pi}{20} = 2 \sin \frac{\frac{\pi}{10} + \frac{\pi}{20}}{2} \sin \frac{\frac{\pi}{10} - \frac{\pi}{20}}{2} = -2 \sin \frac{3\pi}{40} \sin \frac{\pi}{40};$$

$$б) \cos \frac{11\pi}{12} + \cos \frac{3\pi}{4} = 2 \cos \frac{\frac{11\pi}{12} + \frac{3\pi}{4}}{2} \cos \frac{\frac{11\pi}{12} - \frac{3\pi}{4}}{2} = 2 \cos \frac{5\pi}{6} \cos \frac{\pi}{12} = -\sqrt{3} \cos \frac{\pi}{12};$$

$$в) \cos \frac{\pi}{5} - \cos \frac{\pi}{11} = 2 \sin \frac{\frac{\pi}{5} + \frac{\pi}{11}}{2} \sin \frac{\frac{\pi}{5} - \frac{\pi}{11}}{2} = -2 \sin \frac{8\pi}{55} \sin \frac{3\pi}{55};$$

$$r) \cos \frac{3\pi}{8} + \cos \frac{5\pi}{4} = 2 \cos \frac{\frac{3\pi}{8} + \frac{5\pi}{4}}{2} \cos \frac{\frac{3\pi}{8} - \frac{5\pi}{4}}{2} = 2 \cos \frac{13\pi}{16} \cos \frac{7\pi}{16}.$$

$$527. a) \sin 3t - \sin t = 2 \sin \frac{3t - t}{2} \cos \frac{3t + t}{2} = 2 \sin t \cos t;$$

$$б) \cos(\alpha - 2\beta) - \cos(\alpha + 2\beta) = 2 \sin \frac{\alpha - 2\beta + \alpha + 2\beta}{2} \sin \frac{\alpha + 2\beta - \alpha + 2\beta}{2} = 2 \sin \alpha \sin 2\beta;$$

$$в) \cos 6t + \cos 4t = 2 \cos \frac{6t + 4t}{2} \cos \frac{6t - 4t}{2} = 2 \cos 5t \cos t;$$

$$r) \sin(\alpha - 2\beta) - \sin(\alpha + 2\beta) = 2 \sin \frac{\alpha - 2\beta - \alpha + 2\beta}{2} \cos \frac{\alpha - 2\beta + \alpha + 2\beta}{2} = -2 \sin 2\beta \cos \alpha.$$

$$528. a) \operatorname{tg} 25^\circ + \operatorname{tg} 35^\circ = \frac{\sin(25^\circ + 35^\circ)}{\cos 25^\circ \cos 35^\circ} = \frac{\sin 60^\circ}{\cos 25^\circ \cos 35^\circ} = \frac{\sqrt{3}}{\cos 25^\circ \cos 35^\circ};$$

$$б) \operatorname{tg} \frac{\pi}{5} - \operatorname{tg} \frac{\pi}{10} = \frac{\sin\left(\frac{\pi}{5} - \frac{\pi}{10}\right)}{\cos \frac{\pi}{5} \cos \frac{\pi}{10}} = \frac{\sin \frac{\pi}{10}}{\cos \frac{\pi}{5} \cos \frac{\pi}{10}} = \frac{\operatorname{tg} \frac{\pi}{10}}{\cos \frac{\pi}{5}};$$

$$в) \operatorname{tg} 20^\circ + \operatorname{tg} 40^\circ = \frac{\sin(20^\circ + 40^\circ)}{\sin 20^\circ \sin 40^\circ} = \frac{\sin 60^\circ}{\sin 20^\circ \sin 40^\circ} = \frac{\sqrt{3}}{2 \sin 20^\circ \sin 40^\circ};$$

$$r) \operatorname{tg} \frac{\pi}{3} - \operatorname{tg} \frac{\pi}{4} = \frac{\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)}{\cos \frac{\pi}{3} \cos \frac{\pi}{4}} = \frac{\sin \frac{\pi}{12}}{\frac{1}{2} \frac{\sqrt{2}}{2}} = 2\sqrt{2} \sin \frac{\pi}{12}.$$

$$529. a) \frac{\cos 68^\circ - \cos 22^\circ}{\sin 68^\circ - \sin 22^\circ} = \frac{2 \sin \frac{68^\circ + 22^\circ}{2} \sin \frac{22^\circ - 68^\circ}{2}}{2 \sin \frac{68^\circ - 22^\circ}{2} \sin \frac{68^\circ + 22^\circ}{2}} = \frac{-\sin 45^\circ \sin 23^\circ}{2 \sin 23^\circ \cos 45^\circ} =$$

$$= -\operatorname{tg} 45^\circ = -1;$$

$$\begin{aligned}
 \text{б)} \quad \frac{\sin 130^\circ + \sin 110^\circ}{\cos 130^\circ + \cos 110^\circ} &= \frac{2 \sin \frac{130^\circ + 110^\circ}{2} \cos \frac{130^\circ - 110^\circ}{2}}{2 \cos \frac{130^\circ + 110^\circ}{2} \cos \frac{130^\circ - 110^\circ}{2}} = \frac{\sin 120^\circ \sin 10^\circ}{\cos 120^\circ \cos 10^\circ} = \\
 &= \frac{\sin 120^\circ}{\cos 120^\circ} = \operatorname{tg} 120^\circ = -\sqrt{3}.
 \end{aligned}$$

$$530. \text{ а) } \sin 35^\circ + \sin 25^\circ = \cos 5^\circ,$$

$$\sin 35^\circ + \sin 25^\circ = 2 \sin \frac{35^\circ + 25^\circ}{2} \cos \frac{35^\circ - 25^\circ}{2} = 2 \sin 30^\circ \cos 5^\circ = \cos 5^\circ$$

$$\text{б) } \sin 40^\circ + \cos 70^\circ = \cos 10^\circ,$$

$$\begin{aligned}
 \sin 40^\circ + \cos 70^\circ &= \sin 40^\circ + \sin 20^\circ = 2 \sin \frac{40^\circ + 20^\circ}{2} \cos \frac{40^\circ - 20^\circ}{2} = \\
 &= 2 \sin 30^\circ \cos 10^\circ = \cos 10^\circ;
 \end{aligned}$$

$$\text{в) } \cos 12^\circ - \cos 48^\circ = \sin 18^\circ,$$

$$\cos 12^\circ - \cos 48^\circ = 2 \sin \frac{12^\circ + 48^\circ}{2} \sin \frac{48^\circ - 12^\circ}{2} = 2 \sin 30^\circ \sin 18^\circ = \sin 18^\circ;$$

$$\text{г) } \cos 20^\circ - \sin 50^\circ = \sin 10^\circ,$$

$$\begin{aligned}
 \cos 20^\circ - \sin 50^\circ &= \cos 20^\circ - \cos 40^\circ = 2 \sin \frac{20^\circ + 40^\circ}{2} \sin \frac{40^\circ - 20^\circ}{2} = \\
 &= 2 \sin 30^\circ \sin 10^\circ = \sin 10^\circ.
 \end{aligned}$$

$$531. \text{ а) } \frac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \operatorname{tg} 4\alpha,$$

$$\frac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \frac{2 \sin \frac{2\alpha + 6\alpha}{2} \cos \frac{6\alpha - 2\alpha}{2}}{2 \cos \frac{2\alpha + 6\alpha}{2} \cos \frac{6\alpha - 2\alpha}{2}} = \frac{\sin 4\alpha \cos 2\alpha}{\cos 4\alpha \cos 2\alpha} = \frac{\sin 4\alpha}{\cos 4\alpha} = \operatorname{tg} 4\alpha;$$

$$\text{б) } \frac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \operatorname{tg} 3\alpha \operatorname{tg} \alpha,$$

$$\frac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \frac{2 \sin \frac{2\alpha + 4\alpha}{2} \cos \frac{4\alpha - 2\alpha}{2}}{2 \cos \frac{2\alpha + 4\alpha}{2} \cos \frac{4\alpha - 2\alpha}{2}} = \frac{\sin 3\alpha \sin \alpha}{\cos 3\alpha \cos \alpha} = \operatorname{tg} 3\alpha \operatorname{tg} \alpha.$$

$$532. \text{ а) } \cos x + \cos 3x = 0; 2 \cos \frac{x+3x}{2} \cos \frac{3x-x}{2} = 0, \cos 2x \cos x = 0,$$

$$\left[\begin{array}{l} \cos 2x = 0 \\ \cos x = 0 \end{array} \right]; \left[\begin{array}{l} 2x = \frac{\pi}{2} + \pi n \\ x = \frac{\pi}{2} + \pi k \end{array} \right]; \left[\begin{array}{l} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{2} + \pi k \end{array} \right];$$

$$\text{б) } \sin 12x + \sin 4x = 0; 2 \sin \frac{12x+4x}{2} \cos \frac{12x-4x}{2} = 0, \sin 8x \cos 4x = 0,$$

$$\begin{cases} \sin 8x = 0 \\ \cos 4x = 0 \end{cases}; \begin{cases} 8x = \pi n \\ 4x = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi n}{8} \\ x = \frac{\pi}{8} + \frac{\pi k}{4} \end{cases}; x = \frac{\pi n}{8};$$

$$\text{в) } \cos x = \cos 5x; \cos x - \cos 5x = 0, 2 \sin \frac{x+5x}{2} \cos \frac{5x-x}{2} = 0,$$

$$\sin 3x \sin 2x = 0, \begin{cases} \sin 3x = 0 \\ \cos 2x = 0 \end{cases}; \begin{cases} 3x = \pi n \\ 2x = \pi k \end{cases}; \begin{cases} x = \frac{\pi n}{3} \\ x = \frac{\pi k}{2} \end{cases};$$

$$\text{г) } \sin 3x = \sin 17x; \sin 17x - \sin 3x = 0, 2 \sin \frac{17x-3x}{2} \cos \frac{17x+3x}{2} = 0,$$

$$\sin 7x \cos 10x = 0, \begin{cases} \sin 7x = 0 \\ \cos 10x = 0 \end{cases}; \begin{cases} 7x = \pi n \\ 10x = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi n}{7} \\ x = \frac{\pi}{20} + \frac{\pi k}{10} \end{cases}.$$

$$533. \text{ а) } \sin x + \sin 2x + \sin 3x = 0,$$

$$\sin 2x + 2 \sin \frac{x+3x}{2} \cos \frac{3x-x}{2} = 0, \sin 2x + 2 \sin 2x \cos x = 0,$$

$$\sin 2x(1 + 2 \cos x) = 0, \begin{cases} \sin 2x = 0 \\ \cos x = -\frac{1}{2} \end{cases}; \begin{cases} 2x = \pi n \\ x = \pm \frac{2\pi}{3} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ x = \pm \frac{2\pi}{3} + 2\pi k \end{cases};$$

$$\text{б) } \cos 3x - \cos 5x = \sin 4x,$$

$$2 \sin \frac{3x+5x}{2} \cos \frac{5x-3x}{2} = \sin 4x, 2 \sin 4x \sin x - \sin 4x = 0,$$

$$\sin 4x(2 \sin x - 1) = 0, \begin{cases} \sin 4x = 0 \\ \cos x = \frac{1}{2} \end{cases}; \begin{cases} 4x = \pi n \\ x = (-1)^k \frac{\pi}{6} + \pi k \end{cases}; \begin{cases} x = \frac{\pi n}{4} \\ x = (-1)^k \frac{\pi}{6} + \pi k \end{cases}.$$

$$534. \text{ а) } \frac{1}{2} - \cos t = \cos \frac{\pi}{3} - \cos t = 2 \sin \frac{\frac{\pi}{3}+t}{2} \sin \frac{t-\frac{\pi}{3}}{2} = 2 \sin \left(\frac{\pi}{6} + \frac{t}{2} \right) \sin \left(\frac{t}{2} - \frac{\pi}{6} \right);$$

$$\text{б) } \frac{\sqrt{3}}{2} + \sin t = \sin \frac{\pi}{3} + \sin t = 2 \sin \frac{\frac{\pi}{3}+t}{2} \cos \frac{\frac{\pi}{3}-t}{2} = 2 \cos \left(\frac{\pi}{6} + \frac{t}{2} \right) \cos \left(\frac{\pi}{6} - \frac{t}{2} \right);$$

$$\begin{aligned} \text{в) } 1 + 2 \cos t &= 2 \left(\frac{1}{2} + \cos t \right) = 2 \left(\cos \frac{\pi}{3} + \cos t \right) = 4 \cos \frac{\frac{\pi}{3}+t}{2} \cos \frac{\frac{\pi}{3}-t}{2} = \\ &= 4 \cos \left(\frac{\pi}{6} + \frac{t}{2} \right) \cos \left(\frac{\pi}{6} - \frac{t}{2} \right); \end{aligned}$$

$$\begin{aligned} \text{г) } \cos t + \sin t &= \cos t + \cos\left(\frac{\pi}{2} - t\right) = 2 \cos \frac{t + \frac{\pi}{2} - t}{2} \cos \frac{t - \frac{\pi}{2} + t}{2} = \\ &= 2 \cos \frac{\pi}{4} \cos\left(t - \frac{\pi}{4}\right) = \sqrt{2} \cos\left(t - \frac{\pi}{4}\right). \end{aligned}$$

$$\begin{aligned} 535. \text{ а) } \sin 5x + 2 \sin 6x + \sin 7x &= (\sin 5x + \sin 7x) + 2 \sin 6x = \\ &= 2 \sin \frac{5x + 7x}{2} \cos \frac{7x - 5x}{2} + 2 \sin 6x = 2 \sin 6x \cos x + 2 \sin 6x = \\ &= 2 \sin 6x(1 + \cos x) = 4 \sin 6x \cos^2 \frac{x}{2}; \end{aligned}$$

$$\begin{aligned} \text{б) } 2 \cos x + \cos 2x + \cos 4x &= 2 \cos x + 2 \cos \frac{2x + 4x}{2} \cos \frac{4x - 2x}{2} = \\ &= 2 \cos x + 2 \cos x \cos 3x = 2 \cos x(1 + \cos 3x) = 4 \cos x \cos^2 \frac{3x}{2}. \end{aligned}$$

$$\begin{aligned} 536. \text{ а) } \sin t + \sin 2t + \sin 3t + \sin 4t &= (\sin t + \sin 4t) + (\sin 2t + \sin 3t) = \\ &= 2 \sin \frac{t + 4t}{2} \cos \frac{4t - t}{2} + 2 \sin \frac{2t + 3t}{2} \cos \frac{3t - 2t}{2} = 2 \sin \frac{5t}{2} \cos \frac{3t}{2} + 2 \sin \frac{5t}{2} \cos \frac{t}{2} = \\ &= 2 \sin \frac{5t}{2} \left(\cos \frac{3t}{2} + \cos \frac{t}{2} \right) = 4 \sin \frac{5t}{2} \cos \frac{\frac{3t}{2} + \frac{t}{2}}{2} \cos \frac{\frac{3t}{2} - \frac{t}{2}}{2} = 4 \sin \frac{5t}{2} \cos t \cos \frac{t}{2}; \end{aligned}$$

$$\begin{aligned} \text{б) } \cos 2t - \cos 4t - \cos 6t + \cos 8t &= (\cos 2t + \cos 8t) - (\cos 4t + \cos 6t) = \\ &= 2 \cos \frac{2t + 8t}{2} \cos \frac{8t - 2t}{2} - 2 \cos \frac{4t + 6t}{2} \cos \frac{6t - 4t}{2} = \\ &= 2 \cos 5t \cos 3t - 2 \cos 5t \cos t = 2 \cos 5t (\cos 3t - \cos t) = \\ &= 4 \cos 5t \sin \frac{3t + t}{2} \sin \frac{t - 3t}{2} = -4 \cos 5t \sin 2t \sin t. \end{aligned}$$

$$\begin{aligned} 537. \text{ а) } \sin 20^\circ + \sin 40^\circ - \cos 10^\circ &= 0, \\ \sin 20^\circ + \sin 40^\circ - \cos 10^\circ &= 2 \sin \frac{20^\circ + 40^\circ}{2} \cos \frac{40^\circ - 20^\circ}{2} - \cos 10^\circ = \\ &= 2 \sin 30^\circ \cos 10^\circ - \cos 10^\circ = 2 \cdot \frac{1}{2} \cdot \cos 10^\circ - \cos 10^\circ = 0; \\ \text{б) } \cos 85^\circ + \cos 35^\circ - \cos 25^\circ &= 0, \\ \cos 85^\circ + \cos 35^\circ - \cos 25^\circ &= 2 \cos \frac{85^\circ + 35^\circ}{2} \cos \frac{85^\circ - 35^\circ}{2} - \cos 25^\circ = \\ &= 2 \cos 60^\circ \cos 25^\circ - \cos 25^\circ = 2 \cdot \frac{1}{2} \cdot \cos 25^\circ - \cos 25^\circ = 0. \end{aligned}$$

$$\begin{aligned} 538. \text{ а) } \sin 87^\circ - \sin 59^\circ - \sin 93^\circ + \sin 61^\circ &= \sin 1^\circ, \\ \sin 87^\circ - \sin 59^\circ - \sin 93^\circ + \sin 61^\circ &= (\sin 87^\circ - \sin 93^\circ) + (\sin 61^\circ - \sin 59^\circ) = \\ &= 2 \sin \frac{87^\circ - 93^\circ}{2} \cos \frac{87^\circ + 93^\circ}{2} + 2 \sin \frac{61^\circ - 59^\circ}{2} \cos \frac{61^\circ + 59^\circ}{2} = \\ &= -2 \sin 3^\circ \cos 90^\circ + 2 \sin 1^\circ \cos 60^\circ = 0 + 2 \cdot \frac{1}{2} \cdot \sin 1^\circ = \sin 1^\circ; \end{aligned}$$

$$\begin{aligned}
 & \text{б) } \cos 115^\circ - \cos 35^\circ + \cos 65^\circ + \cos 25^\circ = \sin 5^\circ, \\
 & \cos 115^\circ - \cos 35^\circ + \cos 65^\circ + \cos 25^\circ = (\cos 115^\circ + \cos 65^\circ) + (\cos 25^\circ - \cos 35^\circ) = \\
 & = 2 \cos \frac{115^\circ + 65^\circ}{2} \cos \frac{115^\circ - 65^\circ}{2} + 2 \sin \frac{25^\circ + 35^\circ}{2} \sin \frac{35^\circ - 25^\circ}{2} = \\
 & = 2 \sin 90^\circ \cos 25^\circ + 2 \sin 30^\circ \sin 5^\circ = 0 + 2 \cdot \frac{1}{2} \cdot \sin 5^\circ = \sin 5^\circ.
 \end{aligned}$$

$$\begin{aligned}
 539. \text{ а) } \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\cos(\alpha + \beta) + \cos(\alpha - \beta)} &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta + \sin \alpha \sin \beta} = \\
 &= \frac{2 \sin \alpha \cos \beta}{2 \cos \alpha \cos \beta} = \frac{\sin \alpha}{\cos \alpha} = \operatorname{tg} \alpha;
 \end{aligned}$$

$$\begin{aligned}
 \text{б) } \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{\sin(\alpha + \beta) - \sin(\alpha - \beta)} &= \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta - \cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta - \sin \alpha \cos \beta + \cos \alpha \sin \beta} = \\
 &= \frac{2 \sin \alpha \sin \beta}{2 \cos \alpha \sin \beta} = \frac{\sin \alpha}{\cos \alpha} = \operatorname{tg} \alpha.
 \end{aligned}$$

$$\begin{aligned}
 540. \text{ а) } \sin x + \sin y + \sin(x - y) &= 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} + 2 \sin \frac{x-y}{2} \cos \frac{x-y}{2} = \\
 &= 2 \cos \frac{x-y}{2} \left(\sin \frac{x+y}{2} + \sin \frac{x-y}{2} \right) = 4 \cos \frac{x-y}{2} \sin \frac{\frac{x+y}{2} + \frac{x-y}{2}}{2} \cos \frac{\frac{x+y}{2} - \frac{x-y}{2}}{2} = \\
 &= 4 \sin \frac{x}{2} \cos \frac{x}{2} \cos \frac{x-y}{2};
 \end{aligned}$$

$$\begin{aligned}
 \text{б) } \frac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} &= \frac{\sin 2x + 2 \sin \frac{x+3x}{2} \cos \frac{3x-x}{2}}{\cos 2x + 2 \cos \frac{x+3x}{2} \cos \frac{3x-x}{2}} = \frac{\sin 2x + 2 \sin 2x \cos x}{\cos 2x + 2 \cos 2x \cos x} = \\
 &= \frac{\sin 2x(1 + 2 \cos x)}{\cos 2x(1 + 2 \cos x)} = \frac{\sin 2x}{\cos 2x} = \operatorname{tg} 2x.
 \end{aligned}$$

$$\begin{aligned}
 541. \text{ а) } \sin^2(\alpha + \beta) - \sin^2(\alpha - \beta) &= (\sin(\alpha + \beta) - \sin(\alpha - \beta))(\sin(\alpha + \beta) + \sin(\alpha - \beta)) = \\
 &= (2 \cos \alpha \sin \beta)(2 \sin \alpha \cos \beta) = (2 \sin \alpha \cos \alpha)(2 \sin \beta \cos \beta) = \sin 2\alpha \sin 2\beta;
 \end{aligned}$$

$$\begin{aligned}
 \text{б) } \cos^2(\alpha - \beta) - \cos^2(\alpha + \beta) &= (\cos(\alpha - \beta) - \cos(\alpha + \beta))(\cos(\alpha - \beta) + \cos(\alpha + \beta)) = \\
 &= (2 \sin \alpha \sin \beta)(2 \cos \alpha \cos \beta) = (2 \sin \alpha \cos \alpha)(2 \sin \beta \cos \beta) = \sin 2\alpha \sin 2\beta.
 \end{aligned}$$

$$\begin{aligned}
 542. \frac{\sin \alpha + \sin 3\alpha + \sin 5\alpha + \sin 7\alpha}{\cos \alpha + \cos 3\alpha + \cos 5\alpha + \cos 7\alpha} &= \frac{(\sin \alpha + \sin 7\alpha) + (\sin 3\alpha + \sin 5\alpha)}{(\cos \alpha + \cos 7\alpha) + (\cos 3\alpha + \cos 5\alpha)} = \\
 &= \frac{2 \sin 4\alpha \cos 3\alpha + 2 \sin 4\alpha \cos \alpha}{2 \cos 4\alpha \cos 3\alpha + 2 \cos 4\alpha \cos \alpha} = \frac{2 \sin 4\alpha (\cos 3\alpha + \cos \alpha)}{2 \cos 4\alpha (\cos 3\alpha + \cos \alpha)} = \frac{\sin 4\alpha}{\cos 4\alpha} = \frac{1}{\operatorname{ctg} 4\alpha} = \frac{1}{0,2} = 5.
 \end{aligned}$$

$$\begin{aligned}
 543. \text{ а) } \operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma &= \operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg}(\pi - \alpha - \beta) = (\operatorname{tg} \alpha + \operatorname{tg} \beta) - \operatorname{tg}(\alpha + \beta) = \\
 &= \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} - \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \sin(\alpha + \beta) \left(\frac{1}{\cos \alpha \cos \beta} - \frac{1}{\cos(\alpha + \beta)} \right) = \\
 &= \sin(\pi - \alpha - \beta) \left(\frac{1}{\cos \alpha \cos \beta} - \frac{1}{\cos(\pi - \alpha - \beta)} \right) = \sin \gamma \left(\frac{1}{\cos \alpha \cos \beta} + \frac{1}{\cos \gamma} \right) =
 \end{aligned}$$

$$= \sin \gamma \frac{\cos \gamma + \cos \alpha \cos \beta}{\cos \alpha \cos \beta \cos \gamma} = \sin \gamma \frac{-\cos(\alpha + \beta) + \cos \alpha \cos \beta}{\cos \alpha \cos \beta \cos \gamma} = \frac{\sin \alpha \sin \beta \sin \gamma}{\cos \alpha \cos \beta \cos \gamma} = \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma;$$

$$\begin{aligned} 6) \sin \alpha + \sin \beta + \sin \gamma &= \sin \alpha + \sin \beta + \sin(\pi - \alpha - \beta) = (\sin \alpha + \sin \beta) + \sin(\alpha + \beta) = \\ &= 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} + 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha + \beta}{2} = 2 \sin \frac{\alpha + \beta}{2} \left(\cos \frac{\alpha - \beta}{2} + \cos \frac{\alpha + \beta}{2} \right) = \\ &= 4 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha}{2} \cos \frac{\beta}{2} = 4 \cos \left(\frac{\pi}{2} - \frac{\alpha + \beta}{2} \right) \cos \frac{\alpha}{2} \cos \frac{\beta}{2} = 4 \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2}. \end{aligned}$$

$$\begin{aligned} 544. a) \sin^2 10^\circ + \sin^2 130^\circ + \sin^2 110^\circ &= \frac{1}{2} (1 - \cos 20^\circ + 1 - \cos 260^\circ + 1 - \cos 220^\circ) = \\ &= \frac{3}{2} - \frac{1}{2} \cos 20^\circ + \frac{1}{2} \cos 80^\circ + \frac{1}{2} \cos 40^\circ = \frac{3}{2} - \frac{1}{2} \cos 20^\circ + \cos \frac{80^\circ + 40^\circ}{2} \cos \frac{80^\circ - 40^\circ}{2} = \\ &= \frac{3}{2} - \frac{1}{2} \cos 20^\circ + \cos 60^\circ \cos 20^\circ = \frac{3}{2} - \frac{1}{2} \cos 20^\circ + \frac{1}{2} \cos 20^\circ = \frac{3}{2}; \end{aligned}$$

$$\begin{aligned} 6) \cos^2 35^\circ + \cos^2 25^\circ + \cos^2 5^\circ &= \frac{1}{2} (1 + \cos 70^\circ + 1 + \cos 50^\circ - 1 - \cos 10^\circ) = \\ &= \frac{1}{2} (1 + \cos 70^\circ + \cos 50^\circ - \cos 10^\circ) = \frac{1}{2} + \cos \frac{70^\circ + 50^\circ}{2} \cos \frac{70^\circ - 50^\circ}{2} - \frac{1}{2} \cos 10^\circ = \\ &= \frac{1}{2} + \cos 60^\circ \cos 10^\circ - \frac{1}{2} \cos 10^\circ = \frac{1}{2} + \frac{1}{2} \cos 10^\circ - \frac{1}{2} \cos 10^\circ = \frac{1}{2}. \end{aligned}$$

$$545. a) \sin 3x = \cos 2x; \sin 3x - \sin \left(\frac{\pi}{2} - 2x \right) = 0; 2 \sin \frac{3x - \frac{\pi}{2} + 2x}{2} \cos \frac{3x - \frac{\pi}{2} - 2x}{2} = 0;$$

$$\sin \left(\frac{5x}{2} - \frac{\pi}{4} \right) \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) = 0; \begin{cases} \sin \left(\frac{5x}{2} - \frac{\pi}{4} \right) = 0 \\ \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) = 0 \end{cases}; \begin{cases} \frac{5x}{2} - \frac{\pi}{4} = \pi n \\ \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{10} + \frac{2\pi n}{5} \\ x = \frac{\pi}{2} + 2\pi k \end{cases}; x = \frac{\pi}{10} + \frac{2\pi n}{5};$$

$$6) \sin(5x - x) = \cos(2x + 7\pi); \sin x = -\cos 2x; \cos 2x + \cos \left(\frac{\pi}{2} - x \right) = 0;$$

$$2 \cos \frac{2x + \frac{\pi}{2} - x}{2} \cos \frac{2x + \frac{\pi}{2} + x}{2} = 0; \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) \cos \left(\frac{3x}{2} - \frac{\pi}{4} \right) = 0;$$

$$\begin{cases} \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) = 0 \\ \cos \left(\frac{3x}{2} - \frac{\pi}{4} \right) = 0 \end{cases}; \begin{cases} \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi n \\ \frac{3x}{2} - \frac{\pi}{4} = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{2} + 2\pi n \\ x = \frac{\pi}{2} + \frac{2\pi k}{3} \end{cases}; x = \frac{\pi}{2} + \frac{2\pi k}{3};$$

$$b) \cos 5x = \sin 15x; \cos 5x - \cos \left(\frac{\pi}{2} - 15x \right) = 0; 2 \sin \frac{5x + \frac{\pi}{2} - 15x}{2} \sin \frac{\pi - 15x - 5x}{2} = 0;$$

$$\sin \left(\frac{\pi}{4} - 5x \right) \sin \left(\frac{\pi}{4} - 10x \right) = 0; \sin \left(5x - \frac{\pi}{4} \right) \sin \left(10x - \frac{\pi}{4} \right) = 0;$$

$$\begin{cases} \sin\left(5x - \frac{\pi}{4}\right) = 0 \\ \cos\left(10x - \frac{\pi}{4}\right) \end{cases}; \begin{cases} 5x - \frac{\pi}{4} = \pi n \\ 10x - \frac{\pi}{4} = \pi k \end{cases}; \begin{cases} x = \frac{\pi}{10} + \frac{2\pi n}{5} \\ x = \frac{\pi}{2} + 2\pi k \end{cases};$$

$$r) \sin(7\pi + x) = \cos(9\pi + 2x); -\sin x = -\cos 2x; \cos 2x - \sin x = 0; \cos 2x - \cos\left(\frac{\pi}{2} - x\right) = 0;$$

$$2\sin\frac{2x + \frac{\pi}{2} - x}{2} \sin\frac{\frac{\pi}{2} - x - 2}{2} = 0; \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \sin\left(\frac{\pi}{4} - \frac{3x}{2}\right) = 0; \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \sin\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0;$$

$$\begin{cases} \sin\left(\frac{x}{2} - \frac{\pi}{4}\right) = 0 \\ \sin\left(\frac{3x}{2} - \frac{\pi}{4}\right) \end{cases}; \begin{cases} \frac{x}{2} + \frac{\pi}{4} = \pi n \\ \frac{3x}{2} - \frac{\pi}{4} = \pi k \end{cases}; \begin{cases} x = -\frac{\pi}{2} + 2\pi n \\ x = \frac{\pi}{6} + \frac{2\pi k}{3} \end{cases}; x = \frac{\pi}{6} + \frac{2\pi k}{3}.$$

$$546. a) 1 + \cos 6x = 2 \sin^2 5x; 1 + \cos 6x = 1 - \cos 10x; \cos 6x + \cos 10x = 0;$$

$$2\cos\frac{6x+10x}{2} \cos\frac{10x-6x}{2} = 0; \cos 8x \cos 2x = 0; \begin{cases} \cos 8x = 0 \\ \cos 2x = 0 \end{cases}; \begin{cases} 8x = \frac{\pi}{2} + \pi k \\ 2x = \frac{\pi}{2} + \pi n \end{cases}; \begin{cases} x = \frac{\pi}{16} + \frac{\pi k}{8} \\ x = \frac{\pi}{4} + \frac{\pi n}{2} \end{cases};$$

$$b) \cos^2 2x = \cos^2 4x; \frac{1}{2}(1 + \cos 4x) = \frac{1}{2}(1 + \cos 8x); \cos 4x - \cos 8x = 0;$$

$$2\sin\frac{4x+8x}{2} \cos\frac{8x-4x}{2} = 0; \sin 6x \sin 2x = 0; \begin{cases} \sin 6x = 0 \\ \sin 2x = 0 \end{cases}; \begin{cases} 6x = \pi n \\ 2x = \pi k \end{cases}; \begin{cases} x = \frac{\pi n}{6} \\ x = \frac{\pi k}{2} \end{cases}; x = \frac{\pi n}{6};$$

$$в) \sin^2 \frac{x}{2} = \cos^2 \frac{7x}{2}; \frac{1}{2}(1 - \cos x) = \frac{1}{2}(1 + \cos 7x); \cos 7x + \cos x = 0;$$

$$2\cos\frac{7x+x}{2} \cos\frac{7x-x}{2} = 0; \cos 4x \cos 3x = 0; \begin{cases} \cos 4x = 0 \\ \cos 3x = 0 \end{cases}; \begin{cases} 4x = \frac{\pi}{2} + \pi n \\ 3x = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{8} + \frac{\pi n}{4} \\ x = \frac{\pi}{6} + \frac{\pi k}{3} \end{cases};$$

$$r) \sin^2 x + \sin^2 3x = 1; \frac{1}{2}(1 - \cos 2x) = \frac{1}{2}(1 - \cos 6x); \cos 2x - \cos 6x = 0;$$

$$2\cos\frac{2x+6x}{2} \cos\frac{6x-2x}{2} = 0; \cos 4x \cos 2x = 0; \begin{cases} \cos 4x = 0 \\ \cos 2x = 0 \end{cases}; \begin{cases} 4x = \frac{\pi}{2} + \pi n \\ 2x = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{8} + \frac{\pi n}{4} \\ x = \frac{\pi}{4} + \frac{\pi k}{2} \end{cases};$$

$$547. a) 2\sin 2x + \cos 5x = 1; 1 - \cos 2x + 5x = 1; \cos 5x - \cos 2x = 0;$$

$$\begin{cases} \sin\frac{7x}{2} = 0 \\ \sin\frac{3x}{2} = 0 \end{cases}; \begin{cases} \frac{7x}{2} = \pi n \\ \frac{3x}{2} = \pi k \end{cases}; \begin{cases} x = \frac{2\pi n}{7} \\ x = \frac{2\pi k}{3} \end{cases};$$

$$6) 2\sin^2 3x - 1 = \cos^2 4x - \sin^2 4x; -\cos 6x = \cos 8x; \cos 6x + \cos 8x = 0;$$

$$2\cos \frac{6x+8x}{2} \cos \frac{8x-6x}{2} = 0; \cos 7x \cos x = 0; \begin{cases} \cos 7x = 0 \\ \cos x = 0 \end{cases}; \begin{cases} 7x = \frac{\pi}{2} + \pi n \\ x = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{14} + \frac{\pi n}{7} \\ x = \frac{\pi}{6} + \frac{\pi k}{3} \end{cases}; x = \frac{\pi}{14} + \frac{\pi n}{7}.$$

$$548. a) \operatorname{tg} x + \operatorname{tg} 5x = 0; \frac{\sin(x+5x)}{\cos x \cos 5x} = 0; \frac{\sin 6x}{\cos x \cos 5x} = 0;$$

$$\begin{cases} \sin 6x = 0 \\ \cos x \neq 0 \\ \cos 5x \neq 0 \end{cases}; \begin{cases} 6x = \pi n \\ x \neq \frac{\pi}{2} + \pi k \\ 5x \neq \frac{\pi}{2} + \pi l \end{cases}; \begin{cases} x = \frac{\pi n}{6} \\ x \neq \frac{\pi}{2} + \pi k \\ x \neq \frac{\pi}{10} + \frac{\pi l}{5} \end{cases}; \begin{cases} x = \frac{\pi n}{6} \\ n \neq 3 + 6k \\ 5n \neq 3 + 6l \end{cases};$$

$$6) \operatorname{tg} 3x = \operatorname{ctg} x; \frac{\sin 3x}{\cos 3x} = \frac{\cos x}{\sin x}; \begin{cases} \sin 3x \sin x = \cos 3x \cos x \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases}; \begin{cases} \sin 3x \sin x - \cos 3x \cos x = 0 \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases};$$

$$\begin{cases} \cos(3x+x) = 0 \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases}; \begin{cases} \cos 4x = 0 \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases}; \begin{cases} 4x = \frac{\pi}{2} + \pi n \\ 3x \neq \frac{\pi}{2} + \pi k \\ x \neq \pi l \end{cases}; \begin{cases} x = \frac{\pi}{8} + \frac{\pi n}{4} \\ x \neq \frac{\pi}{6} + \frac{\pi k}{3} \\ x \neq \pi l \end{cases}; x = \frac{\pi}{8} + \frac{\pi n}{4};$$

$$b) \operatorname{tg} 2x + \operatorname{tg} 4x = 0; \frac{\sin 2x}{\cos 2x} = \frac{\sin 4x}{\cos 4x}; \begin{cases} \sin 2x \cos 4x = \sin 4x \cos 2x \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \end{cases};$$

$$\begin{cases} \sin(4x-2x) = 0 \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \end{cases}; \begin{cases} \sin 2x = 0 \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \end{cases}; \begin{cases} 2x = \pi n \\ 2x \neq \frac{\pi}{2} + \pi k \\ 4x \neq \frac{\pi}{2} + \pi l \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ x \neq \frac{\pi}{4} + \frac{\pi k}{2} \\ x \neq \frac{\pi}{8} + \frac{\pi l}{4} \end{cases}; x = \frac{\pi n}{2};$$

$$r) \operatorname{ctg} \frac{x}{2} + \operatorname{ctg} \frac{3x}{2} = 0; \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} + \frac{\cos \frac{3x}{2}}{\sin \frac{3x}{2}} = 0; \begin{cases} \cos \frac{x}{2} \sin \frac{3x}{2} + \cos \frac{3x}{2} \sin \frac{x}{2} = 0 \\ \sin \frac{x}{2} \neq 0 \\ \sin \frac{3x}{2} \neq 0 \end{cases};$$

$$\begin{cases} \sin 2x = 0 \\ \sin \frac{x}{2} \neq 0 \\ \sin \frac{3x}{2} \neq 0 \end{cases}; \begin{cases} 2x = \pi n \\ \frac{x}{2} \neq \pi k \\ \frac{3x}{2} \neq \pi l \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ x \neq 2\pi k \\ x \neq \frac{2\pi l}{3} \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ n \neq 4k \\ 3n \neq 4l \end{cases}.$$

$$549. \text{ а) } \sin x + \sin 3x + \cos x + \cos 3x = 0; 2\sin \frac{x+3x}{2} \cos \frac{3x-x}{2} + 2\cos \frac{x+3x}{2} \cos \frac{3x-x}{2} = 0;$$

$$\sin 2x \cos x + \cos 2x \cos x = 0; \cos x (\sin 2x + \cos 2x) = 0;$$

$$\sqrt{2} \cos x \sin \left(\frac{\pi}{4} + 2x \right) = 0; \begin{cases} \cos x = 0 \\ \sin \left(\frac{\pi}{4} + 2x \right) = 0 \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n \\ \frac{\pi}{4} + 2x = \pi k \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n \\ x = -\frac{\pi}{8} + \frac{\pi k}{2} \end{cases};$$

$$\text{б) } \sin 5x + \sin x + 2\sin^2 x = 1; \sin 5x + \sin x - \cos 2x = 0;$$

$$2\sin \frac{5x+x}{2} \cos \frac{5x-x}{2} - \cos 2x = 0; 2\sin 3x \cos 2x - \cos 2x = 0;$$

$$\cos 2x (2\sin 3x - 1) = 0; \begin{cases} \cos 2x = 0 \\ \sin 3x = \frac{1}{2} \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + \pi n \\ 3x = (-1)^k \frac{\pi}{6} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = (-1)^k \frac{\pi}{18} + \frac{\pi k}{3} \end{cases}.$$

$$550. \text{ а) } \sin 2x + \sin 6x = \cos 2x; 2\sin \frac{2x+6x}{2} \cos \frac{6x-2x}{2} - \cos 2x = 0;$$

$$2\sin 4x \cos 2x - \cos 2x = 0; \cos 2x (2\sin 4x - 1) = 0;$$

$$\begin{cases} \cos 2x = 0 \\ \sin 4x = \frac{1}{2} \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + \pi n \\ 4x = (-1)^k \frac{\pi}{6} + \pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{24} + 2\pi k \\ x = \frac{5\pi}{24} + 2\pi k \end{cases}.$$

$$\text{На отрезке } \left[0, \frac{\pi}{2} \right] \text{ данное уравнение имеет три корня: } x = \frac{\pi}{4}, x = \frac{\pi}{24}, x = \frac{5\pi}{24};$$

$$\text{б) } 2\cos^2 x - 1 = \sin 3x; \cos 2x = \sin 3x; \cos 2x - \cos \left(\frac{\pi}{2} - 3x \right) = 0;$$

$$2\sin \frac{2x + \frac{\pi}{2} - 3x}{2} \sin \frac{\frac{\pi}{2} - 3x - 2x}{2} = 0; \sin \left(\frac{\pi}{4} - \frac{x}{2} \right) \sin \left(\frac{5x}{2} - \frac{\pi}{4} \right) = 0;$$

$$\begin{cases} \sin \left(\frac{x}{2} - \frac{\pi}{4} \right) = 0 \\ \cos \left(\frac{5x}{2} - \frac{\pi}{4} \right) = 0 \end{cases}; \begin{cases} \frac{x}{2} - \frac{\pi}{4} = \pi n \\ \frac{5x}{2} - \frac{\pi}{4} = \pi k \end{cases}; \begin{cases} x = \frac{\pi}{2} + 2\pi n \\ x = \frac{\pi}{10} + \frac{2\pi k}{5} \end{cases}.$$

$$\text{На отрезке } \left[0, \frac{\pi}{2} \right] \text{ данное уравнение имеет два корня: } x = \frac{\pi}{10}, x = \frac{\pi}{2}.$$

$$551. \text{ а) } \cos 6x + \cos 8x = \cos 10x + \cos 12x; 2\cos \frac{6x+8x}{2} \cos \frac{8x-6x}{2} = 2\cos \frac{10x+12x}{2} \cos \frac{12x-10x}{2};$$

$$\cos 7x \cos x = \cos 11x \cos x; \cos x (\cos 7x - \cos 11x) = 0;$$

$$2\cos x \sin \frac{7x+11x}{2} \sin \frac{11x-7x}{2} = 0; \cos x \sin 9x \sin 2x = 0;$$

$$\begin{cases} \cos 8x = 0 \\ \cos 2x = 0 \end{cases}; \begin{cases} 8x = \frac{\pi}{2} + \pi k \\ 2x = \frac{\pi}{2} + \pi n \end{cases}; \begin{cases} x = \frac{\pi}{16} + \frac{\pi k}{8} \\ x = \frac{\pi}{4} + \frac{\pi n}{2} \end{cases}.$$

На отрезке $[0, \pi]$ данное уравнение имеет девять корней: $x = \frac{\pi k}{9} (k=1, \dots, 8)$ и $x = \frac{\pi}{2}$;

$$\text{б) } \sin 2x + 5\sin 4x + \sin 6x = 0; (\sin 2x + \sin 6x) + 5\sin 4x = 0;$$

$$2\sin 4x \cos 2x + 5\sin 4x = 0; \sin 4x (2\cos 2x + 5) = 0; \sin 4x = 0 \text{ (т.к. } 2\cos 2x + 5 >$$

0 при любом x); $4x = \pi$; $x = \frac{\pi n}{4}$. На отрезке $[0, \pi]$ данное уравнение имеет три

корня: $x = \frac{\pi n}{4} (n=1, 2, 3)$.

552. Числа a, b, c образуют арифметическую прогрессию, если $b - a = c - d$;

$$\text{а) } \cos 2x - \cos 7x = \cos 11x - \cos 2x; 2\cos 2x - (\cos 7x + \cos 11x) = 0;$$

$$2\cos 2x - 2\cos 9x \cos 2x = 0; \cos 2x (1 - \cos 9x) = 0;$$

$$\begin{cases} \cos 2x = 0 \\ \cos 9x = 1 \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + \pi n \\ 9x = 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{2\pi k}{9} \end{cases};$$

$$\text{б) } \cos x - \sin 3x = \sin 5x - \cos x; 2\cos x - (\sin 3x + \sin 5x) = 0;$$

$$2\cos x - 2\sin 4x \cos x = 0; \cos x (1 - \sin 4x) = 0;$$

$$\begin{cases} \cos x = 0 \\ \sin 4x = 1 \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n \\ 4x = \frac{\pi}{2} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n \\ x = \frac{\pi}{8} + \frac{\pi k}{2} \end{cases}.$$

§27. Преобразование произведений тригонометрических функций в сумму

$$553. \text{ а) } \sin 23^\circ \sin 32^\circ = \frac{1}{2} (\cos (32^\circ - 23^\circ) - \cos (32^\circ + 23^\circ)) = \frac{1}{2} (\cos 9^\circ - \cos 55^\circ);$$

$$\text{б) } \cos \frac{\pi}{12} \cos \frac{\pi}{8} = \frac{1}{2} \left(\cos \left(\frac{\pi}{8} - \frac{\pi}{12} \right) + \cos \left(\frac{\pi}{8} + \frac{\pi}{12} \right) \right) = \frac{1}{2} \left(\cos \frac{\pi}{24} + \cos \frac{5\pi}{24} \right);$$

$$\text{в) } \sin 14^\circ \sin 16^\circ = \frac{1}{2} (\cos (16^\circ - 14^\circ) - \cos (16^\circ + 14^\circ)) = \frac{1}{2} (\cos 2^\circ - \cos 30^\circ) =$$

$$= \frac{1}{2} \left(\cos 2^\circ - \frac{\sqrt{3}}{2} \right);$$

$$r) 2 \sin \frac{\pi}{8} \cos \frac{\pi}{5} = \sin \left(\frac{\pi}{8} + \frac{\pi}{5} \right) + \sin \left(\frac{\pi}{8} - \frac{\pi}{5} \right) = \sin \frac{13\pi}{40} - \sin \frac{3\pi}{40}.$$

$$554. a) \sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta = \cos^2 \beta - \cos^2 \alpha;$$

$$б) \cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \beta - \sin^2 \alpha = \cos^2 \alpha - \sin^2 \beta;$$

$$в) \cos\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) \cos\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \cos^2 \frac{\beta}{2} - \sin^2 \frac{\alpha}{2} = \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\beta}{2};$$

$$r) \cos\left(\alpha + \frac{\pi}{4}\right) \cos\left(\alpha - \frac{\pi}{4}\right) = \cos^2 \frac{\pi}{4} - \sin^2 \alpha = \frac{1}{2} - \sin^2 \alpha.$$

$$555. a) \cos \alpha \sin(\alpha + \beta) = \frac{1}{2} (\sin(2\alpha + \beta) + \sin(\alpha + \beta - \alpha)) = \frac{1}{2} (\sin(2\alpha + \beta) + \sin \beta);$$

$$б) \sin(60^\circ + \alpha) \sin(60^\circ - \alpha) = \sin^2 60^\circ - \sin^2 \alpha = \frac{3}{2} \sin^2 \alpha;$$

$$в) \sin \beta \cos(\alpha + \beta) = \frac{1}{2} (\sin(\alpha + 2\beta) + \sin(\beta - \alpha - \beta)) = \frac{1}{2} (\sin(\alpha + 2\beta) - \sin \alpha);$$

$$r) \cos\left(\alpha + \frac{\pi}{4}\right) \cos\left(\alpha - \frac{\pi}{4}\right) = \cos^2 \frac{\pi}{4} - \sin^2 \alpha = \frac{1}{2} - \sin^2 \alpha.$$

$$556. a) \cos\left(x + \frac{\pi}{3}\right) \cos\left(x - \frac{\pi}{3}\right) - 0,25 = 0; \frac{1}{2} \left(\cos\left(x + \frac{\pi}{3} - x + \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{3} + x - \frac{\pi}{3}\right) \right) = \frac{1}{4};$$

$$\frac{1}{2} \left(\cos^2 \frac{\pi}{3} + \cos 2x \right) = \frac{1}{4}; \frac{1}{2} \left(-\frac{1}{2} + \cos 2x \right) = \frac{1}{4}; \cos 2x = 1; 2x = 2\pi n; x = \pi n;$$

$$б) \sin\left(x + \frac{\pi}{3}\right) \cos\left(x - \frac{\pi}{6}\right) = 1; \frac{1}{2} \left(\sin\left(x + \frac{\pi}{3} - x - \frac{\pi}{6}\right) + \sin\left(x + \frac{\pi}{3} - x + \frac{\pi}{6}\right) \right) = 1;$$

$$\frac{1}{2} \left(\sin\left(2x + \frac{\pi}{6}\right) + \sin \frac{\pi}{2} \right) = 1; \frac{1}{2} \sin\left(2x + \frac{\pi}{6}\right) + \frac{1}{2} = 1; \sin\left(2x + \frac{\pi}{6}\right) = 1;$$

$$2x + \frac{\pi}{6} = \frac{\pi}{2} + 2\pi n; x = \frac{\pi}{6} + \pi n.$$

$$557. a) 2 \sin x \cos 3x + \sin 4x = 0; \sin(x + 3x) + \sin(x - 3x) + \sin 4x = 0;$$

$$2 \sin 4x - \sin 2x = 0; 4 \sin 2x \cos 2x - \sin 2x = 0; \sin 2x (4 \cos 2x - 1) = 0;$$

$$\begin{cases} \sin 2x = 0 \\ \cos 2x = \frac{1}{4} \end{cases} \begin{cases} 2x = \pi n \\ 2x = \pm \arccos \frac{1}{4} + 2\pi k \end{cases} \begin{cases} x = \frac{\pi n}{2} \\ x = \pm \frac{1}{2} \arccos \frac{1}{4} + \pi k \end{cases}.$$

$$б) \sin \frac{x}{2} \sin \frac{3x}{2} = \frac{1}{2}; \frac{1}{2} \left(\cos\left(\frac{x}{2} - \frac{3x}{2}\right) - \cos\left(\frac{x}{2} + \frac{3x}{2}\right) \right) = \frac{1}{2}; \cos x - \cos 2x = 1;$$

$$2 \cos^2 x - \cos x = 0; \cos x (2 \cos x - 1) = 0; \begin{cases} \cos x = 0 \\ \cos x = \frac{1}{2} \end{cases} \begin{cases} x = \frac{\pi}{2} + \pi n \\ x = \pm \frac{\pi}{3} + 2\pi k \end{cases}.$$

$$558. \text{ а) } \sin 10^\circ \cos 8^\circ \cos 6^\circ = \frac{1}{2} \sin 10^\circ (\cos (8^\circ - 6^\circ) + \cos (8^\circ + 6^\circ)) =$$

$$= \frac{1}{2} \sin 10^\circ \cos 2^\circ + \frac{1}{2} \sin 10^\circ = \frac{1}{4} (\sin 12^\circ + \sin 8^\circ + \sin 24^\circ - \sin 4^\circ);$$

$$\text{ б) } 4 \sin 25^\circ \cos 15^\circ \sin 5^\circ = 2 \sin 25^\circ (\sin 20^\circ + \sin (-10^\circ)) = 2 (\sin 25^\circ \sin 20^\circ - \sin 25^\circ \sin 10^\circ) = \cos 5^\circ - \cos 45^\circ - \cos 15^\circ + \cos 35^\circ = \cos 5^\circ - \cos 15^\circ + \cos 35^\circ - \frac{1}{\sqrt{2}}.$$

$$559. \text{ а) } 2 \sin t \sin 2t + \cos 3t = \cos (2t - 1) - \cos (2t + 1) + \cos 3t = \cos t.$$

$$\text{ б) } \sin \alpha - 2 \sin \left(\frac{\alpha}{2} - 15^\circ \right) \cos \left(\frac{\alpha}{2} + 15^\circ \right) = \sin \alpha - \sin \left(\frac{\alpha}{2} - 15^\circ + \frac{\alpha}{2} + 15^\circ \right) -$$

$$- \sin \left(\frac{\alpha}{2} - 15^\circ - \frac{\alpha}{2} - 15^\circ \right) = \sin \alpha - \sin \alpha - \sin (-30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

$$560. \text{ а) } \cos^2 3^\circ + \cos^2 1^\circ - \cos 4^\circ \cos 2^\circ = \cos^2 3^\circ + \cos^2 1^\circ - \cos (3^\circ + 1^\circ) \cos (3^\circ - 1^\circ) =$$

$$= \cos^2 3^\circ + \cos^2 1^\circ - \cos^2 1^\circ + \sin^2 3^\circ = \cos^2 3^\circ + \sin^2 3^\circ = 1;$$

$$\text{ б) } \sin^2 10^\circ + \cos 50^\circ \cos 70^\circ = \sin^2 10^\circ = \cos (60^\circ - 10^\circ) \cos (60^\circ + 10^\circ) =$$

$$= \sin^2 10^\circ + \cos^2 60^\circ - \sin^2 10^\circ = \cos^2 60^\circ = \frac{1}{4}.$$

$$561. \text{ а) } \frac{1}{2 \sin 10^\circ} - 2 \sin 70^\circ = \frac{1 - 4 \sin 10^\circ \sin 70^\circ}{2 \sin 10^\circ} = \frac{1 - 2 \cos (70^\circ - 10^\circ) + 2 \cos (70^\circ + 10^\circ)}{2 \sin 10^\circ} =$$

$$= \frac{1 - 2 \cos 60^\circ + 2 \cos 80^\circ}{2 \sin 10^\circ} = \frac{1 - 2 \cdot \frac{1}{2} + 2 \sin 10^\circ}{2 \sin 10^\circ} = 1;$$

$$\text{ б) } \frac{\operatorname{tg} 60^\circ}{\sin 40^\circ} + 4 \cos 100^\circ = \frac{\operatorname{tg} 60^\circ + 4 \sin 40^\circ \cos 100^\circ}{\sin 40^\circ} = \frac{\operatorname{tg} 60^\circ + 2 \sin 140^\circ + 2 \sin (-60^\circ)}{\sin 40^\circ} =$$

$$= \frac{\sqrt{3} + 2 \sin 40^\circ - 2 \cdot \frac{\sqrt{3}}{2}}{\sin 40^\circ} = \frac{2 \sin 40^\circ}{\sin 40^\circ} = 2.$$

$$562. \text{ а) } \sin 3x \cos x = \sin \frac{5x}{2} \cos \frac{3x}{2}.$$

$$\frac{1}{2} (\sin (3x + x) + \sin (3x - x)) = \frac{1}{2} \sin \left(\sin \left(\frac{5x}{2} + \frac{3x}{2} \right) + \sin \left(\frac{5x}{2} + \frac{3x}{2} \right) \right);$$

$$\sin 4x + \sin 2x = \sin 4x + \sin x; \sin 2x - \sin x = 0; 2 \sin \frac{2x - x}{2} \cos \frac{2x + x}{2} = 0;$$

$$\sin \frac{x}{2} \cos \frac{3x}{2} = 0; \begin{cases} \sin \frac{x}{2} = 0 \\ \cos \frac{3x}{2} \end{cases}; \begin{cases} \frac{x}{2} = \pi n \\ \frac{3x}{2} = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = 2\pi n \\ x = \frac{\pi}{3} + \frac{2\pi k}{3} \end{cases}.$$

$$\text{ б) } 2 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) + \sin^2 x = 0.$$

$$2\left(\sin^2 \frac{\pi}{4} - \sin^2 x\right) + \sin^2 x = 0; 1 - \sin^2 x = 0; -\frac{1}{2}(1 - \cos 2x) + 1 = 0;$$

$$\cos 2x = -1; 2x = \pi + 2\pi n; x = \frac{\pi}{2} + \pi n.$$

$$\text{в) } \sin 2x \cos x = \sin x \cos 2x.$$

$$\frac{1}{2}(\sin 3x + \sin x) = \frac{1}{2}(\sin 3x - \sin x); \sin x = 0; x = \pi n.$$

$$\text{г) } \cos 2x \cos x = \cos 2,5x \cos 0,5x.$$

$$\frac{1}{2}(\cos x \cos 3x) = \frac{1}{2}(\cos 2x + \cos 3x); \cos x = \cos 2x; \cos x - \cos 2x = 0;$$

$$2\sin \frac{x+2x}{2} \sin \frac{2x-x}{2} = 0; \sin \frac{3x}{2} \sin \frac{x}{2} = 0; \begin{cases} \sin \frac{3x}{2} = 0 \\ \sin \frac{x}{2} = 0 \end{cases}; \begin{cases} \frac{3x}{2} = \pi n \\ \frac{x}{2} = \pi k \end{cases}; \begin{cases} x = \frac{2\pi n}{3} \\ x = 2\pi k \end{cases}.$$

$$563. \text{ а) } \sin x \sin 3x = 0,5; \frac{1}{2}(\cos 2x - \cos 4x) = \frac{1}{2}; \cos 2x = 1 + \cos 4x; \cos 2x = 2\cos^2 2x;$$

$$\cos 2x (2\cos 2x - 1) = 0; \begin{cases} \cos 2x = 0 \\ \cos 2x = \frac{1}{2} \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + \pi n \\ 2x = \pm \frac{\pi}{3} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \pm \frac{\pi}{6} + \pi k \end{cases}.$$

$$\text{б) } \cos x \cos 3x = 0,5; \frac{1}{2}(\cos 2x + \cos 4x) = \frac{1}{2}; \cos 2x + (1 + \cos 4x) = 0; \cos 2x +$$

$$2\cos^2 2x = 0; \cos 2x (2\cos 2x + 1) = 0; \begin{cases} \cos 2x = 0 \\ \cos 2x = -\frac{1}{2} \end{cases}; \begin{cases} 2x = \frac{\pi}{2} + \pi n \\ 2x = \pm \frac{2\pi}{3} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \pm \frac{\pi}{3} + \pi k \end{cases}.$$

$$564. \text{ а) } f(x) = \sin\left(x + \frac{\pi}{8}\right) \cos\left(x - \frac{\pi}{24}\right) = \frac{1}{2}\left(\sin\left(x + \frac{\pi}{8} + x - \frac{\pi}{24}\right) + \sin\left(x + \frac{\pi}{8} + x + \frac{\pi}{24}\right)\right) = \\ = \frac{1}{2}\sin\left(2x + \frac{\pi}{12}\right) + \frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{2}\sin\left(2x + \frac{\pi}{12}\right) + \frac{1}{4}.$$

Поскольку наибольшее и наименьшее значения функции $y = \frac{1}{2}\sin\left(2x + \frac{\pi}{12}\right)$

равны 1 и -1 соответственно, то наибольшее и наименьшее значения функции

$f(x)$ равны $\frac{3}{4}$ и $-\frac{1}{4}$ соответственно.

$$\text{б) } f(x) = \sin\left(x - \frac{\pi}{3}\right) \cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}\left(\sin\left(x + \frac{\pi}{8} + x - \frac{\pi}{24}\right) + \sin\left(x + \frac{\pi}{8} - x + \frac{\pi}{24}\right)\right) = \\ = \frac{1}{2}\left(\cos\frac{2\pi}{3} - \cos 2x\right) = -\frac{1}{4} - \frac{1}{2}\cos 2x.$$

Поскольку наибольшее и наименьшее значения функции $y = \cos 2x$ равны 1 и

–1 соответственно, то наибольшее и наименьшее значения функции $f(x)$ равны $\frac{1}{4}$ и $-\frac{3}{4}$ соответственно.

$$\begin{aligned} 565. \cos^2(45^\circ - \alpha) - \cos^2(60^\circ + \alpha) - \cos 75^\circ \sin(75^\circ - 2\alpha) &= \cos^2(45^\circ - \alpha) - \\ &- \cos^2(60^\circ + \alpha) - \frac{1}{2}(\sin(75^\circ - 2\alpha + 75^\circ) + \sin(75^\circ - 2\alpha - 75^\circ)) = \\ &= \frac{1}{2}(1 + \cos(90^\circ - 2\alpha)) - \frac{1}{2}(1 + \cos(120^\circ + 2\alpha)) - \sin(150^\circ - 2\alpha) + \sin 2\alpha = \\ &= \sin 2\alpha - \frac{1}{2}(\cos 120^\circ \cos 2\alpha - \sin 120^\circ \sin 2\alpha) - \frac{1}{2}(\sin 150^\circ \cos 2\alpha - \sin 2\alpha \cos 150^\circ) = \\ &= \sin 2\alpha - \frac{1}{2}\left(-\frac{1}{2}\cos 2\alpha - \frac{\sqrt{3}}{2}\sin 2\alpha + \frac{1}{2}\cos 2\alpha + \frac{\sqrt{3}}{2}\sin 2\alpha\right) = \sin 2\alpha. \end{aligned}$$

566. Числа a, b, c образуют геометрическую прогрессию, если $a \neq 0, b \neq 0, c \neq 0$ и $\frac{b}{a} = \frac{c}{b}$.

$$a) \begin{cases} \frac{\cos 4x}{\cos 6x} = \frac{\cos 2x}{\cos 4x} \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \begin{cases} \cos^2 4x = \cos 2x \cos 6x \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \begin{cases} \frac{1}{2}(1 + \cos 8x) = \frac{1}{2}(\cos 8x + \cos 4x) \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases};$$

$$\begin{cases} \cos 4x = 1 \\ \cos 2x \neq 0 \\ \cos 4x \neq 0 \\ \cos 6x \neq 0 \end{cases}; \begin{cases} 4x = 2\pi n \\ \cos 2x \neq 0; \\ \cos 6x \neq 0 \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ 2x \neq \frac{\pi}{2} + \pi k; \\ 6x \neq \frac{\pi}{2} + \pi \ell \end{cases}; \begin{cases} x = \frac{\pi n}{2} \\ x \neq \frac{\pi}{4} + \frac{\pi k}{2}; \\ x \neq \frac{\pi}{12} + \frac{\pi \ell}{6} \end{cases}; x = \frac{\pi n}{2}.$$

$$b) \begin{cases} \frac{\sin 3x}{\sin 2x} = \frac{\sin 4x}{\sin 3x} \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}; \begin{cases} \sin^2 3x = \sin 2x \sin 4x \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}; \begin{cases} \frac{1}{2}(1 - \cos 6x) = \frac{1}{2}(\cos 2x - \cos 6x) \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases};$$

$$\begin{cases} \cos 2x = 1 \\ \sin 2x \neq 0 \\ \sin 4x \neq 0 \\ \sin 6x \neq 0 \end{cases}; \begin{cases} 2x = 2\pi n \\ 2x \neq \pi k \\ 4x \neq \pi \ell \\ 6x \neq \pi m \end{cases} \Rightarrow a, b, c \text{ не образуют геометрическую прогрессию.}$$

§ 28. Преобразование выражения $A \sin x + B \cos x$ к виду $C \sin(x + t)$

$$567. a) \sqrt{3} \sin x + \cos x = 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right) = 2 \left(\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} \right) = 2 \sin \left(x + \frac{\pi}{6} \right);$$

$$\text{б) } \sin x + \sqrt{3} \cos x = 2 \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right) = 2 \left(\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} \right) = 2 \cos \left(x - \frac{\pi}{6} \right);$$

$$\text{в) } \sin x - \cos x = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x \right) = \sqrt{2} \left(\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} \right) = \sqrt{2} \sin \left(x - \frac{\pi}{4} \right);$$

$$\text{г) } 2 \sin x - \sqrt{12} \cos x = 4 \left(\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x \right) = 4 \left(\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} \right) = 4 \sin \left(x - \frac{\pi}{3} \right).$$

$$\begin{aligned} 568. \text{ а) } 3 \sin x + 4 \cos x &= 5 \left(\frac{3}{5} \sin x + \frac{4}{5} \cos x \right) = \\ &= 5 \left(\sin x \cos \left(\arcsin \frac{4}{5} \right) + \cos x \sin \left(\arcsin \frac{4}{5} \right) \right) = 5 \sin \left(x + \arcsin \frac{4}{5} \right); \end{aligned}$$

$$\begin{aligned} \text{б) } 5 \cos x - 12 \sin x &= 13 \left(\frac{5}{13} \cos x - \frac{12}{13} \sin x \right) = \\ &= 13 \left(\cos x \cos \left(\arcsin \frac{12}{13} \right) - \sin x \sin \left(\arcsin \frac{12}{13} \right) \right) = 13 \cos \left(x + \arcsin \frac{12}{13} \right); \end{aligned}$$

$$\begin{aligned} \text{в) } 7 \sin x - 24 \cos x &= 25 \left(\frac{7}{25} \sin x - \frac{24}{25} \cos x \right) = \\ &= 25 \left(\sin x \cos \left(\arcsin \frac{24}{25} \right) - \cos x \sin \left(\arcsin \frac{24}{25} \right) \right) = 25 \sin \left(x - \arcsin \frac{24}{25} \right); \end{aligned}$$

$$\begin{aligned} \text{г) } 8 \cos x + 15 \sin x &= 17 \left(\frac{8}{17} \cos x + \frac{15}{17} \sin x \right) = \\ &= 17 \left(\cos x \cos \left(\arcsin \frac{15}{17} \right) + \sin x \sin \left(\arcsin \frac{15}{17} \right) \right) = 17 \cos \left(x - \arcsin \frac{15}{17} \right). \end{aligned}$$

$$569. \text{ а) } \sqrt{3} \sin x + \cos x = 1. \quad 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right) = 1; \sin x \sin \frac{\pi}{3} + \cos x \cos \frac{\pi}{3} = \frac{1}{2};$$

$$\cos \left(x - \frac{\pi}{3} \right) = \frac{1}{2}; \begin{cases} x - \frac{\pi}{3} = \frac{\pi}{3} + 2\pi k \\ x - \frac{\pi}{3} = -\frac{\pi}{3} + 2\pi n \end{cases}; \begin{cases} x = \frac{2\pi}{3} + 2\pi k \\ x = 2\pi n \end{cases}$$

$$\text{б) } \sin x + \cos x = \sqrt{2}. \frac{1}{\sqrt{2} \sin x} + \frac{1}{\sqrt{2}} \cos x = 1; \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} = 1;$$

$$\sin \left(x + \frac{\pi}{4} \right) = 1; x + \frac{\pi}{4} = \frac{\pi}{2} + 2\pi n; x = \frac{\pi}{4} + 2\pi n.$$

$$\text{в) } \sin x - \sqrt{3} \cos x = \sqrt{3}. \quad \frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x = \frac{\sqrt{3}}{2}; \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2};$$

$$\sin \left(x - \frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}; \begin{cases} x - \frac{\pi}{3} = \frac{\pi}{3} + 2\pi n \\ x - \frac{\pi}{3} = \frac{2\pi}{3} + 2\pi k \end{cases}; \begin{cases} x = \frac{2\pi}{3} + 2\pi n \\ x = \pi + 2\pi k \end{cases}$$

$$r) \sin x - \cos x = 1 \cdot \frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}; \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}};$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}; \begin{cases} x - \frac{\pi}{4} = \frac{\pi}{4} + 2\pi n \\ x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{2} + 2\pi n \\ x = \pi + 2\pi k \end{cases}.$$

$$570. a) y = \sqrt{3} \sin x + \cos x = 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right) = 2 \left(\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} \right) = 2 \sin \left(x + \frac{\pi}{6} \right).$$

Наибольшее значение 2, наименьшее -2.

$$б) y = \sin x - \sqrt{3} \cos x = 2 \left(\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x \right) = 2 \left(\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} \right) = 2 \sin \left(x - \frac{\pi}{3} \right).$$

Наибольшее значение 2, наименьшее -2.

$$в) y = \sin x - \cos x = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x \right) = \sqrt{2} \sin \left(x - \frac{\pi}{4} \right).$$

Наибольшее значение $\sqrt{2}$, наименьшее $-\sqrt{2}$.

$$г) y = \sqrt{6} \sin x - \sqrt{2} \cos x = 2\sqrt{2} \left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right) = 2\sqrt{2} \left(\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6} \right) = 2\sqrt{2} \sin \left(x - \frac{\pi}{6} \right).$$

Наибольшее значение $2\sqrt{2}$, наименьшее $-2\sqrt{2}$.

$$571. a) y = 3 \sin 2x - 4 \cos 2x = 5 \left(\frac{3}{5} \sin 2x - \frac{4}{5} \cos 2x \right) = 5 \left(\sin 2x \cos \left(\arcsin \frac{4}{5} \right) + \cos 2x \sin \left(\arcsin \frac{4}{5} \right) \right) = 5 \sin \left(2x - \arcsin \frac{4}{5} \right).$$

Область значений $[-5; 5]$.

$$б) y = 5 \cos 3x + 12 \sin 3x = 13 \left(\frac{5}{13} \cos 3x + \frac{12}{13} \sin 3x \right) = 13 \left(\cos 3x \cos \left(\arcsin \frac{12}{13} \right) + \sin 3x \sin \left(\arcsin \frac{12}{13} \right) \right) = 13 \cos \left(3x - \arcsin \frac{12}{13} \right).$$

Область значений $[-13; 13]$.

$$в) y = 7 \sin \frac{x}{2} + 24 \cos \frac{x}{2} = 25 \left(\frac{7}{25} \sin \frac{x}{2} + \frac{24}{25} \cos \frac{x}{2} \right) = 25 \left(\sin \frac{x}{2} \cos \left(\arcsin \frac{7}{25} \right) + \cos \frac{x}{2} \sin \left(\arcsin \frac{7}{25} \right) \right) = 25 \sin \left(\frac{x}{2} + \arcsin \frac{7}{25} \right).$$

Область значений $[-25; 25]$.

$$г) y = 8 \cos \frac{x}{3} - 15 \sin \frac{x}{3} = 17 \left(\frac{8}{17} \cos \frac{x}{3} - \frac{15}{17} \sin \frac{x}{3} \right) = 17 \left(\cos \frac{x}{3} \cos \left(\arcsin \frac{15}{17} \right) - \sin \frac{x}{3} \sin \left(\arcsin \frac{15}{17} \right) \right) = 17 \cos \left(\frac{x}{3} + \arcsin \frac{15}{17} \right).$$

Область значений $[-17; 17]$.

572. а) $\sin 5x + \cos 5x = 1,5$. $\sqrt{2} \sin\left(5x + \frac{\pi}{4}\right) = \frac{3}{2}$; $\sin\left(5x + \frac{\pi}{4}\right) = \frac{3}{2\sqrt{2}} = \sqrt{\frac{9}{8}}$.

Это равенство не выполняется ни при одном значении x , т.к.

$$\sin\left(5x + \frac{\pi}{4}\right) \leq 1 \text{ при всех } x, \text{ а } \sqrt{\frac{9}{8}} > 1.$$

б) $3\sin 2x - 4\cos 2x = \sqrt{26}$. $5\left(\frac{3}{5}\sin 2x - \frac{4}{5}\cos 2x\right) = \sqrt{26}$;

$$5\left(\sin 2x \cos\left(\arcsin \frac{4}{5}\right) - \cos 2x \sin\left(\arcsin \frac{4}{5}\right)\right) = \sqrt{26}; 5\sin\left(2x - \arcsin \frac{4}{5}\right) = \sqrt{26};$$

$$\sin\left(2x - \arcsin \frac{4}{5}\right) = \frac{\sqrt{26}}{5}. \text{ Это равенство не выполняется ни при одном значе-}$$

$$\text{нии } x, \text{ т.к. } \sin\left(2x - \arcsin \frac{4}{5}\right) \leq 1 \text{ при всех } x, \text{ а } \frac{\sqrt{26}}{5} > 1.$$

в) $\sin 7x - \sqrt{3} \cos 7x = \frac{\pi}{2}$. $\frac{1}{2}\sin 7x - \frac{\sqrt{3}}{2}\cos 7x = \frac{\pi}{4}$; $\sin 7x \cos \frac{\pi}{3} - \cos 7x \sin \frac{\pi}{3} = \frac{\pi}{4}$;

$$\sin\left(7x - \frac{\pi}{3}\right) = \frac{\pi}{4}. \text{ Значения } x, \text{ при которых выполняется это равенство, существуют,}$$

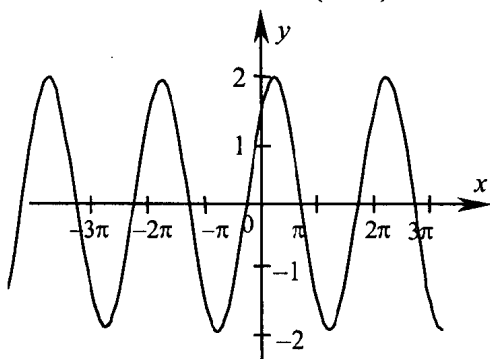
$$\text{т.к. область значений функции } y = \sin\left(7x - \frac{\pi}{3}\right) \text{ — это отрезок } [-1; 1] \text{ и } -1 < \frac{\pi}{4} < 1.$$

г) $5\sin x + 12\cos x = \sqrt{170}$. $\frac{5}{13}\sin x + \frac{12}{13}\cos x = \frac{\sqrt{170}}{13}$; $\sin x \cos\left(\arcsin \frac{12}{13}\right) +$

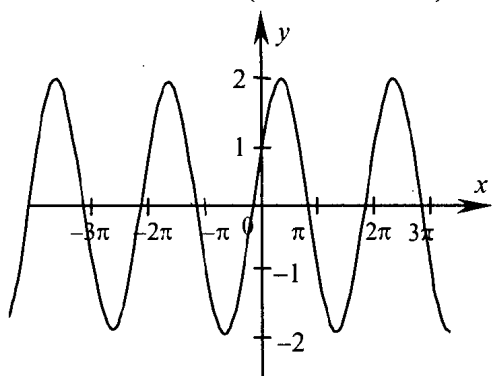
$$+ \cos x \sin\left(\arcsin \frac{12}{13}\right) = \frac{\sqrt{170}}{13}; \sin\left(x + \arcsin \frac{12}{13}\right) = \frac{\sqrt{170}}{13}. \text{ Это равенство не выпол-}$$

$$\text{няется ни при одном значении } x, \text{ т.к. } \sin\left(x + \arcsin \frac{12}{13}\right) \leq 1 \text{ при всех } x, \text{ а } \frac{\sqrt{170}}{13} > 1.$$

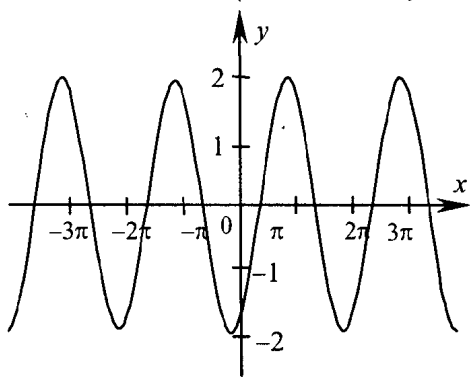
573. а) $y = \sqrt{2}(\sin x + \cos x) = 2\sin\left(x + \frac{\pi}{4}\right)$.



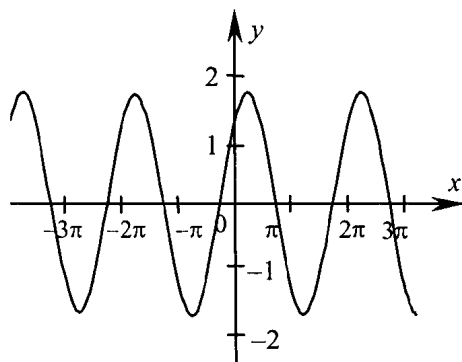
$$\text{б) } y = \sqrt{3} \sin x + \cos x = 2 \left(\sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2} \right) = 2 \left(\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} \right) = 2 \sin \left(x + \frac{\pi}{6} \right).$$



$$\text{в) } y = \sin x - \sqrt{3} \cos x = 2 \left(\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x \right) = 2 \left(\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} \right) = 2 \sin \left(x - \frac{\pi}{3} \right).$$



$$\text{г) } y = \sin x - \cos x = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x \right) = \sqrt{2} \left(\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} \right) = \sqrt{2} \sin \left(x - \frac{\pi}{4} \right).$$



$$574. \text{ а) } \cos 2x + \sqrt{3} \sin 2x = \sqrt{2}; \frac{1}{2} \cos 2x + \frac{\sqrt{3}}{2} \sin 2x = \frac{1}{\sqrt{2}}; \cos 2x \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \sin 2x = \frac{1}{\sqrt{2}};$$

$$\cos\left(2x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}; \begin{cases} 2x - \frac{\pi}{3} = \frac{\pi}{4} + 2\pi n \\ 2x - \frac{\pi}{3} = -\frac{\pi}{4} + 2\pi k \end{cases}; \begin{cases} x = \frac{7\pi}{24} + \pi n \\ x = \frac{\pi}{24} + \pi k \end{cases}.$$

$$\text{ б) } \sin 5x - \cos 5x = \frac{\sqrt{6}}{2}; \frac{1}{\sqrt{2}} \sin 5x - \frac{1}{\sqrt{2}} \cos 5x = \frac{\sqrt{3}}{2}; \sin 5x \cos \frac{\pi}{4} - \cos 5x \sin \frac{\pi}{4} = \frac{\sqrt{3}}{2};$$

$$\sin\left(5x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}; \begin{cases} 5x - \frac{\pi}{4} = \frac{\pi}{3} + 2\pi n \\ 5x - \frac{\pi}{4} = \frac{2\pi}{3} + 2\pi k \end{cases}; \begin{cases} x = \frac{7\pi}{60} + \frac{2\pi n}{5} \\ x = \frac{3\pi}{20} + \frac{2\pi k}{5} \end{cases}.$$

$$\text{ в) } \cos \frac{x}{2} - \sqrt{3} \sin \frac{x}{2} + 1 = 0; \frac{1}{2} \cos \frac{x}{2} - \frac{\sqrt{3}}{2} \sin \frac{x}{2} = -\frac{1}{2}; \sin \frac{x}{2} \cos \frac{\pi}{6} - \cos \frac{x}{2} \sin \frac{\pi}{6} = \frac{1}{2};$$

$$\sin\left(\frac{x}{2} - \frac{\pi}{6}\right) = \frac{1}{2}; \begin{cases} \frac{x}{2} - \frac{\pi}{6} = \frac{\pi}{6} + 2\pi n \\ \frac{x}{2} - \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi k \end{cases}; \begin{cases} x = \frac{2\pi}{3} + 4\pi n \\ x = 2\pi + 2\pi k \end{cases}.$$

$$\text{ г) } \sin \frac{x}{3} + \cos \frac{x}{3} = 1; \frac{1}{\sqrt{2}} \sin \frac{x}{3} + \frac{1}{\sqrt{2}} \cos \frac{x}{3} = \frac{1}{\sqrt{2}}; \sin \frac{x}{3} \cos \frac{\pi}{4} + \cos \frac{x}{3} \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}};$$

$$\sin\left(\frac{x}{3} + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}; \begin{cases} \frac{x}{3} + \frac{\pi}{4} = \frac{\pi}{4} + 2\pi k \\ \frac{x}{3} + \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi n \end{cases}; \begin{cases} x = 6\pi k \\ x = \frac{3\pi}{2} + 6\pi n \end{cases}.$$

$$575. \text{ а) } 4 \sin x - 3 \cos x = 5; \frac{4}{5} \sin x - \frac{3}{5} \cos x = 1; \sin x \cos\left(\arcsin \frac{3}{5}\right) - \cos x \sin\left(\arcsin \frac{3}{5}\right) = 1;$$

$$\sin\left(x - \arcsin \frac{3}{5}\right) = 1; x - \arcsin \frac{3}{5} = \frac{\pi}{2} + 2\pi n; x = \frac{\pi}{2} + \arcsin \frac{3}{5} + 2\pi n.$$

$$\text{ б) } 3 \sin 2x + 4 \cos 2x = 2,5; \frac{3}{5} \sin 2x + \frac{4}{5} \cos 2x = \frac{1}{2}; \sin 2x \cos\left(\arcsin \frac{4}{5}\right) + \cos 2x \sin\left(\arcsin \frac{4}{5}\right) = \frac{1}{2}; \sin\left(2x + \arcsin \frac{4}{5}\right) = \frac{1}{2}; 2x + \arcsin \frac{4}{5} + (-1)^n \frac{\pi}{6} + \pi n;$$

$$x = (-1)^n \frac{\pi}{12} - \frac{1}{2} \arcsin \frac{4}{5} + \frac{\pi n}{2}.$$

$$\text{ в) } 12 \sin x + 5 \cos x + 13 = 0; \frac{12}{13} \sin x + \frac{5}{13} \cos x = -1; \sin x \cos\left(\arcsin \frac{5}{13}\right) + \cos x \sin\left(\arcsin \frac{5}{13}\right) = -1; \sin\left(x + \arcsin \frac{5}{13}\right) = -1; x + \arcsin \frac{5}{13} = -\frac{\pi}{2} + 2\pi n;$$

$$x = -\frac{\pi}{2} - \arcsin \frac{5}{13} + 2\pi n.$$

$$r) 5\cos\frac{x}{2} - 12\sin\frac{x}{2} = 6,5; \frac{5}{13}\cos\frac{x}{2} - \frac{12}{13}\sin\frac{x}{2} = \frac{1}{2}; \cos\frac{x}{2}\cos\left(\arcsin\frac{12}{13}\right) - \sin\left(\arcsin\frac{12}{13}\right) = \frac{1}{2};$$

$$\cos\left(\frac{x}{2} + \arcsin\frac{12}{13}\right) = \frac{1}{2}; \frac{x}{2} + \arcsin\frac{12}{13} = \pm\frac{\pi}{3} + 2\pi n; x = \pm\frac{2\pi}{3} - 2\arcsin\frac{12}{13} + 4\pi n.$$

$$576. a) \sin x + \cos x + \sqrt{2} = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) + \sqrt{2} = \sqrt{2} \left(1 + \cos\left(x - \frac{\pi}{4}\right)\right) = 2\sqrt{2} \cos^2\left(\frac{x}{2} - \frac{\pi}{8}\right);$$

$$6) \cos 2x - \sin 2x = \sqrt{2} \cos\left(2x + \frac{\pi}{4}\right) - \sqrt{2} = -\sqrt{2} \left(1 - \cos\left(2x + \frac{\pi}{4}\right)\right) =$$

$$= -2\sqrt{2} \sin^2\left(\frac{2x + \frac{\pi}{4}}{2}\right) = -2\sqrt{2} \sin^2\left(x + \frac{\pi}{8}\right).$$

$$577. a) 2\sin 17x + \sqrt{3} \cos 5x + \sin 5x = 0; 2\sin 17x + 2\left(\frac{\sqrt{3}}{2} \cos 5x + \frac{1}{2} \sin 5x\right) = 0;$$

$$2\sin 17x + 2\sin\left(5x + \frac{\pi}{3}\right) = 0; \sin 17x + \sin\left(5x + \frac{\pi}{3}\right) = 0; 2\sin\frac{17x+5x+\frac{\pi}{3}}{2} \cos\frac{17x-5x-\frac{\pi}{3}}{2};$$

$$\sin\left(11x + \frac{\pi}{6}\right) \cos\left(6x - \frac{\pi}{6}\right) = 0; \begin{cases} \sin\left(11x + \frac{\pi}{6}\right) = 0 \\ \cos\left(6x - \frac{\pi}{6}\right) = 0 \end{cases}; \begin{cases} 11x + \frac{\pi}{6} = \pi n \\ 6x - \frac{\pi}{6} = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = -\frac{\pi}{66} + \frac{\pi n}{11} \\ x = \frac{\pi}{9} + \frac{\pi k}{6} \end{cases}.$$

$$6) 5\sin x - 12\cos x + 13\sin 3x = 0; \frac{5}{13}\sin x - \frac{12}{13}\cos x + \sin 3x = 0;$$

$$\sin x \cos\left(\arcsin\frac{12}{13}\right) - \cos x \sin\left(\arcsin\frac{12}{13}\right) + \sin 3x = 0; \sin\left(x - \arcsin\frac{12}{13}\right) + \sin 3x = 0;$$

$$2\sin\frac{x - \arcsin\frac{12}{13} + 3x}{2} \cos\frac{x - \arcsin\frac{12}{13} - 3x}{2} = 0;$$

$$\sin\left(2x - \frac{1}{2}\arcsin\frac{12}{13}\right) \cos\left(2x + \frac{1}{2}\arcsin\frac{12}{13}\right) = 0; \begin{cases} \sin\left(2x - \frac{1}{2}\arcsin\frac{12}{13}\right) = 0 \\ \cos\left(x + \frac{1}{2}\arcsin\frac{12}{13}\right) = 0 \end{cases};$$

$$\begin{cases} 2x - \frac{1}{2}\arcsin\frac{12}{13} = \pi n \\ x + \frac{1}{2}\arcsin\frac{12}{13} = \frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = \frac{1}{4}\arcsin\frac{12}{13} + \frac{\pi n}{2} \\ x = \frac{\pi}{2} - \frac{1}{2}\arcsin\frac{12}{13} + \pi k \end{cases}.$$

$$578. \text{ а) } (\sin x + \sqrt{3} \cos x)^2 - 5 = \cos\left(\frac{\pi}{6} - x\right) \cdot 4\left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x\right)^2 - 5 = \cos\left(\frac{\pi}{6} - x\right);$$

$$4\left(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}\right)^2 - 5 = \cos\left(\frac{\pi}{6} - x\right); 4 \sin^2\left(x + \frac{\pi}{3}\right) - 5 = \cos\left(\frac{\pi}{6} - x\right);$$

$$4 \cos^2 x \left(\frac{\pi}{6} - x\right) - 5 = \cos\left(\frac{\pi}{6} - x\right); 4 \cos^2\left(\frac{\pi}{6} - x\right) - \cos\left(\frac{\pi}{6} - x\right) - 5 = 0;$$

$$\cos\left(\frac{\pi}{6} - x\right) = \frac{1 \pm \sqrt{1+80}}{8} = \frac{1 \pm 9}{8} = \begin{cases} \frac{5}{4} \\ -1 \end{cases}; \text{ т.к. } \cos\left(\frac{\pi}{6} - x\right) \leq 1, \text{ то } \cos\left(\frac{\pi}{6} - x\right) = \frac{5}{4}.$$

$$\cos\left(\frac{\pi}{6} - x\right) = -1; x - \frac{\pi}{6} = \pi + 2\pi n; x = \frac{7\pi}{6} + 2\pi n.$$

$$\text{б) } (\sqrt{3} \sin x - \cos x)^2 + 1 = 4 \cos\left(x + \frac{\pi}{3}\right); 4\left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x\right)^2 + 1 = 4 \cos\left(x + \frac{\pi}{3}\right);$$

$$4 \cos^2\left(x + \frac{\pi}{3}\right) + 1 - 4 \cos\left(x + \frac{\pi}{3}\right) = 0; \left(2 \cos\left(x + \frac{\pi}{3}\right) - 1\right)^2 = 0;$$

$$\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}; \begin{cases} x + \frac{\pi}{3} = \frac{\pi}{3} + 2\pi k \\ x + \frac{\pi}{3} = -\frac{\pi}{3} + 2\pi n \end{cases}; \begin{cases} x = 2\pi k \\ x = -\frac{2\pi}{3} + 2\pi n \end{cases}.$$

$$579. \text{ а) } \sqrt{3} \sin x + \cos x + 2 = \frac{12}{\pi} x; \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x + 1 = \frac{6}{\pi} x; \sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} + 1 = \frac{6}{\pi} x;$$

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{6}{\pi} x - 1; x = \frac{\pi}{3}.$$

$$\text{б) } \sqrt{2}(\cos x - \sin x) = 2x - \frac{\pi}{2}; 2\left(\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}\right) = 2x - \frac{\pi}{2};$$

$$\cos\left(x + \frac{\pi}{4}\right) = x - \frac{\pi}{4}; x = \frac{\pi}{4}.$$

$$580. \text{ а) } \sqrt{3} \sin x + \cos x > 1; \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x > \frac{1}{2}; \sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} > \frac{1}{2};$$

$$\sin\left(x + \frac{\pi}{6}\right) > \frac{1}{2}; \frac{\pi}{6} + 2\pi n < x + \frac{\pi}{6} < \frac{5\pi}{6} + 2\pi n; 2\pi n < x < \frac{2\pi}{3} + 2\pi n.$$

$$\text{б) } 3 \sin x - 4 \cos x < 2.5; \frac{3}{5} \sin x - \frac{4}{5} \cos x < \frac{1}{2}; \sin x \cos\left(\arcsin \frac{4}{5}\right) - \cos x \sin\left(\arcsin \frac{4}{5}\right) < \frac{1}{2};$$

$$\sin\left(x - \arcsin \frac{4}{5}\right) < \frac{1}{2}; -\frac{7\pi}{6} + 2\pi n < x - \arcsin \frac{4}{5} < \frac{\pi}{6} + 2\pi n;$$

$$-\frac{7\pi}{6} + 2\pi n + \arcsin \frac{4}{5} < x < \frac{\pi}{6} + 2\pi n + \arcsin \frac{4}{5}.$$

Глава 4. Производная

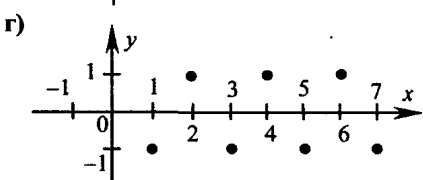
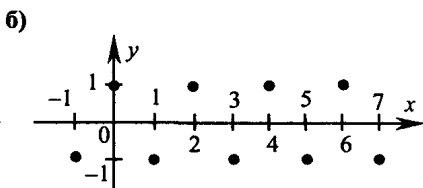
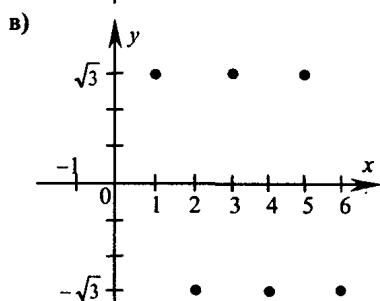
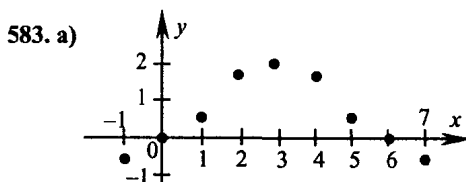
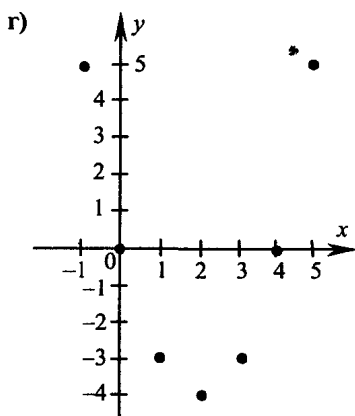
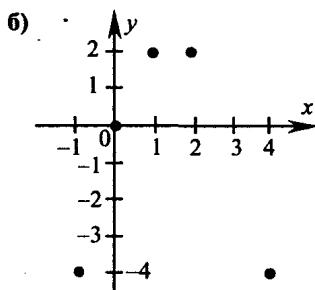
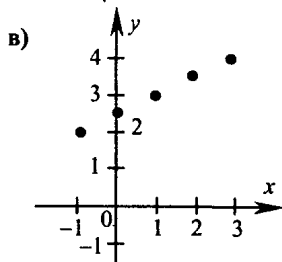
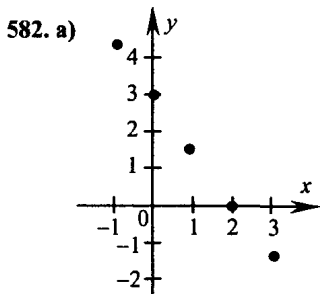
§29. Числовые последовательности

581. а) $y = 3x^2 + 5$, $x \in \mathbb{Z}$, является;

в) $y = 7 - x^2$, $x \in \mathbb{Q}$, не является;

б) $y = \sin x$, $x \in [0; 2\pi]$, не является;

г) $y = \cos \frac{x}{2}$, $x \in \mathbb{N}$, является.



584. а) $y_n = 2^n$; б) 1, 2, 4, 8, 16...; в) $b_1 = 2, b_{n+1} = b_n \cdot 2$.

585. а) $y_n = -n, n \in N, y_1 = -1, y_2 = -2, y_3 = -3, y_4 = -4, y_5 = -5$;

б) $y_n = \sqrt{n}, n \in N, y_1 = 1, y_2 = \sqrt{2}, y_3 = \sqrt{3}, y_4 = 2, y_5 = \sqrt{5}$;

в) $y_n = -5, y_1 = -5, y_2 = -5, y_3 = -5, y_4 = -5, y_5 = -5$;

г) $y_n = \frac{n^2}{2}, n \in N, y_1 = \frac{1}{2}, y_2 = 2, y_3 = \frac{9}{2}, y_4 = 8, y_5 = \frac{25}{2}$.

586. а) $y_7 = 42, y_9 = 54, y_{17} = 72, 6 \cdot n = y_n$; б) $y_6 = 42, y_{10} = 70, y_{31} = 217, 7 \cdot n = y_n$.

587. а) $y_1 = 2, y_2 = 7, y_3 = 12, y_4 = 17, y_5 = 22$; б) $S = 3 + 7 + 11 + 15 + 19 + 23 = 78$.

588. а) $y_n = n^5, n \in N, y_1 = 1, y_2 = 32, y_n = n^5, y_{n+1} = (n+1)^5$;

б) $y_n = 3^5, y_5 = 3^5 = 243, y_7 = 3^7 = 2187, y_{21} = 3^{21}, y_{2n} = 3^{2n}, y_{2n+5} = 3^{2n+5}$.

589. а) y_{732} и $y_{745}, y_{733}, y_{734}, \dots, y_{744}$;

б) y_{n-1} и y_{n+2}, y_n, y_{n+1} ;

в) y_{998} и $y_{1003}, y_{999}, y_{1000}, y_{1001}, y_{1002}$;

г) $y_{2n-2}, y_{2n-1}, y_{2n}, y_{2n+1}, y_{2n+2}$.

590. а) $y_n = 3 - 2n, y_1 = 1, y_2 = -1, y_3 = -3, y_4 = -5, y_5 = -7$;

б) $y_n = 2n^2 - n, y_1 = 1, y_2 = 6, y_3 = 15, y_4 = 28, y_5 = 45$;

в) $y_n = n^3 - 1, y_1 = 0, y_2 = 7, y_3 = 26, y_4 = 63, y_5 = 124$;

г) $y_n = \frac{3n-1}{2n} = \frac{3}{2} - \frac{1}{2n}, y_1 = 1, y_2 = \frac{5}{4} = 1\frac{1}{4}, y_3 = \frac{8}{6} = 1\frac{1}{3}, y_4 = \frac{11}{8} = 1\frac{3}{8}, y_5 = \frac{7}{5} = 1\frac{2}{5}$.

591. а) $y_n = (-1)^n, y_1 = -1, y_2 = 1, y_3 = -1, y_4 = 1, y_5 = -1$;

б) $y_n = \frac{(-2)^n}{n^2 + 1}, y_1 = -1, y_2 = \frac{4}{5}, y_3 = -\frac{8}{10} = -\frac{4}{5}, y_4 = \frac{16}{17}, y_5 = -\frac{32}{26} = -\frac{16}{13} = -1\frac{3}{13}$;

в) $y_n = (-1)^n \frac{1}{10^n}, y_1 = -\frac{1}{10}, y_2 = \frac{1}{100}, y_3 = -\frac{1}{1000}, y_4 = \frac{1}{10000}, y_5 = -\frac{1}{100000}$;

г) $y_n = \frac{(-1)^n + 2}{3n - 2}, y_1 = 1, y_2 = \frac{3}{4}, y_3 = \frac{1}{7}, y_4 = \frac{3}{10}, y_5 = \frac{1}{13}$.

593. а) $y_n = 3 \cos \frac{2\pi}{n}, y_1 = 3, y_2 = -3, y_3 = -\frac{3}{2}, y_4 = 0, y_5 = 3 \cos \frac{2\pi}{5}$;

б) $y_n = \operatorname{tg}\left((-1)^n \frac{\pi}{4}\right), y_1 = -1, y_2 = 1, y_3 = -1, y_4 = 1, y_5 = -1$;

в) $y_n = 1 - \cos^2 \frac{\pi}{n}, y_1 = 0, y_2 = 1, y_3 = \frac{3}{4}, y_4 = \frac{1}{2}, y_5 = \sin^2 \frac{\pi}{5}$;

г) $y_n = \sin \pi n - \cos \pi n = -\cos \pi n, y_1 = 1, y_2 = -1, y_3 = 1, y_4 = -1, y_5 = 1$.

593. $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 = 4 + 9 + 25 + 79 + 121 + 169 + 289 + 361 = 1027$.

594. а) $y_n = n, n = 1, 2, 3, \dots$; б) $y_n = -n, n \in N$; в) $y_n = n + 4, n \in N$; г) $y_n = 11 - n, n \in N$.

595. а) $y_n = 5n, n \in N$; б) $y_n = 6n, n \in N$; в) $y_n = 4n, n \in N$; г) $y_n = 3n, n \in N$.

596. а) $y_n = 3^n, n \in N$; б) $y_n = (n+2)^2, n \in N$; в) $y_n = n^3, n \in N$; г) $y_n = n^3 + 1, n \in N$.

597. а) $y_n = \frac{2}{2^n} = \frac{1}{2^{n-1}}$; б) $y_n = \frac{2n+1}{2n+2}$; в) $y_n = \frac{1}{n^3}$; г) $y_n = \frac{1}{(2n+1)(2n+3)}$.

598. а) $y_n = 1, 5n, y_1 = 1, 5, y_2 = 3, y_3 = 4, 5$; б) $y_n = (-1)^n, y_1 = -1, y_2 = 1, y_3 = -1$;

в) $y_n = \frac{8}{n}, y_1 = 8, y_2 = 4, y_3 = \frac{8}{3}, \dots$; г) $y_n = (-1)^{n+1}n, y_1 = 1, y_2 = -2, y_3 = 3, y_4 = -4$.

599. а) 1; 1,4; 1,41; 1,414; 1,4142.

б) 2; 1,5; 1,42; 1,415; 1,4143.

600. $y_n = \frac{2-n}{5n+1}$

а) $y_n = 0, n = 2;$

б) $y_n = -\frac{3}{26}, n = 5;$

в) $y_n = -\frac{1}{6}, \frac{2-n}{5n+1} = -\frac{1}{6}, 6n-12=5n+1, n=13;$

г) $-\frac{43}{226} = y_n, \frac{2-n}{5n+1} = -\frac{43}{226}, 226n-452=215n+43, 11n=495, n=45.$

601. $a_n = (2n-1)(3n+2)$

а) $a_n = 0$. нет, т.к. n не может быть равным $\frac{1}{2}, -\frac{2}{3}$.

б) $a_n = 24. 6n^2 + n - 2 = 24. 6n^2 + n - 26 = 0.$

$$n = \frac{-1+25}{12} = 2, \text{ или } n = -\frac{26}{12}. \text{ (не подходит, т.к. } n \in \mathbb{N}).$$

Ответ: является.

в) $a_n = 153. 6n^2 + n - 155 = 0. n = \frac{-1+61}{12} = 5. n = -\frac{62}{12}$ (не подходит).

Ответ: является.

г) $a_n = -2. 6n^2 + n = 0$

 $n(6n+1) = 0$. решение в натуральных числах нет \Rightarrow не является.

602. а) $1; \frac{1}{2}; \frac{1}{3}; \frac{1}{4} \dots$ ограничена снизу и сверху.

б) $-1; 2; -3; 4; -5 \dots$ не ограничена. в) $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5} \dots$ снизу ограничена.

г) $5; 4; 3; 2; 1; 0; -1 \dots$ не ограничена снизу.

603. а) $-3; -2; -1; 0; 1 \dots$ не ограничена сверху.

б) $1; -1; 1; -2; 1; -3 \dots$ ограничена сверху.

в) $\frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \frac{1}{5} \dots$ ограничена сверху.

г) $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5} \dots$ ограничена сверху.

604. а) $\frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \dots \frac{1}{n} \dots$ ограничена, т.к. $0 < y_n < 1$.

б) $\frac{1}{2}; \frac{3}{4}; \frac{5}{6}; \dots \frac{2n-1}{2n} \dots$ ограничена, т.к. $0 < y_n < 1$.

в) $5; -5; 5; -5; (-1)^{n-1}5$ ограничена, т.к. $-10 < y_n < 10$,

г) $-2; 3; -4; 5; (-1)^n(n+1)$ не ограничена.

605. а) $x_n = 3 \cdot n + 2$; возьмем $n_2 > n_1$

$x_{n_2} = 3 \cdot 2n_2 + 2 > 3 \cdot n_1 + 2 \Rightarrow$ последов. возрастающая.

б) $x_n = \frac{5}{n+3}$ убывающая, т.к. при $n_2 > n_1, x_{n_2} < x_{n_1}$

в) $x_n = n^3$ возрастающая, т.к. при $n_2 > n_1$, $x_{n_2} > x_{n_1}$

г) $x_n = (-1)^{n-1}$ ни возрастающая, ни убывающая.

606. а) $x_n = \left(\frac{1}{3}\right)^{n+1}$ – убывающая. б) $x_n = 7^{n-5}$ – возрастающая.

в) $x_n = 6^{1-n}$ – убывающая. г) $x_n = \left(-\frac{1}{5}\right)^{2n-1}$ – возрастает, т.к. степень нечетная.

607. а) $y_{n+1} - y_n > 0$; $y_{n+1} > y_n \Rightarrow$ по определению она возрастающая.

б) $\frac{y_{n+1}}{y_n} < 1$, $y_n > 0$; $y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_{n+1} - y_n < 0$; $y_{n+1} < y_n \Rightarrow$ убывающая.

г) $\frac{y_{n+1}}{y_n} < 1$, $y_n < 0$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

608. а) $y_n = 2n - 1$; $y_{n+1} = 2n + 1$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

б) $y_n = 5^{-n}$; $y_{n+1} = 5^{-n-1} = \frac{5^{-n}}{5}$; $y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_n = n^2 + 8$, $y_{n+1} = n^2 + 2n + 9$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

г) $y_n = \frac{2}{3n+1}$, $y_{n+1} = \frac{2}{3n+4}$, $y_{n+1} < y_n \Rightarrow$ убывающая.

609. а) $x_n = (-2)^n$, $x_1 = -2$, $x_2 = 4$, $x_3 = -8$

\Rightarrow последовательность не является монотонной.

б) $y_n = \cos \frac{\pi}{n+5}$; $y_1 = \cos \frac{\pi}{6}$, $y_2 = \cos \frac{\pi}{7}$, $y_3 = \cos \frac{\pi}{8}$. $y_1 < y_2 < y_3 \dots \Rightarrow$ возрастает.

в) $y_n = n^3 - 5$; $y_1 = -4$, $y_2 = 3$, $y_3 = 22$. \Rightarrow возрастает.

г) $y_n = \sqrt{n+8}$, $y_1 = 3$, $y_2 = \sqrt{10}$, $y_3 = \sqrt{11}$.

610. а) $y_n = 2^n$; б) $y_n = 2^n$; в) $y_n = \left(\frac{1}{2}\right)^n$; г) $y_n = -n$.

611. а) $y_n = \sin \frac{\pi n}{2} - \operatorname{ctg} \frac{\pi}{4}(2n+1)$; $y_1 = 2$, $y_2 = -1$, $y_3 = 0$, $y_4 = 1$, $y_5 = 2$.

б) $y_n = \cos \frac{\pi n}{2} + \operatorname{tg} \frac{\pi}{4}(2n+1)$; $y_1 = -1$, $y_2 = 0$, $y_3 = -1$, $y_4 = 2$, $y_5 = -1$.

в) $y_n = n \sin \frac{\pi n}{2} + n^2 \cos \frac{\pi n}{2}$; $y_1 = 1$, $y_2 = -4$, $y_3 = -3$, $y_4 = 16$, $y_5 = 5$.

г) $y_n = \sin \frac{\pi n}{4} - n \cos \frac{\pi n}{4}$; $y_1 = 0$, $y_2 = 1$, $y_3 = 2\sqrt{2}$, $y_4 = 4$, $y_5 = 2\sqrt{2}$.

612. а) $y_n = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdots n}{n^3 + 1}$; $y_1 = \frac{1}{2}$, $y_2 = \frac{2}{9}$, $y_3 = \frac{6}{28} = \frac{3}{14}$, $y_4 = \frac{24}{65}$, $y_5 = \frac{120}{126} = \frac{20}{21}$.

б) $y_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n}$;

$$y_1 = \frac{1}{2}, y_2 = \frac{3}{8}, y_3 = \frac{3 \cdot 5}{2 \cdot 4 \cdot 2 \cdot 3} = \frac{5}{16}, y_4 = \frac{5 \cdot 7}{16 \cdot 8} = \frac{35}{128}, y_5 = \frac{35 \cdot 9}{128 \cdot 10} = \frac{7 \cdot 9}{126 \cdot 5} = \frac{63}{256}.$$

613. а) $x_1 = 2, x_2 = 5 - 2 = 3, x_3 = 2, x_4 = 3, x_5 = 2.$

б) $x_1 = 2, x_2 = 12, x_3 = 22, x_4 = 32, x_5 = 42.$

в) $x_1 = -1, x_2 = 1, x_3 = 3, x_4 = 5, x_5 = 7.$ г) $x_1 = 4, x_2 = 1, x_3 = -2, x_4 = -5, x_5 = -8.$

614. а) $x_1 = 2, x_2 = 4, x_3 = 12, x_4 = 48, x_5 = 240.$

б) $x_1 = -5, x_2 = \frac{5}{2}, x_3 = -\frac{5}{4}, x_4 = \frac{5}{8}, x_5 = -\frac{5}{16}.$

в) $x_2 = -2, x_2 = 2, x_3 = -2, x_4 = 2, x_5 = -2.$

г) $x_1 = 1, x_2 = 10, x_3 = 100, x_4 = 1000, x_5 = 10\,000.$

615. а) $-\frac{1}{2}; \frac{3}{4}; -\frac{5}{6}; \frac{7}{8}; -\frac{9}{10} \dots; \frac{(-1)^n \cdot (2n-1)}{2n} = y_n;$

б) $\frac{2}{\sqrt{3}}; \frac{4}{3}; \frac{6}{3\sqrt{3}}; \frac{8}{9}; \frac{10}{9\sqrt{3}} \dots; y_n = \frac{2n}{(\sqrt{3})^n};$

в) $\frac{3}{4}; \frac{9}{16}; \frac{27}{64}; \frac{81}{256} \dots; y_n = \left(\frac{3}{4}\right)^n;$

г) $\frac{1}{\sqrt{2}}; \frac{3}{2}; \frac{5}{2\sqrt{2}}; \frac{7}{4}; \frac{9}{4\sqrt{2}} \dots; y_n = \frac{2n-1}{(\sqrt{2})^n}.$

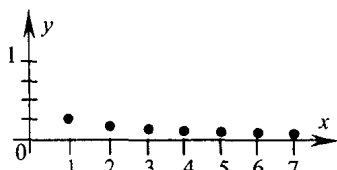
616. а) $y_n = (-1)^{n+1} \frac{n^2}{\sqrt{n(n+1)}};$

в) $y_n = (-1)^{n-1} \frac{5n-1}{n(n+1)(n+2)};$

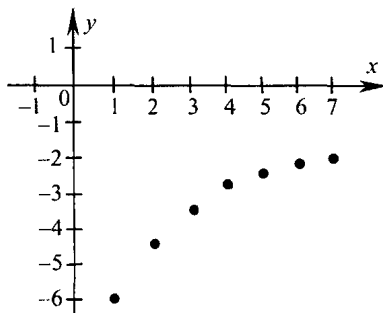
б) $y_n = (-1)^{n+1} \frac{2n+3}{(n+1)^2(n+2)^2};$

г) $y_n = \frac{(-1)^n + 1}{1 \cdot 2 \cdot 3 \cdot 4 \dots n}.$

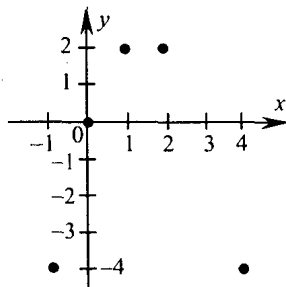
617. а)



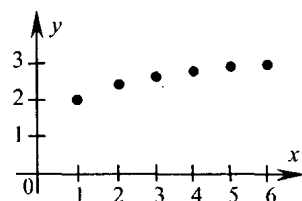
в)



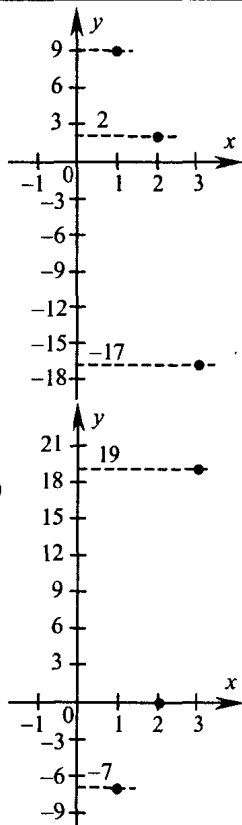
б)



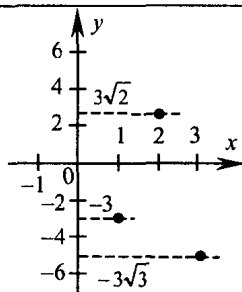
г)



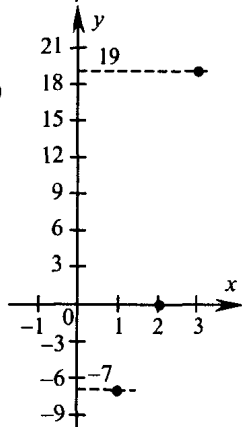
618. а)



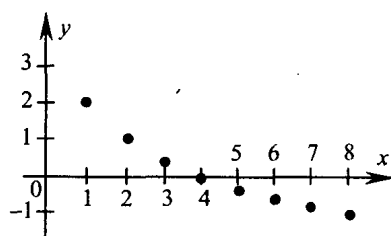
б)



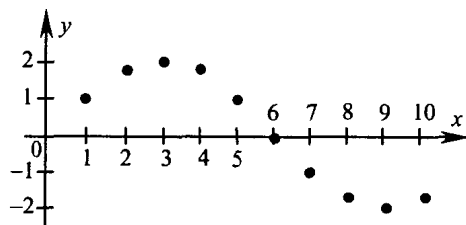
в)



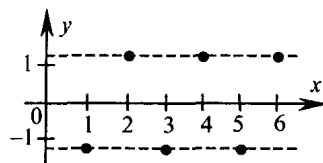
г)



619. а)



б)



$$620. x_1 = 5, x_2 = -3, x_3 = 1, x_4 = 1, x_5 = 0, x_6 = -\frac{1}{2}; x_n = \frac{x_{n-2} + x_{n-1}}{2}.$$

$$621. a_1 = 1, S_1 = 1, P_1 = 4; a_2 = \sqrt{2}, S_2 = 2, P_2 = 4\sqrt{2}; a_3 = 2, S_1 = 4, P_1 = 8; a_3 = 2\sqrt{2}, S_3 = 8, P_3 = 8\sqrt{2}; a_4 = 4, S_4 = 16, P_5 = 16; a_n(\sqrt{2})^{n-1}; S_n = 2^{n-1}; P_n = 4 \cdot (\sqrt{2})^{n-1}; a_{11} = (\sqrt{2})^{10} = 2^5 = 32; S_{17} = 2^{16}.$$

$$622. y_n = 2n^2 - 7n + 5; \begin{cases} 2n^2 - 7n + 5 \leq 5, \\ 2n^2 - 7n + 5 \geq 2. \end{cases} \begin{cases} n(2n-7) \leq 0, \\ 2n^2 - 7n + 3 \geq 0. \end{cases} (x-3)(2x-1) \geq 0;$$

$$\begin{cases} n \in \left[0; \frac{7}{2}\right] \\ n \in \left(-\infty; \frac{1}{2}\right] \cup [3; +\infty) \end{cases} \Rightarrow n \in \left[0; \frac{1}{2}\right] \cup \left[3; \frac{7}{2}\right], \text{ т.к. } x \in N, \text{ то } n = 3, y_3 \in [2; 5].$$

623. а) $x_n = 3n - 2$, $A = 15$. $3n - 2 > 15$, $n > \frac{17}{3}$. $n \geq 6$. Начиная с x_6 .

б) $x_n = 5^{n-1}$, $A = 125$. $5^{n-1} > 125$; $5^{n-1} > 5^3$. $n > 4$. Начиная с x_5 .

в) $x_n = n^2 - 17$, $A = -2$. $n^2 - 17 > -2$. $n^2 > 15$. Начиная с x_4 .

г) $x_n = 3^{n-5}$, $A = 27$. $3^{n-5} > 3^3$. $n > 8$. Начиная с x_9 .

624. а) $x_1 = -14$, $x_n = x_{n-1} + 7$, $A = 25$; $x_1 = -14$, $x_2 = -7$, $x_3 = 0$.

$x_n = -21 + 7n$. $-21 + 7n > 25$. $n > \frac{46}{7}$. Начиная с x_7 .

б) $x_1 = 3$, $x_n = 6x_{n-1}$, $A = 168$. $x_1 = 3$, $x_2 = 18$, $x_3 = 108$.

$x_n = 3 \cdot 6^{n-1}$. $3 \cdot 6^{n-1} > 168$. $6^{n-1} > 56$. Начиная с x_4 .

в) $x_1 = 0$, $x_n = x_{n-1} + 3$, $A = 28$. $x_n = 3(n-1)$. $3(n-1) > 28$. $n > \frac{28}{3} + 1$. Начиная с x_{11} .

г) $x_1 = 3$, $x_n = 7x_{n-1}$, $A = 285$. $x_n = 7^{n-1}$. $7^{n-1} > 285$. Начиная с x_4 .

625. а) $\frac{1}{3125}$; $\frac{1}{625}$; $\frac{1}{125}$...; $x_n = \frac{1}{3125} \cdot 5^{(n-1)} \cdot 5^{-5} \cdot 5^{n-1} \leq 1$. $n-1 \leq 5$. $n \leq 6$. шесть членов.

б) $\frac{6}{377}$; $\frac{11}{379}$; $\frac{16}{381}$... $\frac{6+5(n-1)}{377+2(n-1)} = x_n$. $\frac{5n+1}{375+2n} \leq 1$.

$3n \leq 374$. $n \leq \frac{374}{3}$. 124 члена.

в) $\frac{2}{729}$; $\frac{2}{243}$; $\frac{2}{81}$...; $\frac{2 \cdot 3^{n-1}}{729} = x_n$. $2 \cdot 3^{n-1} \leq 729$. $2 \cdot 3^{n-1} \leq 3^6$. $n < 7$ шесть членов.

г) $\frac{2}{219}$; $\frac{9}{222}$; $\frac{16}{225}$... $\frac{2+7(n-1)}{219+3(n-1)} = x_n$

$\frac{7n-5}{3n+216} \leq 1$; $7n-5 \leq 3n+216$; $4n \leq 221$; $n \leq \frac{221}{4}$ 55 членов.

626. а) $y_n = \frac{n^2}{n+1} = n+1 - \frac{2n+1}{n+1} = n+1-2 + \frac{1}{n+1} = n-1 + \frac{1}{n+1}$.

Ограничена снизу.

б) $y_n = \frac{(-1)^n + 1}{2n}$. $y_1 = 0$ $y_2 = \frac{1}{2}$ $y_3 = 0$ $y_4 = \frac{1}{4}$

Ограничена снизу.

в) $y_1 = 0$, $y_2 = 8$, $y_3 = 0$, $y_4 = 32$; ограничена снизу;

г) $y_1 = 0$, $y_2 = -\frac{3}{4}$, $y_3 = -\frac{8}{27}$; ограничена снизу.

627. а) $x_1 = \frac{1}{2}$, $x_2 = \frac{2}{3}$, $x_3 = \frac{3}{4}$; ограничена сверху;

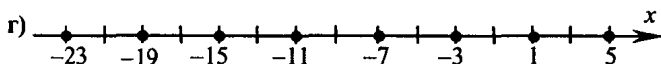
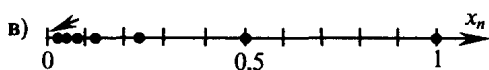
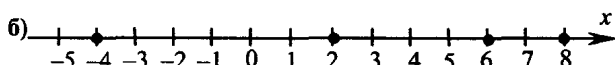
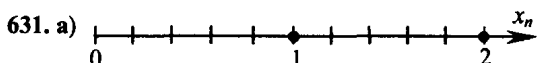
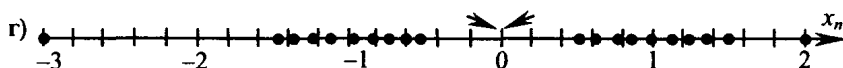
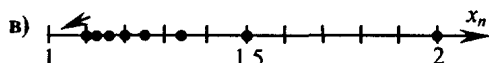
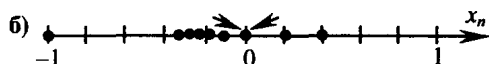
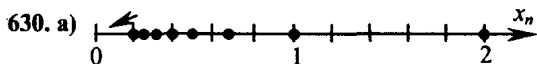
б) $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = \frac{1}{2}$; ограничена сверху;

в) $x_1 = 0, x_2 = 4, x_3 = 0, x_4 = 16$; не ограничена сверху;

г) $x_1 = 0, x_2 = \frac{3}{6}, x_3 = \frac{8}{11}, x_4 = \frac{15}{18}$; ограничена сверху.

628. а) Ограничена; б) ограничена; в) ограничена; г) не ограничена.

629. а) Ограничена; б) ограничена; в) ограничена; г) ограничена.



632. а) $6 \leq a_n \leq 7$; б) $2 \leq b_n \leq 3$; в) $1 \leq p_n \leq 2$; г) $0 \leq q_n \leq 1$.

§30. Предел числовой последовательности

633. а) $a = 0, r = 0,1. (-0,1; 0,1)$;

б) $a = -3, r = 0,5. (-3,5; -2,5)$

в) $a = 2, r = 1. (1; 3)$;

г) $a = 0,2, r = 0,3. (-0,1; 0,5)$.

634. а) $(1; 3), a = 2, r = 1$.

б) $(-0,2; 0,2), a = 0, r = 0,2$.

в) $(2,1; 2,3), a = 2,2, r = 0,1$.

г) $(-7; -5), a = -6, r = 1$.

635. а) $x_1 = 1, a = 2, r = 0,5, 1 \notin (1,5; 2,5)$;

б) $x_1 = 1,1, a = 1, r = 0,2, 1,1 \in (0,8; 1,2)$;

в) $x_1 = -0,2, a = 0, r = 0,3, -0,2 \in (-0,3; 0,3)$;

г) $x_1 = 2,75, a = 2,5, r = 0,3, 2,75 \in (2,2; 2,8)$.

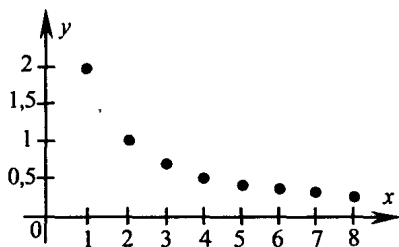
636. а) $x_n = \frac{1}{n^2}, a = 0, r = 0,1. \frac{1}{n^2} \in (-0,1; 0,1)$. При $n_0 \geq 4$.

б) $x_n = \frac{1}{n^2}, a = 1, r = 0,1$. Такого n_0 не существует.

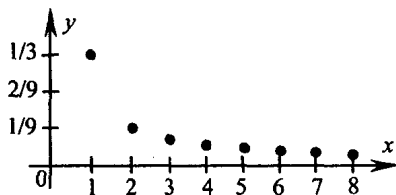
в) $x_n = \frac{n}{n+1}, a = 0, r = 0,1$. Такого n_0 не существует.

г) $x_n = \frac{n}{n+1}$, $a = 1$, $r = 0,1$. $x_n \in (0,9; 1,1)$ при $n_0 \geq 10$.

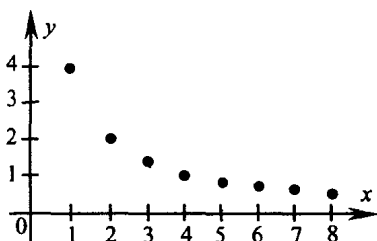
637. а) $y_n = \frac{2}{n}$, $y = 0$.



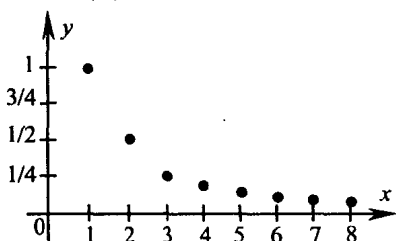
б) $y_n = \left(\frac{1}{3}\right)^n$, $y = 0$.



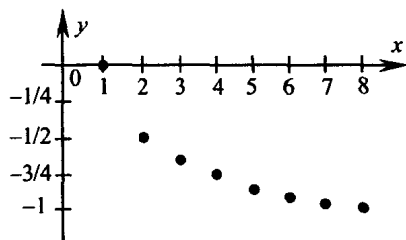
в) $y_n = \frac{4}{n}$, $y = 0$.



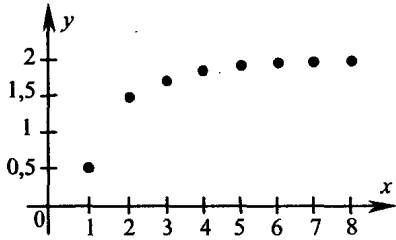
г) $y_n = \left(\frac{1}{2}\right)^{n-1}$, $y = 0$.



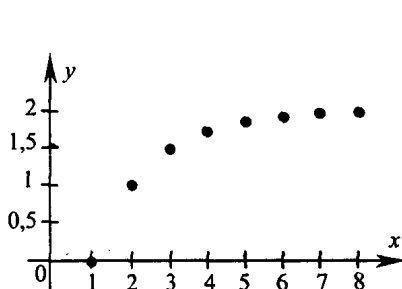
638. а) $y_n = -1 + \frac{1}{n}$, $y = -1$.



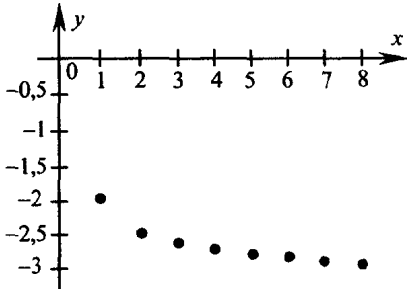
б) $y_n = 2 - \frac{1}{n^2}$, $y = 2$.



в) $y_n = 2 - \frac{2}{n}$, $y = 2$.



г) $y_n = -3 + \frac{1}{n^2}$, $y = -3$.



$$639. \text{ а) } \lim_{n \rightarrow \infty} \frac{5}{n^2} = 0, \quad \text{ б) } \lim_{n \rightarrow \infty} \left(-\frac{17}{n^3} \right) = 0, \quad \text{ в) } \lim_{n \rightarrow \infty} \left(-\frac{15}{n^2} \right) = 0, \quad \text{ г) } \lim_{n \rightarrow \infty} \frac{3}{\sqrt{n}} = 0.$$

$$640. \text{ а) } \lim_{n \rightarrow \infty} \left(\frac{7}{n} + \frac{8}{\sqrt{n}} + \frac{9}{n^3} \right) = 0 + 0 + 0 = 0;$$

$$\text{ б) } \lim_{n \rightarrow \infty} \left(6 - \frac{7}{n^2} - \frac{3}{n} - \frac{3}{\sqrt{n}} \right) = 6 - 0 - 0 - 0 = 6;$$

$$\text{ в) } \lim_{n \rightarrow \infty} \left(\frac{3}{n} + \frac{7}{n^2} - \frac{5}{n^3} + \frac{13}{n^4} \right) = 0 + 0 - 0 + 0 = 0;$$

$$\text{ г) } \lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{3}{\sqrt{n}} - 4 + \frac{7}{n^2} \right) = 0 + 0 - 4 + 0 = -4;$$

$$641. \text{ а) } \lim_{n \rightarrow \infty} \frac{5n+3}{n+1} = \lim_{n \rightarrow \infty} \frac{5 + \frac{3}{n}}{1 + \frac{1}{n}} = 5;$$

$$\text{ б) } \lim_{n \rightarrow \infty} \frac{7n-5}{n+2} = \lim_{n \rightarrow \infty} \frac{7 - \frac{5}{n}}{1 + \frac{2}{n}} = 7;$$

$$\text{ в) } \lim_{n \rightarrow \infty} \frac{3n+1}{n+2} = \lim_{n \rightarrow \infty} \frac{3 + \frac{1}{n}}{1 + \frac{2}{n}} = 3;$$

$$\text{ г) } \lim_{n \rightarrow \infty} \frac{2n+1}{3n-1} = \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{n}}{3 - \frac{1}{n}} = \frac{2}{3}.$$

$$642. \text{ а) } \lim_{n \rightarrow \infty} \frac{5}{2^n} = 0; \quad \text{ б) } \lim_{n \rightarrow \infty} \frac{1}{2} \cdot 5^{-n} = 0; \quad \text{ в) } \lim_{n \rightarrow \infty} 7 \cdot 3^{-n} = 0; \quad \text{ г) } \lim_{n \rightarrow \infty} \frac{4}{3^{n+1}} = 0.$$

$$643. \text{ а) } \lim_{n \rightarrow \infty} \frac{2n^2-1}{n^2} = \lim_{n \rightarrow \infty} \left(2 - \frac{1}{n^2} \right) = 2; \quad \text{ б) } \lim_{n \rightarrow \infty} \frac{(1+2n+n^2)}{n^2} = \lim_{n \rightarrow \infty} \left(\frac{\frac{1}{n^2} + \frac{2}{n} + 1}{1} \right) = 1;$$

$$\text{ в) } \lim_{n \rightarrow \infty} \frac{3-n^2}{n^2} = \lim_{n \rightarrow \infty} \left(\frac{3}{n^2} - 1 \right) = -1; \quad \text{ г) } \lim_{n \rightarrow \infty} \frac{3n-4-2n^2}{n^2} = \lim_{n \rightarrow \infty} \left(\frac{\frac{3}{n} - \frac{4}{n^2} - 2}{1} \right) = -2.$$

$$644. \text{ а) } b_1 = 3 \quad q = \frac{1}{3} \quad S_n = \frac{3}{1 - \frac{1}{3}} = 4\frac{1}{2}. \quad \text{ б) } b_1 = -5 \quad q = -0,1 \quad S_n = \frac{-5}{1+0,1} = -\frac{50}{11} = -4\frac{6}{11}.$$

$$\text{ в) } b_1 = -1 \quad q = 0,2 \quad S_n = \frac{-1}{0,8} = -\frac{5}{4} = -1\frac{1}{4}.$$

$$\text{ г) } b_1 = 2 \quad q = -\frac{1}{3} \quad S_n = \frac{2}{1 - \frac{1}{3}} = \frac{6}{2} = 3.$$

$$645. \text{ а) } 32, 16, 8, 4, \dots; \quad b_1 = 32 \quad q = \frac{1}{2} \quad S_n = \frac{32}{1 - \frac{1}{2}} = 64.$$

$$\text{б) } 24, -8, \frac{8}{3}, -\frac{8}{9} \dots; b_1 = 24 \quad q = -\frac{1}{3} \quad S_n = \frac{24}{1 + \frac{1}{3}} = 36.$$

$$\text{в) } 27, 9, 3, 1, \frac{1}{3} \dots; b_1 = 27 \quad q = \frac{1}{3} \quad S_n = \frac{27}{\frac{2}{3}} = 40,5.$$

$$\text{г) } 18, -6, 2, -\frac{1}{3} \dots; b_1 = 18 \quad q = -\frac{1}{3} \quad S_n = \frac{18}{1 + \frac{1}{3}} = \frac{27}{2}.$$

$$646. \text{ а) } 2 + 1 + \frac{1}{2} + \frac{1}{4} \dots; b_1 = 2 \quad q = \frac{1}{2} \quad S_n = \frac{2}{1 - \frac{1}{2}} = 4.$$

$$\text{б) } 49 + 7 + 1 + \frac{1}{7} \dots; b_1 = 49 \quad q = \frac{1}{7} \quad S_n = \frac{49}{1 - \frac{1}{7}} = 57\frac{1}{6}.$$

$$\text{в) } \frac{3}{2} - 1 + \frac{2}{3} - \frac{4}{9} \dots; b_1 = \frac{3}{2} \quad q = -\frac{2}{3} \quad S_n = \frac{\frac{3}{2}}{1 + \frac{2}{3}} = \frac{3}{2} \cdot \frac{3}{5} = \frac{9}{10}.$$

$$\text{г) } 125 + 25 + 5 + 1 \dots; b_1 = 125 \quad q = \frac{1}{5} \quad S_n = \frac{125}{\frac{4}{5}} = \frac{625}{4} = 156,25.$$

$$647. \text{ а) } -6 + \frac{2}{3} - \frac{2}{27} + \frac{2}{243} \dots; b_1 = -6 \quad q = -\frac{1}{9} \quad S_n = \frac{-6}{1 + \frac{1}{9}} = -\frac{54}{10} = -\frac{27}{5}.$$

$$\text{б) } 3 + \sqrt{3} + 1 + \frac{1}{\sqrt{3}} \dots; b_1 = 3 \quad q = \frac{1}{\sqrt{3}} \quad S_n = \frac{3}{1 - \frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3} - 1} = \frac{3\sqrt{3}(\sqrt{3} + 1)}{2}.$$

$$\text{в) } 4 - 14 + 4 - \frac{8}{7} \dots; b_1 = 49 \quad q = -\frac{2}{7} \quad S_n = \frac{49}{1 + \frac{2}{7}} = \frac{343}{9} = 38\frac{1}{9}.$$

$$\text{г) } 4 + 2\sqrt{2} + 2 + \sqrt{2} \dots; b_1 = 4 \quad q = \frac{1}{\sqrt{2}} \quad S_n = \frac{4}{1 - \frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2} - 1} = 4\sqrt{2}(\sqrt{2} + 1).$$

$$648. \text{ а) } b_1 = -2 \quad b_2 = 1 \quad q = -\frac{1}{2} \quad S_n = \frac{-2}{1 + \frac{1}{2}} = -\frac{4}{3} = -1\frac{1}{3}.$$

$$\text{б) } b_1 = 3 \quad b_2 = \frac{1}{3} \quad q = \frac{1}{9} \quad S_n = \frac{3}{1 - \frac{1}{9}} = \frac{27}{8} = 3\frac{3}{8}.$$

$$\text{в) } b_1 = 7 \quad b_2 = -1 \quad q = -\frac{1}{7} \quad S_n = \frac{7}{1 + \frac{1}{7}} = \frac{49}{8} = 6\frac{1}{8}.$$

$$\text{г) } b_1 = -20, b_2 = 4, q = -\frac{1}{5}, S_n = \frac{-20}{1 + \frac{1}{5}} = -\frac{100}{6} = -\frac{50}{3} = -16\frac{2}{3}.$$

$$649. \text{ а) } S_n = 2, b_1 = 3; S_n = \frac{b_1}{1-q}, 1-q = \frac{b_1}{S_n}, q = 1 - \frac{b_1}{S_n} = 1 - \frac{1}{2} = \frac{1}{2}.$$

$$\text{б) } S_n = -10, b_1 = -5, q = 1 - \frac{5}{10} = \frac{1}{2}.$$

$$\text{в) } S_n = -\frac{9}{4}, b_1 = -3, q = 1 - \frac{3}{\frac{9}{4}} = 1 - \frac{4}{3} = -\frac{1}{3}.$$

$$\text{г) } S_n = 1,5, b_1 = 2, q = 1 - \frac{2}{\frac{3}{2}} = 1 - \frac{4}{3} = -\frac{1}{3}.$$

$$650. \text{ а) } S = 10, q = \frac{1}{10}, S = \frac{b_1}{1-q}, b_1 = S(1-q) = 10 \cdot \frac{9}{10} = 9.$$

$$\text{б) } S = -3, q = -\frac{1}{3}, b_1 = -3 \cdot \left(1 + \frac{1}{3}\right) = -4.$$

$$\text{в) } S = 6, q = -\frac{1}{2}, b_1 = 6 \cdot \frac{3}{2} = 9. \quad \text{г) } S = -21, q = \frac{1}{7}, b_1 = -21 \cdot \left(1 - \frac{1}{7}\right) = -18.$$

$$651. \text{ а) } S = 15, q = -\frac{1}{3}, n = 3, b_1 = 15 \cdot \frac{4}{3} = 20, b_1 = 20 \cdot \left(-\frac{1}{3}\right)^2 = \frac{20}{9} = 2\frac{2}{9}.$$

$$\text{б) } S = -20, b_1 = -16, n = 4, q = 1 - \frac{16}{20} = \frac{1}{5}, b_4 = -16 \cdot \left(\frac{1}{5}\right)^3 = -\frac{16}{125}.$$

$$\text{в) } S = 20, b_1 = 22, n = 4, q = 1 - \frac{22}{20} = -\frac{1}{10}, b_4 = 22 \cdot \left(-\frac{1}{10}\right)^3 = -\frac{11}{500}.$$

$$\text{г) } S = 21, q = \frac{2}{3}, n = 3, b_1 = 21 \cdot \left(1 - \frac{2}{3}\right) = 7, b_1 = 7 \cdot \left(\frac{2}{3}\right)^2 = \frac{28}{9} = 3\frac{1}{9}.$$

$$652. \text{ а) } x_n = \frac{1}{2n}, a = 0, r = 0,1. \frac{1}{2n} < 0,1, 2n > 10, n > 5 \text{ начиная с 6-ого.}$$

$$\text{б) } x_n = 3 + \frac{1}{n^2}, a = 3, r = 0,2. 3 + \frac{1}{n^2} < 3,2, n^2 > 5, n > \sqrt{5} \text{ начиная с 3-его.}$$

$$\text{в) } x_n = 1 + \frac{2}{n^2}, a = 1, r = 0,01. 1 + \frac{2}{n^2} < 1,01, \frac{2}{n^2} < 0,01, \frac{1}{n^2} < \frac{1}{200}, n > 14, \text{ начиная с 15-ого.}$$

$$\text{г) } x_n = -\frac{3}{n}, a = 0, r = 0,1. -\frac{3}{n} > -\frac{1}{10}, \frac{3}{n} < \frac{1}{10}, n > 30, \text{ начиная с 31-ого.}$$

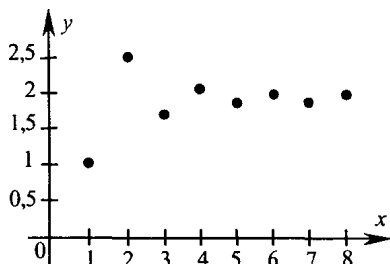
$$653. \text{ а) } x_n = \left(\frac{1}{3}\right)^n, a = 0, r = \frac{1}{27}; \left(\frac{1}{3}\right)^n < \frac{1}{27}, \left(\frac{1}{3}\right)^n < \left(\frac{1}{3}\right)^3, n > 3 \text{ начиная с 4-ого.}$$

б) $x_n = (-1)^n \frac{1}{2^n}$, $a = 0$, $r = \frac{1}{64}$; $\frac{1}{2^n} < \frac{1}{64}$, $\left(\frac{1}{2}\right)^n < \left(\frac{1}{2}\right)^6$, $n > 6$, начиная с 7-ого.

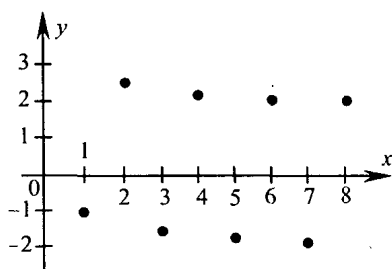
в) $x_n = 2 + \left(\frac{1}{2}\right)^n$, $a = 2$, $r = \frac{1}{128}$; $2 + \left(\frac{1}{2}\right)^n < 2 + \frac{1}{128}$, $\left(\frac{1}{2}\right)^n < \frac{1}{128} = \left(\frac{1}{2}\right)^7$, $n > 7$, начиная с 8-ого.

г) $x_n = 3 - \left(\frac{1}{3}\right)^n$, $a = 3$, $r = \frac{1}{81}$; $3 - \left(\frac{1}{3}\right)^n > 3 - \frac{80}{81}$, $\left(\frac{1}{3}\right)^n < \frac{1}{81} = \left(\frac{1}{3}\right)^4$, начиная с 5-ого.

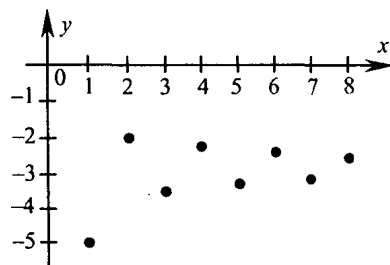
654. а) $y_n = 2 + (-1)^n \frac{1}{n}$, $y = 2$.



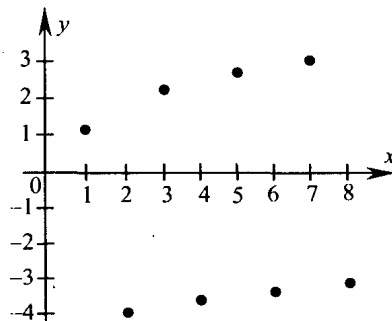
б) $y_n = (-1)^n 2 + \frac{1}{n}$.



в) $y_n = -3 + (-1)^n \frac{2}{n}$, $y = -3$.



г) $y_n = (-1)^{n+1} 3 - \frac{2}{n}$.



655. а) Нет, $y_n = \left(-\frac{1}{n}\right)^n$. б) Нет, $y_n = n$. в) да, $y_n = \frac{1}{n}$. г) нет, $y_n = \left(-\frac{1}{n}\right)^n$.

656. а) $\lim_{n \rightarrow \infty} \frac{(2n+1)(n-3)}{n^2} = \lim_{n \rightarrow \infty} \frac{2n^2 - 5n - 3}{n^2} = \lim_{n \rightarrow \infty} \left(2 - \frac{5}{n} - \frac{3}{n^2}\right) = 2$;

б) $\lim_{n \rightarrow \infty} \frac{(3n+1)(4n-1)}{(n-1)^2} = \lim_{n \rightarrow \infty} \frac{12n^2 + n - 1}{n^2 - 2n + 1} = \lim_{n \rightarrow \infty} \frac{12 + \frac{1}{n} - \frac{1}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^2}} = 12$;

в) $\lim_{n \rightarrow \infty} \frac{(3n-2)(2n+3)}{n^2} = \lim_{n \rightarrow \infty} \frac{6n^2 + 5n - 6}{n^2} = \lim_{n \rightarrow \infty} \left(6 + \frac{5}{n} - \frac{6}{n^2}\right) = 6$;

$$r) \lim_{n \rightarrow \infty} \frac{(1-2n)(1+n)}{(n+2)^2} = \lim_{n \rightarrow \infty} \frac{1-n-2n^2}{n^2+4n+4} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} - \frac{1}{n} - 2}{1 + \frac{4}{n} + \frac{4}{n^2}} = -2.$$

$$657. a) \lim_{n \rightarrow \infty} \frac{(2n+1)(3n-4) - 6n^2 + 12n}{n+5} = \lim_{n \rightarrow \infty} \frac{7n-4}{n+5} = 7;$$

$$b) \lim_{n \rightarrow \infty} \frac{n^2(2n+5) - 2n^3 + 5n^2 - 13}{n(n+1)(n-7) + 1 - n} = \lim_{n \rightarrow \infty} \frac{10n^2 - 13}{n^3 - 6n^2 - 8n + 1} = \\ = \lim_{n \rightarrow \infty} \frac{10/n - 13/n^3}{1 - 6/n - 8/n^2 - 1/n^3} = 0;$$

$$b) \lim_{n \rightarrow \infty} \frac{(1-n)(n^2+1) + n^3}{n^2+2n} = \lim_{n \rightarrow \infty} \frac{n^2+1-n}{n^2+2n} = 1;$$

$$r) \lim_{n \rightarrow \infty} \frac{n(7-n^2) + n^3 - 3n - 1}{(n+1)(n+2) + 2n^2 + 1} = \lim_{n \rightarrow \infty} \frac{7n - n^3 + n^3 - 3n - 1}{n^2 + 3n + 2 + 2n^2 + 1} = \\ = \lim_{n \rightarrow \infty} \frac{4n-1}{3n^2+3n+3} = \lim_{n \rightarrow \infty} \frac{4/n - 1/n^2}{3 + 3/n + 3/n^2} = 0.$$

$$658. a) b_n = \frac{25}{3^n}, b_1 = \frac{25}{3}, b_2 = \frac{25}{9}, q = \frac{1}{3}, S_n = \frac{25/3}{2/3} = 12,5;$$

$$b) b_n = (-1)^n \frac{13}{2^{n-1}}, b_1 = -13, b_2 = \frac{13}{2}, q = -\frac{1}{2}, S_n = \frac{-13}{3/2} = -\frac{26}{3} = -8\frac{2}{3};$$

$$b) b_n = \frac{45}{3^n}, b_1 = \frac{45}{3}, b_2 = \frac{45}{9}, q = \frac{1}{3}, S_n = \frac{45/3}{2/3} = \frac{45}{2} = 22,5;$$

$$r) b_n = (-1)^n \frac{7}{6^{n-2}}, b_1 = -42, b_2 = 7, q = -\frac{1}{6}, S_n = \frac{-42}{1+1/6} = -36.$$

$$659. \begin{cases} b_1 + b_3 = 29 \\ b_2 + b_4 = 11,6 \end{cases}, \begin{cases} b_1 + b_1 q^2 = 29 \\ b_1 q + b_1 q^3 = 11,6 \end{cases}, \begin{cases} b_1(1+q^2) = 29 \\ b_1 q(1+q^2) = 11,6 \end{cases}, q = \frac{2}{5}.$$

$$b_1 = \frac{29}{1 + \frac{4}{25}} = 25, S_n = \frac{25}{1 - \frac{2}{5}} = \frac{25 \cdot 5}{3} = 41\frac{2}{3}.$$

$$660. a) S_n = 24, S_3 = 21. \begin{cases} \frac{b_1}{1-q} = 24 \\ \frac{b_1(q^3-1)}{q-1} = 21 \end{cases} \begin{cases} q^3 - 1 = -\frac{7}{8} \\ q^3 = \frac{1}{8} \\ q = \frac{1}{2} \end{cases} \begin{cases} \frac{b_1}{1/2} = 24 \\ q = \frac{1}{2} \end{cases} b_1 = 12$$

$$b) \begin{cases} \frac{b_1}{1-q} = 31,25 \\ \frac{b_1(q^3-1)}{q-1} = 31 \end{cases} \begin{cases} -(q^3-1) = \frac{31 \cdot 4}{125} = \frac{124}{125} \\ q = \frac{1}{5} \end{cases}$$

$$\frac{b_1}{1-1/5} = \frac{125}{4}, \quad \frac{5b_1}{4} = \frac{125}{4}, \quad b_1 = 25; \quad q = \frac{1}{5}, \quad b_1 = 25, \quad b_7 = 25 \cdot \left(\frac{1}{5}\right)^6 = \frac{1}{625}.$$

$$661. \begin{cases} S_n = 18 \\ b_1^2 + b_1^2 q^2 + b_1^2 q^4 + \dots = 162 \end{cases}$$

$$\begin{cases} \frac{b_1}{q-1} = -18 \\ \frac{b_1^2}{1-q^2} = 162 \end{cases}, \quad \begin{cases} \frac{b_1}{1-q} = 18 \\ \frac{b_1^2}{1-q^2} = 162 \end{cases}, \quad \begin{cases} b_1 = 18(1-q) \\ 324(1-2q+q^2) = 162 - 162q^2 \end{cases}$$

$$2q^2 - 4q + 2 = 1 - q^2, \quad 3q^2 - 4q + 1 = 0; \quad q = \frac{2+1}{3} = 1, \quad b_1 = 0 \text{ не может быть};$$

$$q = \frac{1}{3}, \quad b_1 = 12.$$

$$662. \text{ а) } 2 + 4 + 6 + \dots + 20 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 10 \frac{20+2}{2} + \frac{1/2}{1-1/2} = 110 + 1 = 111;$$

$$\text{б) } 1 + 3 + 5 + \dots + 99 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots = \\ = \frac{1+99}{2} \cdot \frac{99-1+2}{2} + \frac{2/5}{1-2/5} = 2500 + \frac{2}{3} = 2500 \frac{2}{3};$$

$$\text{в) } 21 + 24 + 27 + \dots + 51 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots = \\ = \frac{21+51}{2} \cdot \frac{51-21+3}{3} + \frac{1/3}{1+1/3} = 12 \cdot 33 + \frac{1}{4} = 396 \frac{1}{4};$$

$$\text{г) } 1 + 4 + 7 + \dots + 100 + 0,1 + 0,01 + 0,001 + \dots = \\ = \frac{1+100}{2} \cdot \frac{100-1+3}{3} + \frac{0,1}{1-0,1} = 1717 + \frac{1}{9} = 1717 \frac{1}{9}.$$

$$663. \text{ а) } \sin x + \sin^2 x + \sin^3 x + \dots = \frac{\sin x}{1 - \sin x}; \quad \text{б) } \cos x - \cos^2 x + \cos^3 x + \dots = \frac{\cos x}{1 + \cos x};$$

$$\text{в) } \cos^2 x + \cos^4 x + \cos^6 x + \dots = \frac{\cos^2 x}{1 - \cos^2 x} = \operatorname{ctg}^2 x;$$

$$\text{г) } 1 - \sin^3 x + \sin^6 x - \sin^9 x + \dots = \frac{1}{1 + \sin^3 x}.$$

$$664. \text{ а) } x + x^2 + x^3 + \dots = 4; \quad \frac{x}{1-x} = 4, \quad x = 4 - 4x, \quad x = \frac{4}{5};$$

$$\text{б) } 2x - 4x^2 + 8x^3 - 16x^4 + \dots = \frac{3}{8}; \quad \frac{2x}{1+2x} = \frac{3}{8}, \quad 2x = \frac{3}{8} + \frac{3}{4}x, \quad 10x = 3, \quad x = \frac{3}{10}.$$

$$665. \text{ а) } \frac{1}{x} + 1 + x + x^2 + \dots = \frac{9}{2}; \quad \frac{1}{1-x} = \frac{9}{2} \cdot 2 = 9x - 9x^2, \quad 9x^2 - 9x + 2 = 0, \quad x = \frac{1}{3} \text{ или } x = \frac{2}{3};$$

$$б) 2x+1+x^2-x^3+x^4-\dots=\frac{13}{6}, 2x+1+\frac{x^2}{1+x}=\frac{13}{6}$$

$$2x+1+2x^2+x+x^2-\frac{13}{6}x-\frac{13}{6}=0, 18x^2+18x-13x-7=0, 18x^2+5x-7=0,$$

$$x=\frac{-5+23}{36}=\frac{1}{2}, x=\frac{-5-23}{36}=-\frac{7}{9}.$$

$$666. а) \sin x + \sin^2 x + \sin^3 x + \dots = 5; \frac{\sin x}{1 - \sin x} = 5, 6 \sin x = 5, x = (-1)^n \arcsin \frac{5}{6} + \pi.$$

$$б) \cos x - \cos^2 x + \cos^3 x + \dots = 2$$

$$\frac{\cos x}{1 + \cos x} = 2, \cos x = 2 + 2 \cos x, \cos x = -2 - \text{решений нет}$$

$$в) 1 + \sin^2 x + \sin^4 x + \dots = \frac{4}{3}; \frac{1}{1 - \sin^2 x} = \frac{4}{3} \cdot \cos^2 x = \frac{3}{4}, x = \pm \frac{\pi}{6} + 2\pi n,$$

$$x = \pm \frac{5\pi}{6} + 2\pi n;$$

$$г) 7 \cos^3 x + 7 \cos^6 x + \dots = 1; \frac{\cos^3 x}{1 - \cos^3 x} = \frac{1}{7}, 7 \cos^3 x = 1 - \cos^3 x;$$

$$\cos^3 x = \frac{1}{8}, \cos x = \frac{1}{2}, x = \pm \frac{\pi}{3} + 2\pi n.$$

§31. Предел функции

667. а) при $x \rightarrow +\infty$ рис. 23, 25 учебника.

б) при $x \rightarrow -\infty$ рис. 24, 25 учебника.

в) при $x \rightarrow \infty$ рис. 25 учебника.

668. а) $y = 3$ – горизонт. асимптота на луче $(-\infty; 4]$ $\lim_{x \rightarrow -\infty} f(x) = 3$,

$\lim_{x \rightarrow -\infty} f(x)$, $\lim_{x \rightarrow -\infty} f(x)$ не существуют

б) $y = -2$ – горизонт. асимптота на луче $[-6; +\infty)$ $\lim_{x \rightarrow \infty} f(x)$

$\lim_{x \rightarrow -\infty} f(x)$ не существуют, $\lim_{x \rightarrow +\infty} f(x) = -2$

в) $y = -5$ – горизонт. асимптота на луче $(-\infty; 3]$ $\lim_{x \rightarrow -\infty} f(x) = -5$,

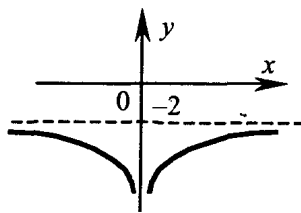
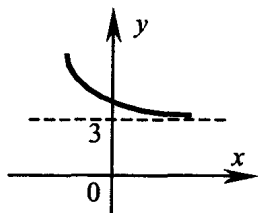
$\lim_{x \rightarrow -\infty} f(x)$, $\lim_{x \rightarrow \infty} f(x)$ не существуют

г) $y = 5$ – горизонт. асимптота на луче $[4; +\infty)$ $\lim_{x \rightarrow \infty} f(x)$

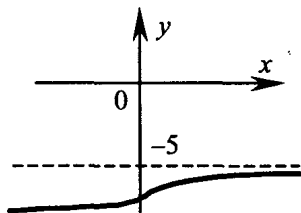
$\lim_{x \rightarrow -\infty} f(x)$ не существуют, $\lim_{x \rightarrow +\infty} f(x) = 5$

669. а) $\lim_{x \rightarrow \infty} f(x) = 3$;

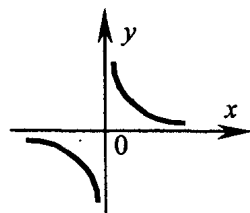
б) $\lim_{x \rightarrow \infty} f(x) = -2$;



$$\text{в) } \lim_{x \rightarrow \infty} f(x) = -5;$$



$$\text{г) } \lim_{x \rightarrow \infty} f(x) = 0.$$



$$670. \lim_{x \rightarrow +\infty} f(x) = -3;$$

$$\text{а) } \lim_{x \rightarrow \infty} 6f(x) = -18; \quad \text{б) } \lim_{x \rightarrow \infty} \frac{f(x)}{3} = -1; \quad \text{в) } \lim_{x \rightarrow \infty} 8f(x) = -24; \quad \text{г) } \lim_{x \rightarrow \infty} 0,4f(x) = -\frac{6}{5}.$$

$$671. \lim_{x \rightarrow \infty} f(x) = 2, \quad \lim_{x \rightarrow \infty} g(x) = -3, \quad \lim_{x \rightarrow \infty} h(x) = 9.$$

$$\text{а) } \lim_{x \rightarrow \infty} (f(x) + g(x)) = \lim_{x \rightarrow \infty} f(x) + \lim_{x \rightarrow \infty} g(x) = 2 - 3 = -1;$$

$$\text{б) } \lim_{x \rightarrow \infty} (f(x) - h(x)) = 2 - 9 = -7; \quad \text{в) } \lim_{x \rightarrow \infty} (g(x) + h(x)) = -3 + 9 = 6;$$

$$\text{г) } \lim_{x \rightarrow \infty} (f(x) + g(x) - h(x)) = 2 - 3 - 9 = -10.$$

$$672. \lim_{x \rightarrow \infty} f(x) = -2, \quad \lim_{x \rightarrow \infty} g(x) = 7, \quad \lim_{x \rightarrow \infty} h(x) = -2.$$

$$\text{а) } \lim_{x \rightarrow \infty} (f(x) \cdot g(x)) = -14;$$

$$\text{б) } \lim_{x \rightarrow \infty} (f(x))^2 = 4;$$

$$\text{в) } \lim_{x \rightarrow \infty} (g(x) \cdot (h(x))^2) = 7 \cdot 4 = 28;$$

$$\text{г) } \lim_{x \rightarrow \infty} (f(x) \cdot g(x) \cdot h(x)) = 7 \cdot 4 = 28.$$

$$673. \lim_{x \rightarrow \infty} f(x) = 6, \quad \lim_{x \rightarrow \infty} g(x) = -10, \quad \lim_{x \rightarrow \infty} L(x) = 25.$$

$$\text{а) } \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{6}{-10} = -\frac{3}{5};$$

$$\text{б) } \lim_{x \rightarrow \infty} \frac{(g(x))^2}{L(x)} = \frac{100}{25} = 4;$$

$$\text{в) } \lim_{x \rightarrow \infty} \frac{f(x)g(x)}{L(x)} = \frac{6 \cdot (-10)}{25} = -\frac{12}{5} = -2,4; \quad \text{г) } \lim_{x \rightarrow \infty} \frac{2L(x)}{3g(x)} = \frac{50}{-30} = -\frac{5}{3}.$$

$$674. \text{ а) } \lim_{x \rightarrow \infty} \left(\frac{1}{x^2} + \frac{3}{x^3} \right) = 0; \quad \text{б) } \lim_{x \rightarrow \infty} \left(\frac{7}{x^5} - \frac{2}{x^3} \right) = 0;$$

$$\text{в) } \lim_{x \rightarrow \infty} \left(\frac{2}{x^2} + \frac{8}{x^3} \right) = 0; \quad \text{г) } \lim_{x \rightarrow \infty} \left(\frac{9}{x^3} - \frac{5}{x^7} \right) = 0.$$

$$675. \text{ а) } \lim_{x \rightarrow \infty} \left(\frac{2}{x^9} + 1 \right) = 1;$$

$$\text{б) } \lim_{x \rightarrow \infty} \left(\frac{4}{x^3} - \frac{7}{x} - 21 \right) = -21;$$

$$\text{в) } \lim_{x \rightarrow \infty} \left(\frac{6}{x^5} + \frac{4}{x^2} + 9 \right) = 9;$$

$$\text{г) } \lim_{x \rightarrow \infty} \left(\frac{7}{x^2} - 7 \right) = -7.$$

$$676. \text{ а) } \lim_{x \rightarrow \infty} \left(12 - \frac{1}{x^2} \right) \frac{16}{x^7} = \lim_{x \rightarrow \infty} \left(\frac{12 \cdot 16}{x^7} - \frac{16}{x^9} \right) = 0;$$

$$\text{б) } \lim_{x \rightarrow \infty} \left(\frac{5}{x^3} + 1 \right) \left(-\frac{8}{x^2} - 2 \right) = 1 \cdot (-2) = -2;$$

$$685. \text{ а) } \lim_{x \rightarrow 0} \frac{x^2}{x^2 - x} = \lim_{x \rightarrow 0} \frac{x}{x - 1} = 0;$$

$$\text{б) } \lim_{x \rightarrow -1} \frac{x+1}{x^2 + x} = \lim_{x \rightarrow -1} \frac{1}{x} = -1;$$

$$\text{в) } \lim_{x \rightarrow 3} \frac{x^2 - 3x}{x - 3} = \lim_{x \rightarrow 3} x = 3;$$

$$\text{г) } \lim_{x \rightarrow 5} \frac{x+5}{x^2 + 5x} = \lim_{x \rightarrow 5} \frac{1}{x} = \frac{1}{5}.$$

$$686. \text{ а) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} (x - 1) = 0;$$

$$\text{б) } \lim_{x \rightarrow -2} \frac{x^2 - 4}{2 + x} = \lim_{x \rightarrow -2} (x - 2) = -4;$$

$$\text{в) } \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} (x + 5) = 10;$$

$$\text{г) } \lim_{x \rightarrow 3} \frac{3 + x}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{1}{x - 3} = -\frac{1}{6}.$$

$$687. y = 2x - 3, x_0 = 3, y(x_0) = 3$$

$$\text{а) } x_1 = 3,2, y(x_1) = 3,4, \Delta y = 0,4;$$

$$\text{б) } x_1 = 2,9, y(x_1) = 2,8, \Delta y = -0,2;$$

$$\text{в) } x_1 = 3,5, y(x_1) = 4, \Delta y = 1;$$

$$\text{г) } x_1 = 2,5, y(x_1) = 2, \Delta y = -1.$$

$$688. y = x^2 + 2x, x_0 = -2, y(x_0) = 0$$

$$\text{а) } x_1 = -1,9, y(x_1) = -0,19, \Delta y = -0,19;$$

$$\text{б) } x_1 = -2,1, y(x_1) = 0,21, \Delta y = 0,21;$$

$$\text{в) } x_1 = -1,5, y(x_1) = -0,75, \Delta y = -0,75;$$

$$\text{г) } x_1 = -2,5, y(x_1) = 1,25, \Delta y = 1,25.$$

$$689. y = \sin x, x_0 = 0, y(x_0) = 0$$

$$\text{а) } x_1 = \frac{\pi}{6}, y(x_1) = \frac{1}{2}, \Delta y = \frac{1}{2};$$

$$\text{б) } x_1 = -\frac{\pi}{6}, y(x_1) = -\frac{1}{2}, \Delta y = -\frac{1}{2};$$

$$\text{в) } x_1 = \frac{\pi}{4}, y(x_1) = \frac{\sqrt{2}}{2}, \Delta y = \frac{\sqrt{2}}{2};$$

$$\text{г) } x_1 = -\frac{\pi}{3}, y(x_1) = -\frac{\sqrt{3}}{2}, \Delta y = -\frac{\sqrt{3}}{2}.$$

$$690. y = 2\sin x \cos x = \sin 2x, x_0 = 0, y(x_0) = 0$$

$$\text{а) } x_1 = -\frac{\pi}{8}, y(x_1) = -\frac{\sqrt{2}}{2}, \Delta y = -\frac{\sqrt{2}}{2};$$

$$\text{б) } x_1 = \frac{\pi}{12}, y(x_1) = \frac{1}{2}, \Delta y = \frac{1}{2};$$

$$\text{в) } x_1 = \frac{\pi}{8}, y(x_1) = \frac{\sqrt{2}}{2}, \Delta y = \frac{\sqrt{2}}{2};$$

$$\text{г) } x_1 = -\frac{\pi}{12}, y(x_1) = -\frac{1}{2}, \Delta y = -\frac{1}{2}.$$

$$691. y = \sqrt{x}, x_0 = 1$$

$$\text{а) } \Delta x = 0,44, y(x + \Delta x) - y(x) = 1,2 - 1 = 0,2;$$

$$\text{б) } \Delta x = -0,19, y(x + \Delta x) - y(x) = 0,9 - 1 = -0,1;$$

$$\text{в) } \Delta x = 0,21, y(x + \Delta x) - y(x) = 1,1 - 1 = 0,1;$$

$$\text{г) } \Delta x = 0,1025, y(x + \Delta x) - y(x) = 1,05 - 1 = 0,05.$$

$$692. \text{ а) } f(x_1) - f(x_0) = 1,4 - 2 = -0,6;$$

$$\text{б) } f(x_1) - f(x_0) = 1 - 6 = -5.$$

$$693. y = 4x^2 - x.$$

$$\text{а) } x = 0, \Delta x = 0,5, y(x + \Delta x) - y(x) = \frac{1}{2};$$

$$\text{б) } x = 1, \Delta x = -0,1, y(x + \Delta x) - y(x) = 2,34 - 3 = -0,66;$$

$$\text{в) } x = 0, \Delta x = -\frac{1}{2}, y(x + \Delta x) - y(x) = \frac{3}{2};$$

$$\text{г) } x = 1, \Delta x = 0,1, y(x + \Delta x) - y(x) = 3,74 - 3 = 0,74.$$

$$694. \text{ а) } f(x) = 3x + 5, f(x + \Delta x) = 3x + 3\Delta x + 5, f(x + \Delta x) - f(x) = 3\Delta x;$$

$$\text{б) } f(x) = -x^2, f(x + \Delta x) = -x^2 - 2x\Delta x - (\Delta x)^2, f(x + \Delta x) - f(x) = -2x\Delta x - (\Delta x)^2;$$

$$\text{в) } f(x) = 4 - 2x, f(x + \Delta x) = 4 - 2x - 2\Delta x, f(x + \Delta x) - f(x) = -2\Delta x;$$

$$r) f(x) = 2x^2, f(x + \Delta x) = 2x^2 + 4\Delta x x + 2\Delta x^2, f(x + \Delta x) - f(x) = 4\Delta x x + 2\Delta x^2.$$

695. $y = x^2 - 4x + 1, x_0 = 2, y(x_0) = -3.$

а) $x = 2,1, y(x) = -2,29, y(x) - y(x_0) = 0,01, \frac{\Delta y}{\Delta x} = \frac{0,01}{0,1} = 0,1;$

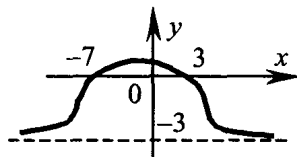
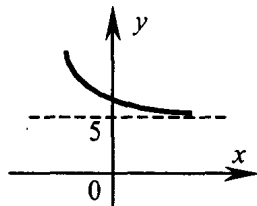
б) $x = 1,9, y(x) = -2,29, y(x) - y(x_0) = 0,1, \frac{\Delta y}{\Delta x} = -\frac{0,01}{0,1} = -0,1;$

в) $x = 2,5, y(x) = -2,75, y(x) - y(x_0) = 0,25, \frac{\Delta y}{\Delta x} = \frac{0,25}{0,5} = 0,5;$

г) $x = 1,5, y(x) = -2,75, y(x) - y(x_0) = 0,25, \frac{\Delta y}{\Delta x} = -\frac{0,25}{0,5} = -0,5.$

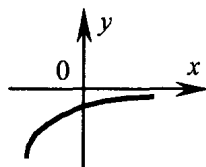
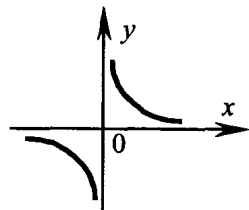
696. а) $\lim_{x \rightarrow +\infty} f(x) = 5, f(x) > 0, x \in \mathbb{R};$

б) $\lim_{x \rightarrow -\infty} f(x) = -3, f(x) \geq 0, x \in [-7; 3];$



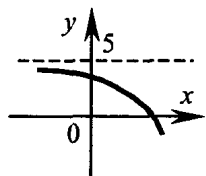
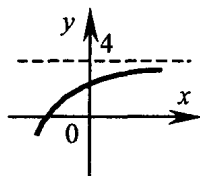
в) $\lim_{x \rightarrow +\infty} f(x) = 0, f(x) > 0, x \in [0; +\infty);$

г) $\lim_{x \rightarrow -\infty} f(x) = 0, f(x) < 0, x \in \mathbb{R}.$



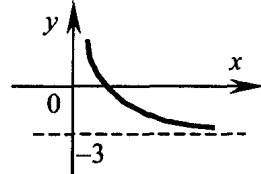
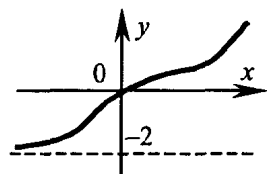
697. а) $\lim_{x \rightarrow +\infty} h(x) = 4$ и функция возрастает;

б) $\lim_{x \rightarrow -\infty} h(x) = 5$ и функция убывает;



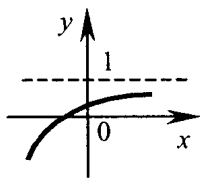
в) $\lim_{x \rightarrow -\infty} h(x) = -2$ и функция возрастает;

г) $\lim_{x \rightarrow +\infty} h(x) = -3$ и функция убывает.

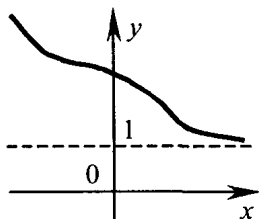
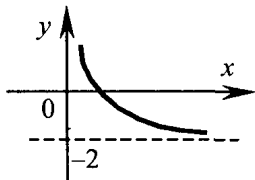


698. а) $\lim_{x \rightarrow +\infty} h(x) = 1$ и функция ограничена сверху;

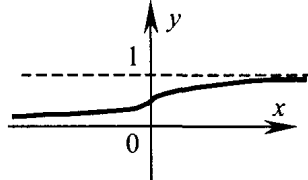
б) $\lim_{x \rightarrow +\infty} h(x) = 1$ и функция ограничена снизу;



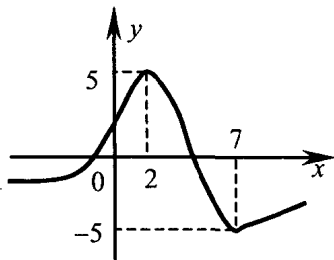
в) $\lim_{x \rightarrow +\infty} h(x) = 1$ и функция ограничена сверху;



г) $\lim_{x \rightarrow +\infty} h(x) = 1$ и функция ограничена снизу.



699.



700. а) $\lim_{x \rightarrow \infty} \frac{4x^2 + 9}{x^2 + 2} = \lim_{x \rightarrow \infty} \frac{4 + \frac{9}{x^2}}{1 + \frac{2}{x^2}} = 4$; б) $\lim_{x \rightarrow \infty} \frac{12x^2 + 5x + 2}{6x^2 + 5x - 3} = \lim_{x \rightarrow \infty} \frac{12 + \frac{5}{x} + \frac{2}{x^2}}{6 + \frac{5}{x} - \frac{3}{x^2}} = 2$;

в) $\lim_{x \rightarrow \infty} \frac{3x^2 - 8}{x^2 - 1} = \lim_{x \rightarrow \infty} \frac{3 - \frac{8}{x^2}}{1 - \frac{1}{x^2}} = 3$; г) $\lim_{x \rightarrow \infty} \frac{10x^2 + 4x - 3}{5x^2 + 2x + 1} = \lim_{x \rightarrow \infty} \frac{10 + \frac{4}{x} - \frac{3}{x^2}}{5 + \frac{2}{x} + \frac{1}{x^2}} = 2$.

701. а) $\lim_{x \rightarrow \infty} \frac{3x - 1}{x^2 + 7x + 5} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - \frac{1}{x^2}}{1 + \frac{7}{x} + \frac{5}{x^2}} = 0$; б) $\lim_{x \rightarrow \infty} \frac{5 - 5x}{2x^2 - 9x} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - \frac{5}{x}}{2 - \frac{9}{x}} = 0$;

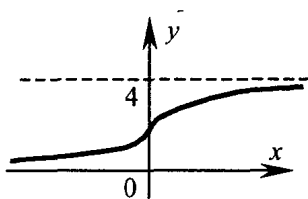
в) $\lim_{x \rightarrow \infty} \frac{-2x - 1}{3x^2 - 4x + 1} = \lim_{x \rightarrow \infty} \frac{-\frac{2}{x} - \frac{1}{x^2}}{3 - \frac{4}{x} + \frac{1}{x^2}} = 0$; г) $\lim_{x \rightarrow \infty} \frac{4x + 3}{12x^2 - 6x} = \lim_{x \rightarrow \infty} \frac{\frac{4}{x} + \frac{3}{x^2}}{12 - \frac{6}{x}} = 0$.

702. а) $\lim_{x \rightarrow \infty} \frac{4x - x^2 + 1}{5x^2 - 2x} = \lim_{x \rightarrow \infty} \frac{\frac{4}{x} - 1 + \frac{1}{x^2}}{5 - \frac{2}{x}} = -\frac{1}{5}$; б) $\lim_{x \rightarrow \infty} \frac{x^3 - 8}{x^3 + 18} = \lim_{x \rightarrow \infty} \frac{1 - \frac{8}{x^3}}{1 + \frac{18}{x^3}} = 1$;

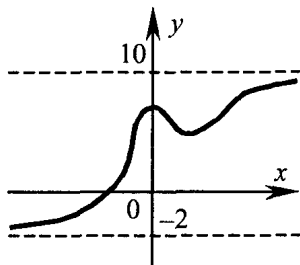
в) $\lim_{x \rightarrow \infty} \frac{3x - 2x^2 + 4}{3x^2 + 2x} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - 2 + \frac{4}{x^2}}{3 + \frac{2}{x}} = -\frac{2}{3}$;

$$г) \lim_{x \rightarrow \infty} \frac{x^3 - 3x^2}{x^4 + 2x + 1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{3}{x^2}}{1 + \frac{2}{x^3} + \frac{1}{x^4}} = 0.$$

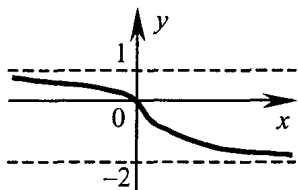
703. а) $\lim_{x \rightarrow +\infty} f(x) = 4$ и $\lim_{x \rightarrow -\infty} f(x) = 0$;



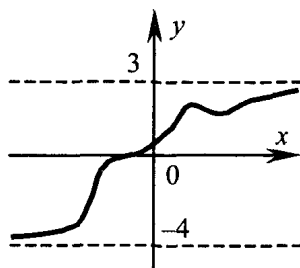
б) $\lim_{x \rightarrow +\infty} f(x) = 10$ и $\lim_{x \rightarrow -\infty} f(x) = -2$;



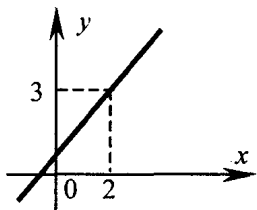
в) $\lim_{x \rightarrow +\infty} f(x) = -2$ и $\lim_{x \rightarrow -\infty} f(x) = 1$;



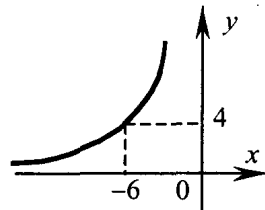
г) $\lim_{x \rightarrow +\infty} f(x) = 3$ и $\lim_{x \rightarrow -\infty} f(x) = -4$.



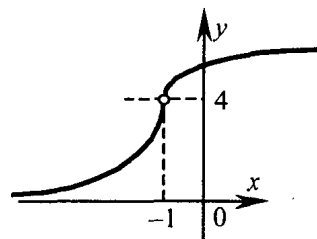
704. а) $\lim_{x \rightarrow 2} f(x) = 3$ и $f(2) = -3$;



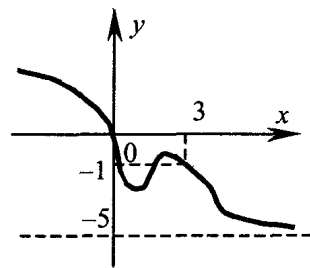
б) $\lim_{x \rightarrow -6} f(x) = 4$ и $\lim_{x \rightarrow -\infty} f(x) = 0$;



в) $\lim_{x \rightarrow -1} f(x) = 4$ и $f(-1)$ — не существует;



г) $\lim_{x \rightarrow 3} f(x) = -1$ и $\lim_{x \rightarrow +\infty} f(x) = -5$.



705. а) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+3)}{x-1} = 1 + 3 = 4$;

$$6) \lim_{x \rightarrow -2} \frac{x+2}{2x^2-x-6} = \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(2x-3)} = -\frac{1}{7};$$

$$в) \lim_{x \rightarrow -1} \frac{x+1}{x^2-2x-3} = \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x-3)} = \frac{1}{-1-3} = -\frac{1}{4};$$

$$г) \lim_{x \rightarrow 9} \frac{x^2-11x+18}{x-9} = \lim_{x \rightarrow 9} \frac{(x-9)(x-2)}{x-9} = 7.$$

$$706. а) \lim_{x \rightarrow -2} \frac{x+2}{x^3+8} = \lim_{x \rightarrow -2} \frac{1}{x^2-2x+4} = \frac{1}{4+4+4} = \frac{1}{12};$$

$$б) \lim_{x \rightarrow -1} \frac{1+x^3}{1-x^2} = \lim_{x \rightarrow -1} \frac{1-x+x^2}{1-x} = \frac{3}{2};$$

$$в) \lim_{x \rightarrow 3} \frac{x-3}{x^3-27} = \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x^2+3x+9)} = \frac{1}{9+9+9} = \frac{1}{27};$$

$$г) \lim_{x \rightarrow 4} \frac{16-x^2}{64-x^3} = \lim_{x \rightarrow 4} \frac{(4-x)(4+x)}{(4-x)(16+4x+x^2)} = \frac{8}{16+16+16} = \frac{1}{6}.$$

$$707. а) \lim_{x \rightarrow 0} \frac{\sin x}{\operatorname{tg} x} = \lim_{x \rightarrow 0} \cos x = 1;$$

$$б) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x \cos 2x}{\cos 2x \cos x} = \lim_{x \rightarrow \frac{\pi}{2}} \operatorname{tg} x - \text{предела не существует};$$

$$в) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\operatorname{ctg} x} = \lim_{x \rightarrow \frac{\pi}{2}} \sin x = 1; \quad г) \lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{\sin 5x + \sin 3x} = \lim_{x \rightarrow 0} -\frac{\sin 4x \sin x}{\sin 4x \cos x} = 0.$$

$$708. а) f(x) = Kx + m, f(x + \Delta x) = Kx + m + K\Delta x, f(x + \Delta x) - f(x) = K\Delta x;$$

$$б) f(x) = ax^2, f(x + \Delta x) = ax^2 + 2a\Delta x x + a\Delta x^2, f(x + \Delta x) - f(x) = 2a\Delta x x + a\Delta x^2;$$

$$в) f(x) = \frac{1}{x}, f(x + \Delta x) = \frac{1}{x + \Delta x}, f(x + \Delta x) - f(x) = \frac{x - x - \Delta x}{(x + \Delta x)x} = -\frac{\Delta x}{x^2 + x\Delta x};$$

$$г) f(x) = \sqrt{x}, f(x + \Delta x) = \sqrt{x + \Delta x}, f(x + \Delta x) - f(x) = \sqrt{x + \Delta x} - \sqrt{x}.$$

$$709. а) f(x) = Kx + m, \Delta f = K\Delta x, \frac{\Delta f}{\Delta x} = K;$$

$$б) f(x) = ax^2, \Delta f = \Delta x(2ax + a\Delta x), \frac{\Delta f}{\Delta x} = 2ax + a\Delta x;$$

$$в) f(x) = \frac{1}{x}, \Delta f = -\frac{\Delta x}{x^2 + x\Delta x}, \frac{\Delta f}{\Delta x} = -\frac{1}{x^2 + x\Delta x};$$

$$г) f(x) = \sqrt{x}, \Delta f = \frac{\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}, \frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}.$$

$$710. а) f(x) = Kx + m, \frac{\Delta f}{\Delta x} = K, \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = K;$$

$$б) f(x) = ax^2, \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \rightarrow 0} (2ax + a\Delta x) = 2ax;$$

$$в) f(x) = \frac{1}{x}, \frac{\Delta f}{\Delta x} = -\frac{1}{x^2 + x\Delta x}, \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = -\frac{1}{x^2};$$

$$г) f(x) = \sqrt{x}, \quad \frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}, \quad \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

$$711. а) \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x^2 - 3x} = \lim_{x \rightarrow 3} \frac{x+6-9}{x(x-3)(\sqrt{x+6}+3)} = \lim_{x \rightarrow 3} \frac{1}{x(\sqrt{x+6}+3)} = \frac{1}{3(3+3)} = \frac{1}{18};$$

$$б) \lim_{x \rightarrow \infty} (\sqrt{2x+3} - \sqrt{2x-7}) = \lim_{x \rightarrow \infty} \frac{10}{\sqrt{2x+3} + \sqrt{2x-7}} = 0.$$

$$712. а) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 - \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 - \cos x)} = \lim_{x \rightarrow 0} \frac{1}{1 - \cos x} = \frac{1}{2};$$

$$б) \lim_{x \rightarrow 0} \frac{\sin 7x - \sin 3x}{\sin 8x - \sin 2x} = \lim_{x \rightarrow 0} \frac{\sin 2x \cos 5x}{\sin 3x \cos 5x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} =$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{3 \sin x - 4 \sin^3 x} = \lim_{x \rightarrow 0} \frac{2 \cos x}{3 - 2 \sin^2 x} = \frac{2}{3}.$$

§32. Определение производной

$$713. а) \Delta S = S(3) - S(2) = 7 - 5 = 2. \Delta t = 3 - 2 = 1. v_{cp} = \frac{\Delta S}{\Delta t} = \frac{2}{1} = 2 \text{ м/с.}$$

$$б) \Delta S = S(2,5) - S(2) = 6 - 5 = 1. \Delta t = 2,5 - 2 = 0,5. v_{cp} = \frac{\Delta S}{\Delta t} = \frac{1}{0,5} = 2 \text{ м/с.}$$

$$в) \Delta S = S(2,1) - S(2) = 5,2 - 5 = 0,2. \Delta t = 2,1 - 2 = 0,1. v_{cp} = \frac{\Delta S}{\Delta t} = \frac{0,2}{0,1} = 2 \text{ м/с.}$$

$$г) \Delta S = S(2,05) - S(2) = 5,1 - 5 = 0,1. \Delta t = 2,05 - 2 = 0,05. v_{cp} = \frac{\Delta S}{\Delta t} = \frac{0,1}{0,05} = 2 \text{ м/с.}$$

В момент времени $t = 2$ с мгновенная скорость $v_{мгн} = 2$ м/с.

$$714. а) \Delta S = S(0,1) - S(0) = 0,01. v_{cp} = \frac{\Delta S}{\Delta t} = 0,1 \text{ м/с.}$$

$$б) \Delta S = S(0,01) - S(0) = 0,0001. v_{cp} = \frac{\Delta S}{\Delta t} = 0,01 \text{ м/с.}$$

$$в) \Delta S = S(0,2) - S(0) = 0,04. v_{cp} = \frac{\Delta S}{\Delta t} = 0,2 \text{ м/с.}$$

$$г) \Delta S = S(0,02) - S(0) = 0,0004. v_{cp} = \frac{\Delta S}{\Delta t} = 0,02 \text{ м/с.}$$

В момент времени $t = 1$ с мгновенная скорость $v_{мгн} = 2$ м/с.

$$715. а) v_{cp} = 4 \text{ м/с; } б) v_{cp} = 6 \text{ м/с; } в) v_{cp} = 3 \text{ м/с; } г) v_{cp} = 5 \text{ м/с.}$$

$$716. а) y_1 = \sqrt{3}x + K; y'_1(x_1) = \sqrt{3} \Rightarrow f'(x) = \sqrt{3}; y_2 = x + K_2; y'_2(x_2) = 1 \Rightarrow f'(x_2) = 1;$$

$$б) f'(x_1) = 0; y = \frac{\sqrt{3}}{3}x + K; y'(x_2) = \frac{\sqrt{3}}{3} \Rightarrow f'(x_2) = \frac{\sqrt{3}}{3};$$

$$в) f'(x_1) = 0; f'(x_2) = -\frac{\sqrt{3}}{3}; \quad г) f'(x_1) = 0; f'(x_2) = 0.$$

717. а) $y = 9,5x - 3$. $v = 9,5$ м/с; б) $y = -16x + 3$. $v = -16$ м/с;
 в) $y = 6,7x - 13$. $v = 6,7$ м/с; г) $y = -9x + 4$. $v = -9$ м/с.
718. а) $f(x) = x^2$. $x_0 = 2$. $f'(x_0) = 2x_0 = 4$. б) $f(x) = x^2$. $x_0 = -1$. $f'(x_0) = -2$.
 в) $f(x) = x^2$. $x_0 = -2$. $f'(x_0) = -4$. г) $f(x) = x^2$. $x_0 = 9$. $f'(x_0) = 18$.
719. а) $f(x) = \frac{1}{x}$. $x_0 = 2$. $f'(x_0) = -\frac{1}{4}$. б) $f(x) = \frac{1}{x}$. $x_0 = -1$. $f'(x_0) = 1$.
 в) $f(x) = \frac{1}{x}$. $x_0 = 5$. $f'(x_0) = -\frac{1}{25}$. г) $f(x) = \frac{1}{x}$. $x_0 = -0,5$. $f'(x_0) = -4$.
720. $S(t) = t^2$. $S'(t) = 2t$. $S''(t) = 2$.
 а) $t = 1$. $v = 2$ (м/с). $a = 2$ (м/с²). б) $t = 2,1$. $v = 4,2$ (м/с). $a = 2$ (м/с²).
 в) $t = 2$. $v = 4$ (м/с). $a = 2$ (м/с²). г) $t = 3,5$. $v = 7$ (м/с). $a = 2$ (м/с²).
721. $y = x^2$. $y' = 2x$. а) $y' > 0$, при $x > 0$. б) $y' < 0$, при $x < 0$.
722. $S(t) = 2t^2 + 1$. $t_1 = 0$.
 а) $t_2 = 0,6$. $S(t_2) - S(t_1) = 1,32$. $\frac{\Delta S}{\Delta t} = \frac{1,32}{0,6} = 2,2$ (м/с);
 б) $t_2 = 0,2$. $S(t_2) - S(t_1) = 0,28$. $\frac{\Delta S}{\Delta t} = \frac{0,28}{0,2} = 1,4$ (м/с);
 в) $t_2 = 0,5$. $S(t_2) - S(t_1) = 1$. $\frac{\Delta S}{\Delta t} = \frac{1}{0,5} = 2$ (м/с);
 г) $t_2 = 0,1$. $S(t_2) - S(t_1) = 0,12$. $\frac{\Delta S}{\Delta t} = \frac{0,12}{0,1} = 1,2$ (м/с).
723. а) $S(t) = t^2 + 3$. $S'(t) = 2t$. $v_{\text{мгн}} = 2t$ (м/с); б) $S(t) = t^2 - t$. $S'(t) = 2t - 1$. $v_{\text{мгн}} = 2t - 1$ (м/с);
 в) $S(t) = t^2 + 4$. $S'(t) = 2t$. $v_{\text{мгн}} = 2t$ (м/с); г) $S(t) = t^2 - 2t$. $S'(t) = 2t - 2$. $v_{\text{мгн}} = 2t - 2$ (м/с).
724. а) $f'(-7) < f'(-2)$. б) $f'(-4) < f'(2)$. в) $f'(-9) < f'(0)$. г) $f'(-1) > f'(5)$.
725. а) $f'(x_1) > 0$, $f'(x_2) > 0$. $x_1 = 0$, $x_2 = 1$. б) $f'(x_1) < 0$, $f'(x_2) > 0$. $x_1 = -6$, $x_2 = 0$.
 в) $f'(x_1) < 0$, $f'(x_2) < 0$. $x_1 = -5$, $x_2 = -4$. г) $f'(x_1) > 0$, $f'(x_2) < 0$. $x_1 = 2$, $x_2 = 4$.
726. а) $\varphi'(x) > 0$; $x = -7, -6, -5$. б) $\varphi'(x) < 0$ и $x > 0$; $x = 4, 5$.
 в) $\varphi'(x) < 0$; $x = -3, -2$. г) $\varphi'(x) > 0$ и $x < 0$; $x = -5, -6$.
727. $S(t) = t^2 + 4t$. $S'(t) = 2t + 4$. $S''(t) = 2$.
 а) $t = 1$. $v = 2 + 4 = 6$ (м/с); $a = 2$ (м/с²); б) $t = 2,1$. $v = 4,2 + 4 = 8,2$ (м/с); $a = 2$ (м/с²);
 в) $t = 2$. $v = 8$ (м/с); $a = 2$ (м/с²); г) $t = 3,5$. $v = 7 + 4 = 11$ (м/с); $a = 2$ (м/с²).

§33. Вычисление производных

728. а) $y = 7$; б) $y = 2x$; в) $y = -6$; г) $y = -\frac{1}{x^2}$.
729. а) $y = \cos x$; б) $y = \frac{1}{2\sqrt{x}}$; в) $y = -\sin x$; г) $y = 0$.

$$730. a) g'(x) = \frac{1}{2\sqrt{x}}; g'(x_0) = \frac{1}{4};$$

$$б) g'(x) = 2x; g'(x_0) = -14;$$

$$в) g'(x) = -3; g'(x_0) = -3;$$

$$г) g'(x) = -\frac{1}{x^2}; g'(x_0) = -4.$$

$$731. a) g'(x) = \cos x; g'(x_0) = 0;$$

$$б) g'(x) = -\sin x; g'(x_0) = -\frac{1}{2};$$

$$в) g'(x) = -\sin x; g'(x_0) = 0;$$

$$г) g'(x) = \cos x; g'(x_0) = 1.$$

$$732. a) h'(x_0) = 7; б) h'(x) = \frac{1}{2\sqrt{x}}; h'(x_0) = \frac{1}{8}; в) h'(x) = -6; г) h'(x) = \frac{1}{2\sqrt{x}}; h'(x_0) = \frac{1}{6}.$$

$$733. a) h'(x) = \frac{1}{x^2}; h'(x_0) = \frac{1}{4};$$

$$б) h'(x) = \cos x; h'(x_0) = 0;$$

$$в) h'(x) = 2x; h'(x_0) = -\frac{1}{5};$$

$$г) h'(x) = -\sin x; h'(x_0) = 0.$$

$$734. a) f'(x) = 2x; f'(x_0) = -8;$$

$$б) f'(x) = -\frac{1}{x^2}; f'(x_0) = -9;$$

$$в) f'(x) = -\frac{1}{x^2}; f'(x_0) = -4;$$

$$г) f'(x) = 2x; f'(x_0) = 4.$$

$$735. a) f'(x) = \cos x; f'(x_0) = \frac{1}{2};$$

$$б) f'(x) = -\sin x; f'(x_0) = \frac{\sqrt{2}}{2};$$

$$в) f'(x) = -\sin x; f'(x) = -\frac{\sqrt{3}}{2};$$

$$г) f'(x) = \cos x; f'(x_0) = \frac{\sqrt{3}}{2}.$$

$$736. a) f(x) = x^2 + c; б) f(x) = \sin x + c; в) f'(x) = 3x + c; г) f(x) = \cos x + c.$$

$$737. a) y' = 2x - 7; б) y' = -6x - 13; в) y' = 14x + 3\$, г) y' = -2x + 8.$$

$$738. a) y' = 12 + \frac{1}{2\sqrt{x}}; б) y' = \frac{1}{2\sqrt{x}} - 18x; в) y' = 15 + \frac{1}{2\sqrt{x}}; г) y' = \frac{1}{2\sqrt{x}} - 10x.$$

$$739. a) y' = -\frac{1}{x^2} + 4; б) y' = -4x + \frac{1}{x^2}; в) y' = -\frac{1}{x^2} - 6; г) y' = 20x - \frac{1}{x^2}.$$

$$740. a) y' = \frac{3}{\sqrt{x}} - \frac{3}{x^2}; б) y' = -\frac{1}{\sqrt{x}} + \frac{1}{x^2}; в) y' = \frac{5}{\sqrt{x}} - \frac{5}{x^2}; г) y' = -\frac{4}{\sqrt{x}} + \frac{1}{x^2}.$$

$$741. a) y' = \cos x; б) y' = -4\sin x; в) y' = -\sin x; г) y' = -2\cos x.$$

$$742. a) y' = -\sin x + 2; б) y' = 2\cos x - 6; в) y' = \cos x - 3; г) y' = -3\sin x + 15.$$

$$743. a) y' = 5\cos x - \sin x; б) y' = 3\cos x - \sin x; в) y' = \cos x + \sin x; г) y' = -2\sin x + \cos x.$$

$$744. a) y' = 5x^4; б) y' = 10x^9; в) y' = 4x^3; г) y' = 201x^{200}.$$

$$745. a) y' = 3x^2 + 10x^4; б) y' = 4x^3 - 9x^8; в) y' = 3x^2 + 400x^{99}; г) y' = 4x^3 - 63x^8.$$

$$746. a) y' = 5x^4 + 180x^{19}; б) y' = 7x^6 - 64x^{15}; в) y' = 6x^5 - 130x^9; г) y' = 9x^8 - 126x^{20}.$$

$$747. a) y' = (x^4 + 2)(2x) + (x^2 - 1)(4x^3); б) y' = (x^2 + 3)6x^5 + (x^6 - 1)2x.$$

$$в) y' = 2x(x^4 - 1) + (x^2 + 3)4x^3; г) y' = 2x(x^7 + 4) + (x^2 - 2)(7x^6).$$

$$748. \text{ а) } y' = \frac{x-2}{\sqrt{x}} + 2\sqrt{x} = \frac{3x-2}{\sqrt{x}};$$

$$\text{б) } y' = \frac{x^3+1}{2\sqrt{x}} + 3x^2\sqrt{x} = \frac{7x^3+1}{2\sqrt{x}};$$

$$\text{в) } y' = \frac{4x-5}{\sqrt{x}} + 8\sqrt{x} = \frac{12x-5}{\sqrt{x}};$$

$$\text{г) } y' = \frac{x^4+2}{2\sqrt{x}} + 4x^3\sqrt{x} = \frac{5x^4+2}{2\sqrt{x}}.$$

$$749. \text{ а) } y' = \sin x + x \cos x;$$

$$\text{б) } y' = \frac{\cos x}{2\sqrt{x}} - \sqrt{x} \sin x = \frac{\cos x - 2x \sin x}{2\sqrt{x}};$$

$$\text{в) } y' = \cos x - x \sin x;$$

$$\text{г) } y' = \frac{\sin x}{2\sqrt{x}} + \sqrt{x} \cos x - \frac{\sin x + 2x \cos x}{2\sqrt{x}}.$$

$$750. \text{ а) } y' = -\frac{1}{x^2}(2x-3) + \frac{1}{x} + 2 = 2 + \frac{3}{x^2};$$

$$\text{б) } y' = \frac{1}{x^2}(6x+1) - \frac{6}{x} + 42 = 42 + \frac{1}{x^2};$$

$$\text{в) } y' = -\frac{1}{x^2}(5x-2) + \frac{5}{x} + 40 = 40 + \frac{2}{x^2}; \quad \text{г) } y' = \frac{1}{x^2}(3x+2) - \frac{3}{x} + 27 = 27 + \frac{2}{x^2}.$$

$$751. \text{ а) } y' = \frac{3x^2(2x+4)-2x^3}{4x^2+16x+16} = \frac{4x^3+12x^2}{4x^2+16x+16} = \frac{x^3+3x^2}{x^2+4x+4} = \frac{x^2(x+3)}{(x+2)^2};$$

$$\text{б) } y' = \frac{2x(x^2-1)-2x^3}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2};$$

$$\text{в) } y' = \frac{2x(3-4x)+4x^2}{(3-4x)^2} = \frac{6x-4x^2}{(3-4x)^2} = \frac{2x(3-2x)}{(3-4x)^2}; \quad \text{г) } y' = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}.$$

$$752. \text{ а) } y' = \frac{\frac{3}{2\sqrt{x}}(2x+9)-6\sqrt{x}}{(2x+9)^2} = \frac{6x+27-12x}{2\sqrt{x}(2x+9)^2} = \frac{27-6x}{2\sqrt{x}(2x+9)^2};$$

$$\text{б) } y' = \frac{x \cos x - \sin x}{x^2}; \quad \text{в) } y' = \frac{-\frac{1}{\sqrt{x}}(8-3x)+3(-2\sqrt{x})}{(8-3x)^2} = \frac{3x-8-6x}{\sqrt{x}(8-3x)^2} = -\frac{8+3x}{\sqrt{x}(8-3x)^2};$$

$$\text{г) } y' = \frac{-x \sin x - \cos x}{x^2} = \frac{\cos x + x \sin x}{x^2}.$$

$$753. \text{ а) } y' = \frac{1}{\cos^2 x};$$

$$\text{б) } y' = -\frac{1}{\sin^2 x};$$

$$\text{в) } y' = \frac{1}{\cos^2 x};$$

$$\text{г) } y' = -\frac{1}{\sin^2 x}.$$

$$754. \text{ а) } y' = 3 \cos x - \frac{1}{\sin^2 x};$$

$$\text{б) } y' = \frac{1}{\cos^2 x} + \sin x;$$

$$\text{в) } y' = -\sin x + \frac{1}{\cos^2 x};$$

$$\text{г) } y' = \frac{6}{\cos^2 x} - \cos x.$$

$$755. \text{ а) } y' = \operatorname{tg} x + \frac{x}{\cos^2 x};$$

$$\text{б) } y' = \sin x + \frac{\sin x}{\cos^2 x};$$

$$\text{в) } y' = \operatorname{ctg} x - \frac{x}{\sin^2 x};$$

$$\text{г) } y' = -\cos x - \frac{\cos x}{\sin^2 x}.$$

756. а) $y' = 6$; $y'(x_0) = 6$;

в) $y' = 5$; $y'(x_0) = 5$;

757. а) $y' = 2x + 2$; $y(x_0) = 2$;

в) $y' = 2x + 3$; $y'(x_0) = 5$;

758. а) $y' = -\frac{2}{x^2}$; $y'(x_0) = -\frac{1}{8}$;

в) $y' = -\frac{8}{x^2}$; $y'(x_0) = -8$;

759. а) $y' = 2\cos x$; $y'(x_0) = 0$;

в) $y' = -\cos x$; $y(x_0) = -\frac{\sqrt{3}}{2}$;

б) $y' = -11$; $y'(x_0) = -11$;

г) $y' = -20$; $y'(x_0) = -20$.

б) $y' = 3x^2 - 3$; $y'(x_0) = 3 - 3 = 0$;

г) $y' = 3x^2 - 18x$; $y'(x_0) = 12 - 36 = -24$.

б) $y' = \frac{1}{2\sqrt{x}}$; $y'(x_0) = \frac{1}{6}$;

г) $y' = \frac{1}{2\sqrt{x}}$; $y'(x_0) = \frac{1}{4}$.

б) $y' = 2\sin x$; $y(x_0) = \frac{\sqrt{3}}{2}$;

г) $y' = -4\sin x$; $y'(x_0) = -2\sqrt{2}$.

760. а) $y' = \frac{1}{\cos^2 x}$; $y'(x_0) = 2$;

б) $y' = \frac{-2}{\sin^2 x}$; $y' = -\frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} = -\frac{8}{3} = -2\frac{2}{3}$;

в) $y' = -\frac{1}{\sin^2 x}$; $y'(x_0) = -4$;

г) $y' = \frac{-4}{\cos^2 x}$; $y'(x_0) = -4$.

761. а) $y' = \frac{x \cos x - \sin x}{x^2}$; $y'(x_0) = -\frac{4}{\pi}$;

б) $y' = \frac{x-1-x-1}{(x-1)^2} = -\frac{2}{(x-1)^2}$; $y'(x_0) = -2$;

в) $y' = \frac{-x \sin x - \cos x}{x^2}$; $y'(x_0) = \frac{1}{\pi^2}$;

г) $y' = \frac{2x+2-2x}{(x+1)^2} = \frac{2}{(x+1)^2}$; $y'(x_0) = 2$.

762. а) $g'(x) = 3x^2 + 2$; $g'(x_0) = 14$;

б) $g'(x) = \frac{\sqrt{x}}{2\sqrt{x}} + \frac{\sqrt{x}+1}{2\sqrt{x}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2\sqrt{x}}$; $g'(x_0) = \frac{3}{2}$;

в) $g'(x) = 2x - 3$; $g'(x_0) = 5$;

г) $g'(x) = -\frac{1}{x^2} \left(\frac{4}{x} - 2 \right) + \frac{1}{x} \left(-\frac{4}{x^2} \right) = -\frac{4}{x^3} + \frac{2}{x^2} - \frac{4}{x^3}$; $g'(x_0) = 4 \cdot 8 + 2 \cdot 4 + 4 \cdot 8 = 72$.

763. а) $g'(x) = -\frac{6}{x^2} - 2$; $g'(x_0) = -6 - 2 = -8$;

б) $g'(x) = 6x^2 - 4$; $g'(x_0) = 20$;

в) $g'(x) = 8x + \frac{12}{x^2}$; $g'(x_0) = -16 + 3 = -13$;

г) $g'(x) = -3x^2 + 4x$; $g'(x_0) = -4$.

764. а) $g'(x) = 2\cos x - 4$; $g'(x_0) = -4$;

б) $g'(x) = \frac{1}{3\cos^2 x}$; $g'(x_0) = \frac{4}{3}$;

в) $g'(x) = 3\sin x + 1$; $g'(x_0) = -\frac{1}{2}$;

г) $g'(x) = \frac{1}{5\sin^2 x}$; $g'(x_0) = -\frac{4}{15}$.

765. а) $h'(x) = 6x^5 - 4$; $h'(x_0) = 2$; $\operatorname{tg} \alpha = 2$;

б) $h'(x) = \frac{1}{2\sqrt{x}}$; $h'(x_0) = 1$; $\operatorname{tg} \alpha = 1$;

в) $h'(x) = -5x^4 - 4x$; $h'(x_0) = -5 + 1 = -4$; $\operatorname{tg} \alpha = -1$;

$$\text{г) } h'(x) = -\frac{25}{x^2}; h'(x_0) = -16; \operatorname{tg} \alpha = -16.$$

$$766. \text{ а) } h'(x) = \sin x; h'(x_0) = -1; \operatorname{tg} \alpha = -1; \quad \text{б) } h'(x) = \frac{2}{\cos^2 x}; h'(x_0) = 4; \operatorname{tg} \alpha = 4;$$

$$\text{в) } h'(x) = -\cos x; h'(x_0) = -1; \operatorname{tg} \alpha = -1; \quad \text{г) } h'(x) = \frac{4}{\sin^2 x}; h'(x_0) = 8; \operatorname{tg} \alpha = 8.$$

$$767. \text{ а) } f'(x_0) = 2\sin x + x^2\cos x; f'\left(\frac{\pi}{2}\right) = \pi;$$

$$\text{б) } f'(x) = \sqrt{3}\cos x + \frac{2x}{\pi} + \frac{1}{2}; f'\left(\frac{\pi}{6}\right) = \frac{3}{2} + \frac{1}{3} + \frac{1}{2} = 2\frac{1}{3};$$

$$\text{в) } f'(x) = 1 + \cos x + x(-\sin x); f'(\pi) = 1 - 1 + 0 = 0;$$

$$\text{г) } f'(x) = -\sqrt{3}\sin x - \frac{\sqrt{3}}{2} + \frac{2x}{\pi}; f'\left(\frac{\pi}{3}\right) = -\frac{3}{2} - \frac{\sqrt{3}}{2} + \frac{2}{3} = \frac{4-9-3\sqrt{3}}{6} = -\frac{5+3\sqrt{3}}{6}.$$

$$768. \text{ а) } f(x) = x^3 + x^2 + c; \quad \text{б) } f(x) = \frac{7}{x} + c; \quad \text{в) } f(x) = x^5 - x + c; \quad \text{г) } f(x) = 9\sqrt{x} + c.$$

$$769. \text{ а) } f'(x) = \frac{1}{\sqrt{x}} - 5 = 2 \Rightarrow x = \frac{1}{49}; \quad \text{б) } f'(x) = 3 - \frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{16}.$$

$$770. \text{ а) } y' = 7(4x - 9)^6 \cdot 4 = 28(4x - 9)^6; \quad \text{б) } y' = 12\left(\frac{x}{3} + 2\right)^{11} \cdot \frac{1}{3} = 4\left(\frac{x}{3} + 2\right)^{11};$$

$$\text{в) } y' = 9(5x + 1)^8 \cdot 5 = 45(5x + 1)^8; \quad \text{г) } y' = 14\left(\frac{x}{4} - 3\right)^{13} \cdot \frac{1}{4} = \frac{7}{2}\left(\frac{x}{4} - 3\right)^{13}.$$

$$771. \text{ а) } y' = -5(3 - x)^4; \quad \text{б) } y' = -240(7 - 24x)^9;$$

$$\text{в) } y' = -\frac{6}{5}\left(12 - \frac{x}{5}\right)^5; \quad \text{г) } y' = -117(15 - 9x)^{12}.$$

$$772. \text{ а) } y' = 3\cos(3x - 9); \quad \text{б) } y' = -2\cos(7 - 2x);$$

$$\text{в) } y' = \frac{1}{2}\cos\left(\frac{x}{2} + 1\right); \quad \text{г) } y' = -3\cos(5 - 3x).$$

$$773. \text{ а) } y' = -5\sin(5x + 9); \quad \text{б) } y' = 4\sin\left(\frac{\pi}{3} - 4x\right);$$

$$\text{в) } y' = -9\sin(9x - 10); \quad \text{г) } y' = \frac{1}{2}\sin\left(\frac{\pi}{4} - \frac{x}{2}\right).$$

$$774. \text{ а) } y' = \frac{5}{\cos^2\left(5x - \frac{\pi}{4}\right)}; \quad \text{б) } y' = \frac{4}{\sin^2\left(\frac{\pi}{6} - 4x\right)};$$

$$\text{в) } y' = \frac{2}{\cos^2\left(2x + \frac{\pi}{3}\right)}; \quad \text{г) } y' = \frac{5}{\sin^2\left(\frac{\pi}{4} - 5x\right)}.$$

$$775. \text{ а) } y' = \frac{-7}{2\sqrt{15-7x}}; \text{ б) } y' = \frac{1}{4\sqrt{42+0,5x}}; \text{ в) } y' = \frac{9}{2\sqrt{4+9x}}; \text{ г) } y' = \frac{-1}{10\sqrt{50-0,2x}}.$$

$$776. \text{ а) } y' = 21(3x-2)^6; y'(3) = 3 \cdot 7^7; \quad \text{ б) } y' = -35(4-5x)^6; y'(-2) = -35 \cdot 14^6;$$

$$\text{ в) } y' = 10(2x+3)^4; y'(2) = 10 \cdot 7^4; \quad \text{ г) } y' = -21(5-3x)^6; y'(-1) = -21 \cdot 8^6.$$

$$777. \text{ а) } y' = 2\cos\left(2x - \frac{\pi}{3}\right); y'\left(\frac{\pi}{6}\right) = 2; \quad \text{ б) } y' = -2\cos\left(\frac{\pi}{6} - 2x\right); y'\left(\frac{\pi}{12}\right) = -2;$$

$$\text{ в) } y' = 4\sin\left(\frac{\pi}{3} - 4x\right); y'\left(\frac{\pi}{8}\right) = -4\sin\frac{\pi}{6} = -2;$$

$$\text{ г) } y' = -6\sin\left(6x - \frac{\pi}{4}\right); y'\left(-\frac{\pi}{12}\right) = -6\sin\left(-\frac{3\pi}{4}\right) = 6\frac{\sqrt{2}}{2} = 3\sqrt{2}.$$

$$778. \text{ а) } y' = \frac{2}{\cos^2\left(2x + \frac{\pi}{8}\right)}; y'(x_0) = \frac{2}{\cos^2\frac{\pi}{4}} = 4;$$

$$\text{ б) } y' = \frac{1}{\sin^2\left(\frac{\pi}{6} - x\right)}; y'(x_0) = \frac{1}{\sin^2\left(-\frac{\pi}{6}\right)} = 4;$$

$$\text{ в) } y' = \frac{2}{\cos^2\left(3x - \frac{\pi}{4}\right)}; y'(x_0) = \frac{3}{\cos^2 0} = 3;$$

$$\text{ г) } y' = \frac{1}{\sin^2\left(\frac{\pi}{3} - x\right)}; y'(x_0) = \frac{1}{\sin^2\frac{\pi}{6}} = 4.$$

$$779. \text{ а) } y' = \frac{3}{\sqrt{6x-1}}; y'(x_0) = \frac{3}{\sqrt{29}} = \frac{3\sqrt{29}}{29}; \quad \text{ б) } y' = -\frac{4}{\sqrt{4-8x}}; y'(x_0) = -2;$$

$$\text{ в) } y' = \frac{7}{2\sqrt{7x+4}}; y'(x_0) = \frac{7}{10}; \quad \text{ г) } y' = -\frac{9}{2\sqrt{25-9x}}; y'(x_0) = -\frac{9}{8} = -1\frac{1}{8}.$$

$$780. \text{ а) } y' = 10(2x+1)^4; y'(x_0) = 10; \quad \text{ б) } y' = \frac{7}{2\sqrt{7x-3}}; y'(x_0) = \frac{7}{4} = 1\frac{3}{4};$$

$$\text{ в) } y' = \frac{-12 \cdot 4}{(12x-5)^2}; y'(x_0) = -\frac{48}{19^2} = -\frac{48}{361}; \quad \text{ г) } y' = \frac{-5}{2\sqrt{11-5x}}; y'(x_0) = -\frac{5}{8}.$$

$$781. \text{ а) } y' = \cos\left(3x - \frac{\pi}{4}\right); y'(x_0) = 0; \quad \text{ б) } y' = \frac{6}{\cos^2 6x}; y'(x_0) = \frac{6}{\cos^2\frac{\pi}{4}} = 12;$$

$$\text{ в) } y' = 2\sin\left(\frac{\pi}{3} - 2x\right); y'(x_0) = 2\sin\left(-\frac{\pi}{3}\right) = -\sqrt{3};$$

$$\text{ г) } y' = \frac{-1}{6\sin^2\frac{x}{3}}; y'(x_0) = -\frac{1}{6\sin^2\frac{\pi}{3}} = -\frac{4}{9}.$$

$$782. \text{ а) } h'(x) = \frac{7}{2}(0,5x+3)^6; h'(x_0) = \frac{7}{2} = 3,5; \quad \text{ б) } h'(x) = \frac{8}{\sqrt{16x+21}}; h'(x_0) = \frac{8}{5} = 1,6;$$

$$\text{ в) } h'(x) = \frac{-18 \cdot 4}{(4x+1)^2}; h'(x_0) = -\frac{72}{9} = -8; \quad \text{ г) } h'(x) = -\frac{1}{\sqrt{6-2x}}; h'(x_0) = -\frac{1}{2}.$$

$$783. \text{ а) } y = (x-1)(x^2+x+1) = x^3-1; y' = 3x^2;$$

$$\text{ б) } y = (x^2+2x+4)(x-2) = x^3-8; y' = 3x^2;$$

$$\text{ в) } y = (x+1)(x^2-x+1) = x^3+1; y' = 3x^2;$$

$$\text{ г) } y = (x^2-9x+9)(x+3) = x^3+27; y' = 3x^2.$$

$$784. \text{ а) } y = \frac{x^9-3}{x^3} = x^6 - \frac{3}{x^3}; y' = 6x^5 + \frac{9}{x^4};$$

$$\text{ б) } y = \frac{x^{15}}{x^{10}+1}; y' = \frac{15x^{14}(x^{10}+1) - 10x^9 \cdot x^{15}}{(x^{10}+1)^2} = \frac{5x^{24} + 15x^{14}}{(x^{10}+1)^2};$$

$$\text{ в) } y = \frac{x^5+x}{x^5-1}; y' = \frac{(5x^4+1)(x^5-1) - 5x^4(x^5+x)}{(x^5-1)^2} = \frac{5x^9+x^5-5x^4-1-5x^9+5x^5}{(x^5-1)^2} = \frac{-4x^5-5x^4-1}{(x^5-1)^2};$$

$$\text{ г) } y = \frac{x^{13}}{x^4-2}; y' = \frac{13x^{12}(x^4-2) - 4x^3 \cdot x^{13}}{(x^4-2)^2} = \frac{9x^{16} - 26x^{12}}{(x^4-2)^2}.$$

$$785. \text{ а) } y = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x; y' = -\sin x; \quad \text{ б) } y = 2 \sin \frac{x}{2} \cos \frac{x}{2} = \sin x; y' = \cos x;$$

$$\text{ в) } y = \cos^2 3x + \sin^2 3x = 1; y' = 0; \quad \text{ г) } y = -\sin \frac{x}{2} \cos \frac{x}{2} = -\frac{1}{2} \sin x; y' = -\frac{1}{2} \cos x.$$

$$786. \text{ а) } y = \sin 2x \cos x - \cos 2x \sin x = \sin x; y' = \cos x;$$

$$\text{ б) } y = \sin \frac{x}{3} \cos \frac{2x}{3} + \cos \frac{x}{3} \sin \frac{2x}{3} = \sin x; y' = \cos x;$$

$$\text{ в) } y = \cos 3x \cos 2x - \sin 3x \sin 2x = \cos x; y' = -\sin x;$$

$$\text{ г) } y = \cos \frac{x}{5} \cos \frac{4x}{6} - \sin \frac{x}{5} \sin \frac{4x}{5} = \cos x; y' = -\sin x.$$

$$787. \text{ а) } f(x) = 2a \cos 2x - b \sin x;$$

$$\begin{cases} 2a \cos \frac{\pi}{3} - b \sin \frac{\pi}{6} = 2 \\ 2a \cos 9\pi - b \sin \frac{9\pi}{2} = -4 \end{cases}; \begin{cases} a - \frac{b}{2} = 2 \\ -2a - b = -4 \end{cases}; \begin{cases} 2a - b = 4 \\ 2a + b = 4 \end{cases}; \begin{cases} a = 2, \\ b = 0. \end{cases}$$

$$\text{ б) } f(x) = -2a \sin 2x + 4b \cos 4x;$$

$$\begin{cases} -2a \sin \frac{7\pi}{6} + 4b \cos \frac{7\pi}{3} = 4 \\ -2a \sin \frac{3\pi}{2} + 4b \cos 3\pi = 2 \end{cases}; \begin{cases} a + 2b = 4 \\ 2a - 4b = 2 \end{cases}; \begin{cases} a + 2b = 4 \\ a - 2b = 1 \end{cases}; \begin{cases} a = \frac{5}{2}, \\ b = \frac{3}{4}. \end{cases}$$

$$788. \text{ а) } f'(x) = \frac{1}{2\sqrt{x}} - 1 = 1; 1 - 4\sqrt{x} = 0; \sqrt{x} = \frac{1}{4}; x = \frac{1}{16};$$

$$6) f'(x) = \frac{1}{2\sqrt{x}} + 3 = 4; 1 - 2\sqrt{x} = 0; \sqrt{x} = \frac{1}{2}; x = \frac{1}{4}.$$

$$789. a) f'(x) = \cos 2x = -\frac{\sqrt{2}}{2}; x = \pm \frac{3\pi}{8} + \pi n;$$

$$6) f'(x) = -2 \cos x \sin x = -\sin 2x = \frac{1}{2}; x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi k}{2}.$$

$$790. a) f(x) 3x^2 - 4x^3 < 0; x^2(3 - 4x) < 0; x > \frac{3}{4};$$

$$6) f'(x) = x^4 - 5x^2 + 6 < 0; (x^2 - 2)(x^2 - 3) < 0; (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3}) < 0; \\ x \in (-\sqrt{3}; -\sqrt{2}) \cup (\sqrt{2}; \sqrt{3}).$$

$$791. a) f'(x) = 2 \cos 2x < 0; 2x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right); x \in \left(\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n\right);$$

$$6) f'(x) = 4 \sin x + 2 < 0; \sin x < -\frac{1}{2}; x \in \left(-\frac{5\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n\right).$$

$$792. a) g'(x) = 3x^2 + 4x^3 > 0; x^2(3 + 4x) > 0; x \in \left(-\frac{3}{4}; 0\right) \cup (0; +\infty);$$

$$6) g'(x) = \frac{20}{(2-5x)^2} > 0; x \in \mathbb{R}, \text{ но } x \neq \frac{2}{5}.$$

$$793. a) f'(x) = -2 \sin 2x > 0; \sin 2x < 0; 2x \in (-\pi + 2\pi n; 2\pi n); x \in \left(-\frac{\pi}{2} + \pi n; \pi n\right);$$

$$6) g'(x) = 2 \sin x \cos x = \sin 2x > 0; 2x \in (2\pi n; \pi + 2\pi n); x \in \left(\pi n; \frac{\pi}{2} + \pi n\right).$$

$$794. a) f'(x) = -2 \cos x \sin x + \cos x = 0; \cos x(1 - 2 \sin x) = 0; \cos x = 0;$$

$$x = \frac{\pi}{2} + \pi n; x = \frac{\pi}{2}; \sin x = \frac{1}{2}; x = (-1)^k \frac{\pi}{6} + \pi k; x = \frac{\pi}{6};$$

$$6) f'(x) = 2 \sin x \cos x + \sin x = 0; \sin x(2 \cos x + 1) = 0; \sin x = 0; x = \pi n; x = \pi;$$

$$\cos x = -\frac{1}{2}; x = \pm \frac{2\pi}{3} + 2\pi n; x = \frac{2\pi}{3}; x = 4\pi.$$

$$795. a) h'(x) = 3x^2 - 6x > 0; x(3x - 6) > 0; x \in (-\infty; 0) \cup (2; +\infty);$$

$$6) h'(x) = \frac{2}{\sqrt{x}} - 1 > 0; \frac{2}{\sqrt{x}} > 1; \sqrt{x} < 2; x \in (0; 4);$$

$$b) y'(x) = 3x^2 - 4x^3 = x^2(3 - 4x) > 0; x \in (-\infty; 0) \cup \left(0; \frac{3}{4}\right);$$

$$r) h'(x) = \frac{1}{\cos^2 x} - 4 > 0; \cos^2 x < \frac{1}{4}; \cos x \in \left(-\frac{1}{2}; \frac{1}{2}\right); x \in \left(\frac{\pi}{3}; \frac{2\pi}{3}\right).$$

$$796. \text{ а) } \varphi'(x) = \cos x < 0; x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n \right);$$

$$\text{б) } \varphi'(x) = x^4 - 10x^2 + 9 < 0; (x^2 - 9)(x^2 - 1) < 0; (x - 3)(x + 3)(x - 1)(x + 1) < 0;$$

$$\text{в) } \varphi'(x) = -\frac{1}{\sin^2 x} + 9 < 0; \sin^2 x < \frac{1}{9}; \sin x \in \left(-\frac{1}{3}; \frac{1}{3} \right); x \in \left(-\arcsin \frac{1}{3} + \pi n; \arcsin \frac{1}{3} + \pi n \right);$$

$$\text{г) } \varphi'(x) = -\frac{1}{\sqrt{4-2x}} < 0; x < 2.$$

$$797. \text{ а) } f'(x) = x^2 - 2x; g'(x) = 15x - 16;$$

$$x^2 - 2x = 15x - 16; x^2 - 17x + 16 = 0; x = \frac{17 \pm 15}{2} = 16; x = 1;$$

$$\text{б) } f'(x) = \frac{1}{2\sqrt{x}}; g'(x) = \frac{1}{x^2};$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{x^2}; x \neq 0; x^2 = 2\sqrt{x}; x^4 = 4x; x(x^3 - 4) = 0; x = 0 \text{ не подходит; } x = \sqrt[3]{4}.$$

$$798. \text{ а) } f'(x) = -2\sin 2x; g'(x) = \cos x;$$

$$-2\sin 2x = \cos x; \cos x (4\sin x + 1) = 0; \cos x = 0;$$

$$x = \frac{\pi}{2} + \pi n; \sin x = -\frac{1}{4}; x = (-1)^{k+1} \arcsin \frac{1}{4} + \pi k;$$

$$\text{б) } f'(x) = \frac{1}{\cos^2 x}; g'(x) = -\frac{1}{\sin^2 x};$$

$$\frac{1}{\cos^2 x} = -\frac{1}{\sin^2 x}; \cos^2 x + \sin^2 = 0 - \text{решений нет (т.к. } \cos^2 x + \sin^2 = 1).$$

$$799. \text{ а) } g'(x) = 3x^2 - 6x; h'(x) = 3x;$$

$$3x^2 - 6x > 3x; x^2 - 3x > 0; x \in (-\infty; 0) \cup (3; +\infty);$$

$$\text{б) } g'(x) = 3\cos\left(3x - \frac{\pi}{6}\right); h'(x) = 6;$$

$$3\cos\left(3x - \frac{\pi}{6}\right) > 6; \cos\left(3x - \frac{\pi}{6}\right) > 2 - \text{таких значений нет;}$$

$$\text{в) } g'(x) = \frac{1}{\cos^2 x}; h'(x) = 4;$$

$$\frac{1}{\cos^2 x} > 4; \cos^2 x < \frac{1}{4}; \cos^2 x \neq 0; \cos x \in \left(-\frac{1}{2}; \frac{1}{2} \right); \cos \neq 0;$$

$$x \in \left(\frac{\pi}{3} + \pi n; \frac{\pi}{2} + \pi n \right) \cup \left(\frac{\pi}{2} + \pi n; \frac{2\pi}{3} + \pi n \right);$$

$$\text{г) } g'(x) = 2\sin\left(\frac{\pi}{4} - 2x\right); h'(x) = -\sqrt{2};$$

$$2\sin\left(\frac{\pi}{4} - 2x\right) > -\sqrt{2}; \sin\left(-\frac{\pi}{4} + 2x\right) < \frac{\sqrt{2}}{2}; 2x - \frac{\pi}{4} \in \left(-\frac{5\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n \right); x \in \left(-\frac{\pi}{2} + \pi n; \frac{\pi}{4} + \pi n \right).$$

800. а) $f'(x) = 2\cos(2x-3)$; $g'(x) = -2\sin(2x-3)$;

$$\cos(2x-3) + \sin(2x-3) = 0; \sin\left(2x-3 + \frac{\pi}{4}\right) = 0; x = \frac{3}{2} - \frac{\pi}{8} + \frac{\pi n}{2};$$

б) $g'(x) = \frac{15}{(7-5x)^2}$; $f'(x) = -\frac{30}{(5x-9)^2}$;

$$\frac{2}{(5x-9)^2} = \frac{1}{(7-5x)^2}; (5x-9)^2 + 2(5x-7)^2 = 0 - \text{решений нет};$$

в) $f'(x) = \frac{3}{2\sqrt{3x-10}}$; $g'(x) = \frac{3}{\sqrt{6x+14}}$;

$$\frac{1}{2\sqrt{3x-10}} = \frac{1}{\sqrt{6x+14}}; 2\sqrt{3x-10} = \sqrt{6x+14}; 12x-40 = 6x+14;$$

$$6x = 54; x = 9; \text{проверка: } 2\sqrt{17} = \sqrt{68};$$

г) $f'(x) = -\frac{1}{\sin^2 x}$; $g'(x) = 2$; $\sin^2 = -\frac{1}{2}$ - решений нет.

801. а) $f(x) = \sin x \cos x = \frac{1}{2} \sin 2x$; $f'(x) = \cos 2x$; $g'(x) = \frac{1}{2}$;

$$\cos 2x \leq \frac{1}{2}; 2x \in \left(\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right); x \in \left(\frac{\pi}{6} + \pi n; \frac{5\pi}{6} + \pi n\right);$$

б) $f(x) = \sin x \cos 2x + \sin 2x \cos x = \sin 3x$; $f'(x) = 3\cos 3x$; $g'(x) = -3$;

$$3\cos 3x \leq -3; \cos 3x \leq -1; 3x = \pi + 2\pi n; x = \frac{\pi}{3} + \frac{2\pi}{3};$$

в) $f(x) = \sin^2 x - \cos^2 x = -\cos 2x$; $f'(x) = 2\sin 2x$; $g'(x) = -2$;

$$2\sin 2x \leq -2; \sin 2x \leq -1; 2x = \frac{3\pi}{2} + 2\pi n; x = \frac{3\pi}{4} + \pi n;$$

г) $f'(x) = \cos x - x\sin x$; $g'(x) = \cos x$;

$$\cos x - x\sin x \leq \cos x; x\sin x \geq 0.$$

I: $\begin{cases} x \geq 0, \\ \sin x \geq 0; \end{cases} x \in [2\pi n; \pi + 2\pi n]; n = 0, 1, 2, \dots$

II: $\begin{cases} x \leq 0, \\ \sin x \leq 0; \end{cases} x \in [-\pi + 2\pi n; 2\pi n]; n = 0, -1, -2, \dots$

802. а) $h'(x) = 2x-3 = \operatorname{tg} 45^\circ$; $2x-3 = 1$; $x = 2$;

б) $h'(x) = \frac{-4}{(x+2)^2} = \operatorname{tg} 135^\circ$; $4 = (x+2)^2$; $x = 0$ или $x = 4$;

в) $h'(x) = \frac{2}{\sqrt{2x-4}} = \operatorname{tg} 60^\circ$; $2 = \sqrt{3(2x-4)}$; $4 = 6x-12$; $6x = 16$; $x = \frac{8}{3} = 2\frac{2}{3}$;

г) $h'(x) = \cos\left(4x - \frac{\pi}{3}\right) = 0$; $4x - \frac{\pi}{3} = \frac{\pi}{2} + \pi n$; $x = \frac{5\pi}{24} + \frac{\pi n}{4}$.

$$803. \text{ а) } y'_1 = 7x^6; y'_2 = 8x^7; 7a^6 = 8a^7; a = \frac{7}{8};$$

$$\text{ б) } y' = \frac{1}{2\sqrt{x}}; y' = \frac{1}{\sqrt{x+8}}; 2\sqrt{a} = a+8; a = \frac{8}{3} = 2\frac{2}{3}.$$

$$804. \text{ а) } f(x) = (2x-1)^3 + c; \quad \text{ б) } f(x) = (4-5x)^4 + c.$$

$$805. \text{ а) } f(x) = \frac{1}{2x+3} + c; \quad \text{ б) } f(x) = \sqrt{5x-7} + c.$$

$$806. \text{ а) } f(x) = -\frac{1}{3} \cos\left(3x - \frac{\pi}{3}\right) + c; \quad \text{ б) } f(x) = \frac{4}{5} \lg(5x-1) + c.$$

$$807. \text{ а) } y = ax^2 + bx + c; \text{ по рисунку видно, что } b = 0, a < 0. y = ax^2 + c; y'(x) = 2ax;$$

$$A(-100; 0); B(100; 0); y'(100) = \operatorname{tg}(-15^\circ); 200a = -\operatorname{tg} 15^\circ; a = \frac{\operatorname{tg} 15^\circ}{200} x^2 + c.$$

$$\text{Подставим точку } (100; 0); 0 = -50 \operatorname{tg} c; c = 50 \operatorname{tg} 15^\circ.$$

$$\text{Уравнение профиля моста: } y = -\frac{\operatorname{tg} 15^\circ}{200} x^2 + 50 \operatorname{tg} 15^\circ.$$

$$808. \text{ а) } y = 4x^2 - |a|x; x(4x - |a|) = 0; x_1 = 0; x_2 = \frac{|a|}{4}. \text{ Т.к. оси параболы направлены}$$

вверх и $x_2 > x_1$, то

$$1) y'(x_1) = -\operatorname{tg} 60^\circ \text{ и } y'(x_2) = \operatorname{tg} 30^\circ; 2) y'(x_1) = -\operatorname{tg} 30^\circ \text{ и } y'(x_2) = \operatorname{tg} 30^\circ.$$

$$1) y'(0) = -|a| = -\operatorname{tg} 60^\circ; a = \pm\sqrt{3}; y'\left(\frac{\pi}{4}\right) = |a| = \operatorname{tg} 60^\circ; a = \pm\sqrt{3};$$

$$2) y'(0) = -|a| = -\operatorname{tg} 30^\circ; a = \pm\frac{\sqrt{3}}{3}.$$

б) $y = x^2 + |a|x; y' = 2x + |a|; x(4x - |a|) = 0; x_1 = 0; x_2 = -|a|$. Т.к. оси параболы направлены вверх и $x_1 > x_2$, то

$$1) y'(x_2) = -\operatorname{tg} \frac{3\pi}{8}; 2) y'(x_2) = -\operatorname{tg} \frac{\pi}{8}; \operatorname{tg} \frac{3\pi}{4} = \frac{2\operatorname{tg} \frac{3\pi}{8}}{1 - \operatorname{tg}^2 \frac{3\pi}{8}}; \operatorname{tg}^2 \frac{3\pi}{8} - 2\operatorname{tg} \frac{3\pi}{8} - 1 = 0;$$

$$\operatorname{tg} \frac{3\pi}{8} = 1 \pm \sqrt{2}, \text{ но т.к. } 0 < \frac{3\pi}{8} < \frac{\pi}{2}, \text{ то } \operatorname{tg} \frac{3\pi}{8} = 1 + \sqrt{2}; \operatorname{tg} \frac{\pi}{4} = \frac{2\operatorname{tg} \frac{\pi}{8}}{1 - \operatorname{tg}^2 \frac{\pi}{8}}; \operatorname{tg}^2 \frac{\pi}{8} + 2\operatorname{tg} \frac{\pi}{8} - 1 = 0;$$

$$\operatorname{tg} \frac{\pi}{8} = -1 \pm \sqrt{2}, \text{ но т.к. } 0 < \frac{\pi}{8} < \frac{\pi}{2}, \text{ то } \operatorname{tg} \frac{\pi}{8} = -1 + \sqrt{2}.$$

$$1) y'(x_2) = -1 - \sqrt{2}; -|a| = -1 - \sqrt{2}; a = \pm(1 + \sqrt{2}); 2) y'(x_2) = 1 - \sqrt{2}; a = \pm(\sqrt{2} - 1).$$

§ 34. Уравнение касательной к графику функции

809. а) $\operatorname{tg} \alpha = 0$. б) $\operatorname{tg} \alpha < 0$. в) $\operatorname{tg} \alpha > 0$. г) $\operatorname{tg} \alpha < 0$. б) $\operatorname{tg} \alpha < 0$. в) $\operatorname{tg} \alpha > 0$.

810. а) $y' = 0$ при $x = 0, x = 3,5$; y' не существует при $x = -1$;
 б) $y' = 0$ при $x = -4, x = -1,5$; y' не существует при $x = 4$;
 в) $y' = 0$ при $x = -4$; y' не существует при $x = -2$;
 г) $y' \neq 0$ при $x \in R$.

811. а) $f(x) = 4 + x^2, a = 2. f(x) = 2x; f'(a) = 4 \Rightarrow$ острый;

б) $f(x) = 1 - \frac{1}{x}, a = 3; f'(x) = \frac{1}{x^2}; f'(a) = \frac{1}{9} \Rightarrow$ острый;

в) $f(x) = (1 - x)^3, a = -3. f'(x) = -3(1 - x)^2; f'(a) = -48 \Rightarrow$ тупой;

г) $f(x) = 2x - x^2, a = 1. f'(x) = 2 - 2x; f'(a) = 0 \Rightarrow$ острый.

812. $y = 1 - x^2; y'(x) = -2x$.

а) $A(0; 1); y'(0) = 0 \Rightarrow \operatorname{tg} \alpha = 0$.

б) $B(2; -3); y'(2) = -4 \Rightarrow \operatorname{tg} \alpha = -4$.

в) $C\left(\frac{1}{2}; \frac{3}{4}\right); y'\left(\frac{1}{2}\right) = -1 \Rightarrow \operatorname{tg} \alpha = -1$; г) $D(-1; 0); y'(-1) = 2 \Rightarrow \operatorname{tg} \alpha = 2$.

813. а) $f(x) = \frac{1}{2}x^2, a = 1. f'(x) = x; f'(a) = 1, \operatorname{tg} \alpha = 1$.

б) $f(x) = -2x^3, a = 2. f'(x) = -6x^2; f'(a) = -24, \operatorname{tg} \alpha = -24$.

в) $f(x) = 0,25x^4, a = -1. f'(x) = x^3; f'(a) = -1, \operatorname{tg} \alpha = -1$.

г) $f(x) = -x^5, a = 1. f'(x) = -5x^4; f'(a) = -5. \operatorname{tg} \alpha = -5$.

814. а) $f(x) = 3x^2 - 4x; f'(a) = 7; \operatorname{tg} \alpha = 7$;

б) $f'(x) = \frac{x+3-x+1}{(x+3)^2} = \frac{4}{(x+3)^2}; f'(a) = \frac{1}{4}; \operatorname{tg} \alpha = \frac{1}{4}$;

в) $f(x) = 4x^3 - 21x^3 + 12; f'(a) = 12; \operatorname{tg} \alpha = 12$;

г) $f'(x) = \frac{2x+2-2x+1}{(x+1)^2} = \frac{3}{(x+1)^2}; f'(a) = \frac{3}{4}; \operatorname{tg} \alpha = \frac{3}{4}$.

815. а) $f'(x) = \frac{1}{2\sqrt{x-7}}; f'(a) = \frac{1}{2}; \operatorname{tg} \alpha = \frac{1}{2}$;

б) $f'(x) = -\frac{5}{2\sqrt{4-5x}}; f'(a) = -\frac{5}{4}; \operatorname{tg} \alpha = -\frac{5}{4}$;

в) $f'(x) = \frac{1}{\sqrt{x+10}}; f'(a) = \frac{1}{\sqrt{5}}; \operatorname{tg} \alpha = \frac{\sqrt{5}}{5}$;

г) $f'(x) = -\frac{1}{4\sqrt{3,5-0,5x}}; f'(a) = -\frac{1}{8}; \operatorname{tg} \alpha = -\frac{1}{8}$.

816. а) $f(x) = \cos x; f'(a) = 1; \operatorname{tg} \alpha = 1$; б) $f'(x) = \frac{2}{\cos^2 2x}; f'(a) = 4; \operatorname{tg} \alpha = 4$;

в) $f(x) = -3\sin 3x; f'(a) = 3; \operatorname{tg} \alpha = 3$; г) $f'(x) = -\frac{1}{\sin^2 x}; f'(a) = -\frac{4}{3}; \operatorname{tg} \alpha = -\frac{4}{3}$.

$$817. \text{ а) } f(x) = 2x; f'(a) = 1; \alpha = \frac{\pi}{4}; \quad \text{б) } f(x) = -9x^2; f'(a) = -1; \alpha = \frac{3\pi}{4};$$

$$\text{в) } f(x) = x^4; f'(a) = 1; \alpha = \frac{\pi}{4}; \quad \text{г) } f(x) = -x^3; f'(a) = 0; \alpha = 0.$$

$$818. \text{ а) } f(x) = 3x^2 - 6x + 2; f'(a) = -1; \alpha = \frac{3\pi}{4}; \quad \text{б) } f(x) = -21x^2 + 20x + 1; f'(a) = 1; \alpha = \frac{\pi}{4}.$$

$$819. \text{ а) } f'(x) = \frac{6-4x+4x-2}{(3-2x)^2} = \frac{4}{(3-2x)^2}; f'(a) = \frac{1}{4}; \alpha = \arctg \frac{1}{4};$$

$$\text{б) } f'(x) = \frac{x-2-x+1}{(x-2)^2} = -\frac{1}{(x-2)^2}; f'(a) = -1; \alpha = \frac{3\pi}{4}.$$

$$820. \text{ а) } f'(x) = \frac{3}{\sqrt{6x+7}}; f' = \frac{1}{\sqrt{3}}; \alpha = \frac{\pi}{4}; \quad \text{б) } f'(x) = -\frac{1}{\sqrt{5-2x}}; f'(a) = -1; \alpha = \frac{3\pi}{4}.$$

$$821. \text{ а) } f'(x) = -\frac{\sqrt{3}}{3} \sin \frac{x}{3}; f'(a) = -\frac{\sqrt{3}}{3}; \alpha = \frac{5\pi}{6}; \quad \text{б) } f(x) = \cos 2x; f'(a) = -1; \alpha = \frac{3\pi}{4}.$$

$$822. \text{ а) } f'(x) = -\frac{2}{3\cos^2 x} + \frac{1}{3} \cos \frac{x}{3}; f'(a) = -\frac{2}{3} - \frac{1}{3} = -1; \alpha = \frac{3\pi}{4};$$

$$\text{б) } f'(x) = -\sin x - \frac{\sqrt{3}}{8\sin^2 \frac{x}{2}}; f'(a) = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = -\sqrt{3}; \alpha = \frac{2\pi}{3}.$$

$$823. \text{ а) } f(x) = 2x; f'(a) = 6; f(a) = 9; y = 9 + 6(x-3) = 6x - 9;$$

$$\text{б) } f(x) = -1 - 3x^2; f'(a) = -1; f(a) = 2; y = 2 - x;$$

$$\text{в) } f(x) = 3x^2; f'(a) = 3; f(a) = 1; y = 1 + 3(x-1) = 3x - 2;$$

$$\text{г) } f(x) = 2x - 3; f'(a) = -5; f(a) = 1 + 3 + 5 = 9; y = 9 - 5(x+1) = -5x + 4.$$

$$824. \text{ а) } f'(x) = \frac{9-3x+3x-2}{(3-x)^2} = \frac{7}{(x-3)^2}; f'(a) = 7; f(a) = 4; y = 4 + 7(x-2) = 7x - 10;$$

$$\text{б) } f'(x) = \frac{10-2x+2x-5}{(5-x)^2} = \frac{5}{(5-x)^2}; f'(a) = 5; f(a) = 3; y = 3 + 5(x-4) = 5x - 17.$$

$$825. \text{ а) } f'(x) = -\frac{3}{(x+2)^4}; f'(a) = -3; f(a) = -1; y = -1 - 3(x+3) = -3x - 10;$$

$$\text{б) } f'(x) = \frac{-4}{4(2x-1)^3} = -\frac{1}{(2x-1)^3}; f'(a) = -1; f(a) = \frac{1}{4}; y = \frac{1}{4} - x + 1 = -x + \frac{5}{4}.$$

$$826. \text{ а) } f'(x) = -\frac{3}{\sqrt{3x-5}}; f'(a) = 3; f(a) = 2; y = 2 + 3(x-2) = 3x - 4;$$

$$\text{б) } f'(x) = -\frac{1}{\sqrt{7-2x}}; f'(a) = -1; f(a) = 1; y = 1 - x + 3 = -x + 4.$$

$$827. \text{ а) } f'(x) = -\frac{1}{3} \sin \frac{x}{3}; f'(a) = 0; f(a) = 1; y = 1;$$

$$\text{б) } f'(x) = -\frac{2}{\sin^2 2x}; f'(a) = -2; f(a) = 0; y = -2\left(x - \frac{\pi}{4}\right) = \frac{\pi}{2} - 2x;$$

$$\text{в) } f(x) = 2\cos 2x; f'(a) = 0; f(a) = 1; y = 1;$$

$$\text{г) } f'(x) = \frac{2}{3\cos^2 \frac{x}{3}}; f'(a) = \frac{3}{2}; f(a) = 0; y = \frac{2}{3}x.$$

$$828. y = 9 - x^2; 9 - x^2 = 0; x = \pm 3; y' = -2x; y'(3) = -6; y'(-3) = 6; \\ y = 6(x + 3) = 6x + 18; y = -6(x - 3) = 18 - 6x.$$

$$829. y = x^2 - 3x; x^2 - 3x - 4 = 0; x = 4; x = -1; y' = 2x - 3; y'(4) = 8 - 3 = 5; y'(-1) = -5; \\ y = 4 + 5(x - 4) = 5x - 16; y = 4 - 5(x + 4) = -5x - 1.$$

$$830. y = 3x^3 - 4x^2 + 1; y' = 9x^2 - 8x = 1; 9x^2 - 8x - 1 = 0; x = \frac{4 \pm 5}{9}; x = 1; x = \frac{1}{9}$$

$$y(1) = 0; y\left(-\frac{1}{9}\right) = -\frac{1}{243} - \frac{4}{81} + 1 = \frac{230}{243}; y = x - 1; y = \frac{230}{243} + \left(x + \frac{1}{9}\right) = x + \frac{257}{243}.$$

$$831. y = x^2; y' = 2x; y = x_0^2 + 2x_0(x - x_0) = 2x_0x - x_0^2.$$

$$\text{а) } y = 2x + 1; x_0 = 1; y_0 = 1; \quad \text{б) } y = -\frac{1}{2}x + 5; x_0 = -\frac{1}{4}; y_0 = \frac{1}{16};$$

$$\text{в) } \frac{3}{4}x - 2 = y; x_0 = \frac{3}{8}; y_0 = \frac{9}{64}; \quad \text{г) } y = -x + 5; x_0 = -\frac{1}{2}; y_0 = \frac{1}{4}.$$

$$832. \text{ а) } f'(x) = x^2 - 6x + 10; y = \frac{x_0^3}{3} - 3x_0^2 + 10x_0 - 4 + (x_0^2 - 6x_0 + 10)(x - x_0);$$

$$x_0^2 - 6x_0 + 10 = 1; x_0^2 - 6x_0 + 9 = 0; x_0 = 3;$$

$$\text{б) } f'(x) = x^3 - 2x; y = \frac{x_0^4}{4} - x_0^2 + 8 + (x_0^2 - 2x_0)(x - x_0); x_0^3 - 2x_0^2 = 0; x_0 = 0; x_0^2 = 2; x_0 = \pm\sqrt{2};$$

$$\text{в) } f'(x) = x^2 - 2x + 2; y = \frac{x_0^3}{3} - x_0^2 + 2x_0 - 7 + (x_0^2 - 2x_0 + 2)(x - x_0); x_0^2 - 2x_0 + 2 = 1; x_0 = 1;$$

$$\text{г) } f'(x) = 5x^3 - 3x^2; x_0 = 0; 5x_0^3 - 3 = 0; x_0 = \frac{3}{5}.$$

$$833. \text{ а) } f'(x) = \cos x; \cos x_0 = -1; x_0 = \pi + 2\pi n; \text{ б) } f'(x) = -3\sin 3x; \sin 3x_0 = 0; x_0 = \frac{\pi n}{3};$$

$$\text{в) } f'(x) = \frac{1}{\cos^2 x}; \frac{1}{\cos^2 x} = 1; \cos x = \pm 1; x = \pi n;$$

$$\text{г) } f'(x) = \frac{1}{2}\cos \frac{x}{2}; \cos \frac{x}{2} = 0; x_0 = \pi + 2\pi n.$$

$$834. y = \frac{x^3}{3} - 2; y' = x^2; y = \frac{x_0^3}{3} - 2 + x_0^2(x - x_0) = x_0x - \frac{2}{3}x_0^3 - 2;$$

$$\text{а) } y = x - 3; x_0^2 = 1; x_0 = \pm 1; y = x - \frac{2}{3} - 2 = x - \frac{8}{3}; x + \frac{2}{3} - 2 = x - 1\frac{1}{3};$$

$$6) y = 9x - 5; x_0^2 = \pm 3; y = 9x - 18 - 2 = 9x - 20; y = 9x + 18 - 2 = 9x + 16.$$

$$835. a) y' = x^2 + 5x; x_0^2 + 5x_0 = -1; x_0^2 + 5x_0 + 1 = 0; x_0 = \frac{-5 \pm \sqrt{21}}{2};$$

$$y = \left(\frac{-5 \pm \sqrt{21}}{2} \right)^3 \cdot \frac{1}{3} + \frac{5}{2} \left(\frac{-5 \pm \sqrt{21}}{2} \right)^2 + 8 + \left(\left(\frac{-5 \pm \sqrt{21}}{2} \right)^2 + 5 \left(\frac{-5 \pm \sqrt{21}}{2} \right) \right) \left(x - \frac{\pm \sqrt{21} - 5}{2} \right);$$

$$6) y' = x^2 + 2x - 1; x_0^2 + 2x_0 = 0; x_0 = 0, x_0 = -2; y(0) = 0, y(-2) = \frac{10}{3};$$

$$y = x'; y = -\frac{8}{3} + 4 + 2 - (x + 2) = -x + \frac{4}{3}; y = 0 - (x - 0) = -x.$$

$$836. a) y' = \frac{3x - 9 - 3x - 1}{(x - 3)^2} = -\frac{10}{(x - 3)^2}; -\frac{10}{(x_0 - 3)^2} = -1; (x_0 - 3)^2 = 10; x_0 = \pm \sqrt{10} + 3;$$

$$y(x_0) = \frac{3(\pm \sqrt{10} + 3) + 1}{\pm \sqrt{10} + 3 - 3} = \frac{\pm 3\sqrt{10} + 10}{\pm 10} = 3 \pm \sqrt{10}; y = 3 \pm \sqrt{10} - (x \mp \sqrt{10} - 3);$$

$$y_1 = 3 + \sqrt{10} - x + \sqrt{10} + 3 = 6 + 2\sqrt{10} - x; y_2 = 3 - \sqrt{10} - x - \sqrt{10} + 3 = 6 - 2\sqrt{10} - x;$$

$$6) y' = -\frac{1}{(x + 8)^2}; -\frac{1}{(x + 8)^2} = -1; x_0 + 8 = \pm 1; x_0 = -9, x_0 = -7; y(-9) = 0, y(-7) = 2;$$

$$y = 2 - (x + 7) = -x - 5, y = -x - 9.$$

$$837. a) y' = -\frac{2}{\sqrt{x + 7}}; -\frac{2}{\sqrt{x + 7}} = -1; x_0 = -3; y(-3) = -8; y = -8 - (x + 3) = -x - 11;$$

$$6) y' = -\frac{1}{\sqrt{1 - 2x}}; -\frac{1}{\sqrt{1 - 2x}} = -1; x_0 = 0; y(0) = 1; y = 1 - x.$$

$$838. a) 0,998^5: y = x^5 = f(x); x = 0,998; a = 1; f(a) = 1; f'(x) = 5x^4; f'(a) = f'(1) = 5;$$

$$0,998^5 \approx 1 - 5 \cdot 0,002 = 0,99;$$

$$6) 1,03^7 \approx 1 + 7 \cdot 0,03 = 1,21.$$

$$839. a) \sqrt{1,05} \approx 1 + \frac{1}{2} \cdot 0,05 = 1,025;$$

$$6) \sqrt{3,99} \approx 2 - \frac{1}{2\sqrt{4}} \cdot 0,01 = 1,9975.$$

$$840. a) f'(x) = \sqrt{3}x^2 - 3\sqrt{3} = \sqrt{3}; x^2 = 4; x = \pm 2; f(-2) = \frac{8}{\sqrt{3}} - 6\sqrt{3} = -\frac{10}{\sqrt{3}}; f(-2) = \frac{10}{\sqrt{3}};$$

$$y = \pm \frac{10}{\sqrt{3}} + \sqrt{3}(x \pm 2); y_1 = \sqrt{3}x - 2\sqrt{3} + \frac{10}{\sqrt{3}} = \sqrt{3}x + \frac{4}{\sqrt{3}}; y_2 = \sqrt{3}x + 2\sqrt{3} - \frac{10}{\sqrt{3}} = \sqrt{3}x - \frac{4}{\sqrt{3}}.$$

$$6) f'(x) = \frac{4}{\sqrt{3}} - \sqrt{3}x^2 = \frac{\sqrt{3}}{3}; x^2 = \frac{4}{3} - \frac{1}{3}; x = \pm 1; f(1) = \sqrt{3}; f(-1) = -\sqrt{3};$$

$$y = \sqrt{3} + \frac{\sqrt{3}}{3}(x - 1) = \frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}; y = -\sqrt{3} + \frac{\sqrt{3}}{3}(x + 1) = \frac{\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3}.$$

$$841. a) y' = \frac{3x + 24 - 3x + 1}{(x + 8)^2} = \frac{25}{(x + 8)^2}; 25 = (x + 8)^2; x + 8 = \pm 5. x = -3. x = -13;$$

$$y(-3) = \frac{-9-1}{5} = -2; y(-13) = \frac{-39-1}{-5} = 8; y_1 = -2 + (x+3) = x+1.$$

$$y_2 = 8 + (x+13) = x+21. (-1; 0); (-21; 0).$$

$$\text{б) } x-5 = \pm 3; x=8; x=2; y(8) = \frac{12}{3} = 4; y(2) = \frac{6}{-3} = -2.$$

$$y_1 = 4 - (x-8) = -x+12. y_2 = -2 - (x-2) = -x; (0; 0); (12; 0).$$

842. Условие приведено не полностью.

$$843. y = x_0^2 + 1 + 2x_0(x-x_0) = 2x_0x - x_0^2 + 1.$$

$$\text{а) } A(-1; -2); -2 = -2x_0 - x_0^2 + 1; x_0^2 + 2x_0 - 3 = 0; x_0 = -3, x_0 = 1; y_1 = -6x - 8; y_2 = 2x.$$

$$\text{б) } A(0; 0); 0 = -x_0^2 + 1; x_0 = \pm 1; y = 2x; y = -2x.$$

$$\text{в) } A(0; -3); -3 = -x_0^2 + 1; x_0 = \pm 2; y = 4x - 3; y = -4x - 3.$$

$$\text{г) } A(-1; 1); 1 = 2x_0 - x_0^2 + 1; x_0 = 0; x_0 = -2; y = 1; y = -4x - 3.$$

$$844. \text{ а) } f'(x) = -2x - 7; y = -x_0^2 - 7x_0 + 8 + (-2x_0 - 7)(x - x_0) = (-2x_0 - 7)x + x_0^2 + 8;$$

$$x_0 = 0, x_0 = 2; f(0) = 8, f(2) = -10; y = -10 - 11(x-2) = -11x + 12; y = 8 - 7x;$$

$$\text{б) } 9 = x_0^2 + 8; x = \pm 1; f(1) = -1 - 7 + 8 = 0; f(-1) = -1 + 7 + 8 = 14;$$

$$y = 9(x-1) = -9x + 9; y = 14 - 5(x+1) = -5x + 9.$$

$$845. \text{ а) } f'(x) = \frac{1}{2\sqrt{3-x}}; 3 = \sqrt{3-x_0} - \frac{1}{2\sqrt{3-x_0}}(-2-x_0); 3 = \frac{6-2x_0+2+x_0}{2\sqrt{3-x_0}}; 6\sqrt{3-x_0} = 8-x_0;$$

$$108 - 36x_0 = 64 - 16x_0 + x_0^2; x_0^2 + 20x_0^2 - 44 = 0; x_0 = -22; x_0 = 2; f(-22) = 5; f(2) = 1;$$

$$y = 5 - \frac{1}{10}(x+22) = -\frac{x}{10} - \frac{11}{5} + 5 = -\frac{x}{10} + \frac{14}{5}; y = 1 - \frac{1}{2}(x-2) = -\frac{1}{2}x + 2;$$

$$\text{б) } f'(x) = -\frac{1}{2\sqrt{3-x}}; 0 = \sqrt{3-x_0} - \frac{1}{2\sqrt{3-x_0}}(4-x_0); 6-2x_0-4+x_0 = 0; x_0 = 2;$$

$$f(2) = 1; y = 1 - \frac{1}{2}(x-2) = -\frac{1}{2}x + 2.$$

$$846. \text{ а) } f'(x) = \frac{1}{2\sqrt{4x-3}}; y = \frac{2}{\sqrt{4x_0-3}}(x-x_0) + \sqrt{4x_0-3}; 3\sqrt{4x_0-3} = 2(2-x_0) + 4x_0-3;$$

$$36x_0 - 27 = 4x_0^2 + 4x_0 + 1; 4x_0^2 - 32x_0 + 28 = 0; x_0^2 - 8x_0 + 7 = 0;$$

$$x_0 = 7; x_0 = 1; y = \frac{2}{5}(x-7) + 5 = \frac{2}{5}x + \frac{11}{5}; y = 2x - 2 + 1 = 2x - 1.$$

$$\text{б) } f'(x) = \frac{1}{\sqrt{2x+1}}; y = \frac{x-x_0}{\sqrt{2x_0+1}} + \sqrt{2x_0+1}; 2\sqrt{2x_0+1} = 1-x_0+2x_0+2x_0+1;$$

$$8x_0 + 4 = x_0^2 + 4x_0 + 4; x_0^2 - 4x_0 = 0; x_0 = 0, x_0 = 4; y = x+1; y = \frac{x-4}{3} + 3 = \frac{x}{3} + \frac{5}{3}.$$

$$847. \text{ а) } y' = -7\sin 7x - 7\sin x; y\left(\frac{\pi}{6}\right) = -\frac{7\sqrt{3}}{2} + \frac{7\sqrt{3}}{2} = 0; y'\left(\frac{\pi}{6}\right) = \frac{7}{2} - \frac{7}{2} = 0;$$

$$y = (-7\sin 7a - 7\sin a)(x-a) + \cos 7a + 7\cos a; -7\sin 7a - 7\sin a = 0; \sin 4a \cos 3a = 0;$$

$$\sin 4a = 0; a = \frac{n}{4}; \cos 3a = 0; 3a = \frac{\pi}{2} + \pi n; a = \frac{\pi}{6} + \frac{\pi n}{3};$$

$$\text{б) } y_1 = 2 - 14\sin 3x; y_2 = 6\sin 7x; y_1' = -42\cos 3x; y_2' = 42\cos 7x;$$

$$y = 2 - 14\sin 3a - 42\cos 3a(x - a), y = 6\sin 7a + 42\cos 7a(x - a); 42\cos 7a = -42\cos 3a;$$

$$\cos 7a + \cos 3a = 0; \cos 5a \cos 2a = 0; 5a = \frac{\pi}{2} + \pi n; a = \frac{\pi}{10} + \frac{\pi n}{5}; a = \frac{\pi}{4} + \frac{\pi n}{2}.$$

$$848. \text{ а) } y = \frac{1}{x^2}; \left| \frac{1}{2} xy \right| = 2,25; x > 0; y' = -\frac{2}{x^3}; y = \frac{2}{x_0^3}(x_0 - x) + \frac{1}{x_0^2} = \frac{3}{x_0^2} - \frac{2x}{x_0^3};$$

$$\begin{cases} y = \frac{3}{x_0^2} - \frac{2x}{x_0^3} & \text{при } x = 0 \quad y = \frac{3}{x_0^2}; \frac{3}{x_0^2} \cdot \frac{3x_0}{2} = \frac{9}{2}; \frac{1}{x_0} = 1; x_0 = 1; y = 3 - 2x; \\ xy = \frac{9}{2} & \text{при } y = 0 \quad x = \frac{3x_0}{2} \end{cases}$$

$$\text{б) } y = \frac{1}{x^2}, x < 0, -\frac{1}{2}xy = \frac{9}{8}; y = \frac{3}{x_0^2} - \frac{2x}{x_0^3}; \text{при } x = 0 \quad y = \frac{3}{x_0^2}, \text{при } y = 0 \quad x = \frac{3x_0}{2};$$

$$\frac{3 \cdot 3}{2} \cdot \frac{1}{x_0} = -\frac{9}{4} \cdot \frac{1}{x_0} = -\frac{1}{2}; x_0 = -2; y = \frac{3}{4} + \frac{2x}{8} = \frac{3+x}{4}.$$

$$849. \text{ а) } y = x^3, x > 0, \left| \frac{1}{2} xy \right| = \frac{2}{3}; \text{т.к. это кубическая парабола, то } xy = -\frac{4}{3}.$$

$$y = x_0^2 + 3x_0^2(x - x_0) = -2x_0^3 + 3x_0^2; \text{при } x = 0, y = -2x_0^3, \text{при } y = 0, x = \frac{2x_0^3}{3x_0^2} = \frac{2}{3}x_0;$$

$$xy = -\frac{4}{3}x_0^4 = -\frac{4}{3}; x_0^4 = 1, x_0 = \pm 1, \text{но } x_0 > 0, \text{значит } x_0 = 1; y = 1 + 3(x - 1) = 3x - 2.$$

$$\text{б) } y = x^3, x < 0 \Rightarrow xy = -\frac{27}{4}; y = -x_0^3 + 3x_0^2; xy = -\frac{4}{3}x_0^4 = -\frac{27}{4};$$

$$x_0^4 = \left(\frac{3}{2}\right)^4, x_0 = \pm \frac{3}{2}, \text{но } x_0 < 0 \Rightarrow -\frac{3}{2}, y = \frac{4}{3} \cdot \frac{81}{16} + 3x \cdot \frac{9}{4} = \frac{27}{4} + \frac{27}{4}x.$$

$$850. \text{ а) } y = 3 - \frac{1}{2}x^2; B(0; z); y' = -x; y = 3 - \frac{1}{2}x_0^2 - x_0(x - x_0) = -x_0x + \frac{1}{2}x_0^2 + 3; z = \frac{1}{2}x_0^2 + 3.$$

Т.к. график симметричен относительно Oy , тангенс угла наклона касательной ± 1 .

$$y' = -x_0; x_0 = \pm 1; z = \frac{1}{2}x_0^2 + 3 = \frac{1}{2} + 3 = 3,5; B(0; 3,5);$$

$$\text{б) } y = 0,5x^2 - 2,5, \alpha = 90^\circ; y' = x; y = \frac{1}{2}x_0^2 - 2,5 + x_0(x - x_0) = x_0x - \frac{1}{2}x_0^2 - \frac{5}{2}.$$

Рассуждения аналогичны пункту а. $y' = x_0 = \pm 1; y = -x - 3; y = x - 3$.

$$851. \text{ а) } y' = \sqrt{3}x; y = \frac{\sqrt{3}}{2}(x_0^2 + 1) + \sqrt{3}x_0(x - x_0). \text{ Рассуждения аналогичны № 850.}$$

$$1. y' = \sqrt{3}x_0 = \pm \operatorname{tg} 60^\circ; \sqrt{3}x_0 = \pm \sqrt{3}; x_0 = \pm 1; y = \frac{\sqrt{3}}{2} \cdot 2 - \sqrt{3} = 0; B(0; 0);$$

$$2. y' = \sqrt{3}x_0 = \pm \operatorname{tg} 30^\circ; \sqrt{3}x_0 = \pm \frac{\sqrt{3}}{3}; x_0 = \pm \frac{1}{3}; y = \frac{\sqrt{3}}{2} \left(1 - \frac{1}{9}\right) = \frac{4\sqrt{3}}{9}; B\left(0; \frac{4\sqrt{3}}{9}\right);$$

$$6) y' = -\frac{\sqrt{3}}{3}x; y = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{3}x_0^2 - \frac{\sqrt{3}x_0^2}{6} + \frac{\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}x_0x + \frac{\sqrt{3}}{6}x_0^2 + \frac{\sqrt{3}}{6};$$

$$1. y' = -\frac{\sqrt{3}}{3}x_0 = \pm \operatorname{tg} 60^\circ; \frac{\sqrt{3}}{3}x_0 = \pm \sqrt{3}; x_0 = \pm 3;$$

$$y = -\sqrt{3}x + \frac{3\sqrt{3}}{3} + \frac{\sqrt{3}}{6} = -\sqrt{3}x + \frac{5\sqrt{3}}{3}; y = \sqrt{3}x + \frac{5\sqrt{3}}{3};$$

$$2. y' = -\frac{\sqrt{3}}{3}x_0 = \pm \operatorname{tg} 30^\circ; x_0 = \pm 1; y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}; y = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}.$$

$$852. \text{ а) I: } y = x^2 - 2x + 6, x = 5.$$

$$y' = 2x - 2; y = 25 - 10 + 6 + (10 - 2)(x - 5) = 8x - 10.$$

$$\text{II: } y = x^2 + 2x - 6, x = -5.$$

$$y' = 2x + 2; y = 25 - 10 - 6 + (-10 + 2)(x + 5) = -8x - 31; 8x - 10 = -8x - 31;$$

$$x = -\frac{3}{4} \cdot \left(-\frac{3}{4}; -25\right).$$

$$6) \text{ I: } y = x^3 - (x - 1), x = 2; y' = 3x^2 + 1.$$

$$\text{II: } y = x^3 - x + 1, x = -2; y' = 3x^2 - 1.$$

$$y = -5 + 11(x + 2) = 11x + 17. 13x - 17 = 11x + 17; x = 17. (17, 204).$$

$$853. \text{ а) } y' = 3x^2 - p. y(1) = 1 - p; y'(1) = 3 - p; y = 1 - p + (3 - p)(x - 1) =$$

$$= (3 - p)x + p - 3 - p + 1 = (3 - p)x - 2; 3 = 2(3 - p) - 2; 5 = -2p + 6; p = \frac{1}{2}.$$

$$6) y' = 3x^2 + 2px; y(1) = 1 + p. y'(1) = 3 + 2p; y = 1 + p + (3 + 2p)(x - 1); 2 = 1 + p + (3 + 2p)(3 - 1), 5p = -5; p = -1.$$

§ 35. Применение производной для исследования функций на монотонность и экстремумы

$$854. \text{ а) } f'(a) > 0; f'(b) > 0; f'(c) < 0; f'(d) > 0;$$

$$\text{б) } f'(a) > 0; f'(b) = 0; f'(c) = 0; f'(d) > 0.$$

$$855. \text{ а) функция возрастает: } \left(-\infty; \frac{c+b}{2}\right) \cup \left[d - \frac{c-b}{2}; +\infty\right), \text{ убывает:}$$

$$\left[\frac{b+c}{2}; d - \frac{c-d}{2}\right];$$

$$\text{б) возрастает: } (-\infty; b] \cup [c; +\infty); \text{ убывает: } [b; c].$$

$$856. \text{ а) убывает: } [-2; 2]; \text{ возрастает: } (+\infty; -2] \text{ и } [2; +\infty);$$

$$\text{б) убывает: } [-4; 0] \cup [3; +\infty); \text{ возрастает: } (-\infty; -4] \cup [0; 3];$$

$$\text{в) убывает: } [-4, 5; +\infty); \text{ возрастает: } (-\infty; -4, 5];$$

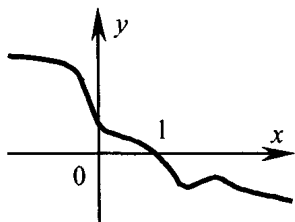
$$\text{г) убывает: } (-\infty; -2, 5] \cup [2, 5; +\infty); \text{ возрастает: } [-2, 5; 2, 5].$$

$$857. \text{ Функция убывает на промежутке } (4, +\infty) \text{ (в).}$$

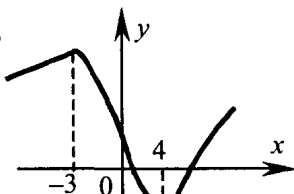
$$858. \text{ Для функции } g(x).$$

$$859. \text{ а) } f(x); \quad \text{б) } h(x).$$

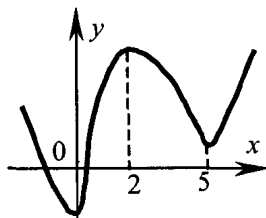
860.



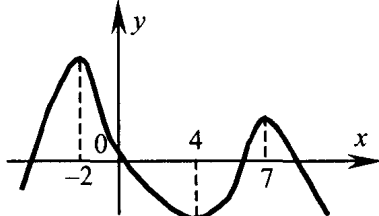
861. а)



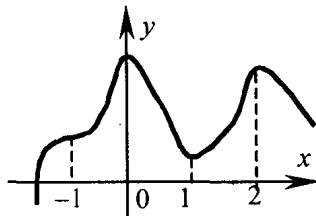
б)



в)



г)

862. а) $y' = -\sin x + 2$; $y' > 0$ при любых x .б) $y' = 5x^4 + 3x^2 + 7$; $y' > 0$ при любых x .в) $y' = \cos x + 3x^2 + 1$; $y' > 0$ при любых x .г) $y' = 5x^4 + 12x^2 + 8$; $y' > 0$ при любых x .863. а) $y = 2\cos 2x - 3$. $y < 0$ при любых x .б) $y' = -3\cos 3x + 4$; $y > 0$ при $\forall x \Rightarrow$ утверждение неверно.864. а) $y' = 5x^4 + 18x^2 \geq 0$, функция не убывает.б) $y' = \cos x - 2 < 0$, монотонно убывает.в) $y' = 1 + \sin x \geq 0$, функция не убывает.г) $y' = -5 - 3x^2 \leq 0$, функция не возрастает.865. а) $y' = 2x - 5$; при $x \geq \frac{5}{2}$ ф-ция возрастает, при $x \leq \frac{5}{2}$ ф-ция убывает.б) $y' = 10x + 15$; при $x \geq -\frac{3}{2}$ ф-ция возрастает; при $x \leq -\frac{3}{2}$ ф-ция убывает.в) $y' = -2x + 8$; при $x \geq 4$ ф-ция возрастает, при $x \leq 4$ ф-ция убывает.г) $y' = 2x - 1$; при $x \geq \frac{1}{2}$ ф-ция возрастает, при $x \leq \frac{1}{2}$ ф-ция убывает.866. а) $y' = 3x^2 + 2$ возрастает при любых x ;б) $y' = 45 - 6x - 3x^2$; $-3(x^2 + 2x - 15) = 0$; $x \in [-5; 3]$ возрастает, $x \in (-\infty; -5]$ и $[3; +\infty)$ убывает;в) $y' = -6x^2 - 6x - 36 = 6(x^2 - x - 6) = 0$; $x \in [-2; 3]$ убывает, $x \in (-\infty; -2]$ и $[3; +\infty)$ возрастает;г) $y' = 5x^4 + 5 = -5(x^4 - 1)$; $x \in [-1; 1]$ возрастает, $x \in (-\infty; -1]$ и $[1; +\infty)$ убывает.

867. а) $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$; $x \in (-\infty; -1] \cup [0; 1]$ убывает, $x \in [-1; 0] \cup [1; +\infty)$ возрастает.
 б) $y' = -5x^4 - 1 = -(5x^4 + 1) < 0$; убывает при всех x .
 в) $y' = -12x^3 + 12x^2 = -12x^2(x - 1)$; $x \in (-\infty; 1]$ возрастает, $x > 1$ убывает.
 г) $y' = 25x^4$ возрастает при всех x .

868. а) $y' = -\frac{1}{(x+3)^2}$ — убывает на всей ОДЗ ($x \neq -3$);

б) $y' = \frac{9x+3-9x+3}{(3x+1)^2} = \frac{6}{(x+3)^2}$ — возрастает на всей ОДЗ ($x \neq -\frac{1}{3}$);

в) $y' = -\frac{2}{x^2}$ — убывает на всей ОДЗ ($x \neq 0$);

г) $y' = \frac{-6-4x-2+4x}{(3+2x)^2} = -\frac{8}{(3+2x)^2}$ — убывает на всей ОДЗ ($x \neq -\frac{3}{2}$).

869. а) $y' = \frac{3}{2\sqrt{3x-1}}$ — возрастает на всей ОДЗ ($x \geq \frac{1}{3}$);

б) $y' = -\frac{1}{2\sqrt{1-x}} + 2 = 0$; $\frac{4\sqrt{1-x}-1}{2\sqrt{1-x}} = 0$; $x = \frac{15}{16}$; возрастает при $x \in \left(-\infty; \frac{15}{16}\right]$,

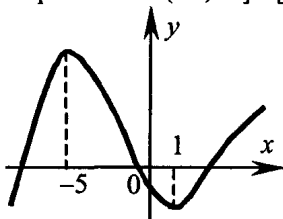
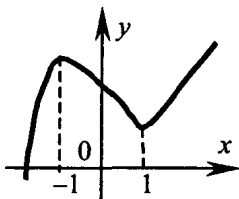
убывает при $x \in \left[\frac{15}{16}; 1\right)$;

в) $y' = -\frac{1}{\sqrt{1-2x}}$; $x \in \left(-\infty; \frac{1}{2}\right)$ — убывает;

г) $y' = \frac{1}{\sqrt{2x-1}} - 1 = 0$; $\frac{1-2\sqrt{2x-1}}{\sqrt{2x-1}} = 0$; возрастает при $x \in \left[\frac{1}{2}; 1\right]$, убывает при $x \in [1; +\infty)$.

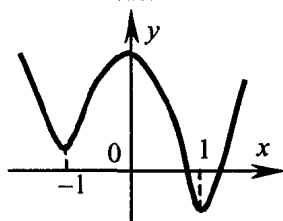
870. а) $y' = 3x^2 - 3$; убывает $x \in [-1; 1]$; возрастает $x \in (-\infty; -1] \cup [1; +\infty)$;

б) $y' = 3x^2 + 12x - 15$; убывает $x \in [-5; 1]$; возрастает $x \in (-\infty; -5] \cup [1; +\infty)$.

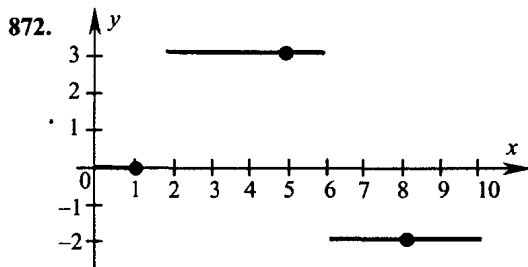
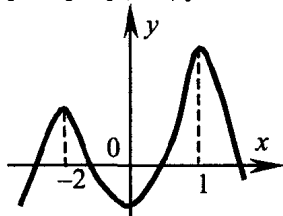


871. а) $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$;

$(-\infty; -1] \cup [0; 1]$ убывает; $[-1; 0]$ и $[1; +\infty)$ возрастает;



6) $y' = -4x^3 + 16x = -4x(x^2 - 4) = -4x(x - 2)(x + 2)$;
 $[-2; 0] \cup [2; +\infty)$ убывает; $[-\infty; -2] \cup [0; 2]$ возрастает.



873. а) $f(b) = f(d) = 0$; б) $f'(c) = 0$;
 в) $f(a) = f(0) = 0$; г) нет точек, в которых производная равна нулю.

874. а) $f(e)$; б) $f(a), f(b)$; в) $f(b), f(c)$; г) $f(a), f(b), f(c), f(d), f(e)$.

875. а) 1; б) 2; в) 2; г) 2.

876. а) 2; б) 1; в) 2; г) 2.

877. а) $(-\infty; -5] \cup [-2; +\infty)$; б) $[-5; -2]$; в) -5; г) -2.

878. а) да; б) да; в) да; г) да.

879. а) да; б) нет; в) нет; г) да.

880. а) $y' = 12 - 3x^2$; $3x^2 = 12$; $x = \pm 2$. б) $y' = 9x^2 + 4x$; $x(9x + 4) = 0$; $x = 0$, $x = -\frac{4}{9}$.

в) $y' = 4x - 4x^3$; $4x(1 - x^2) = 0$; $x = 0$, $x = \pm 1$.

г) $y' = 4x^3 - 16x$; $4x(x^2 - 4) = 0$; $x = 0$, $x = \pm 2$.

881. а) $y' = 2 - \frac{8}{x^2} = 0$; $x = \pm 2$;

б) $y' = \frac{1}{\sqrt{2x-1}}$; $x = \frac{1}{2}$;

в) $y' = \frac{1}{5} - \frac{5}{x^2} = 0$; $x^2 - 25 = 0$; $x = \pm 5$;

г) $y' = 4(x - 3)^3 = 0$; $x = 3$.

882. а) $y' = 4\cos 2x - 4\cos 4x = 0$; $\sin 3x \sin x = 0$; $x = \frac{\pi n}{3}$;

$$\text{б) } y' = -2\sin 2x - 1 = 0; \sin 2x = -\frac{1}{2}; 2x = (-1)^{k+1} \frac{\pi}{6} + \pi k; x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi k}{2};$$

$$\text{в) } y' = -3\sin 3x + 3\sin 5x; \sin 3x - \sin 5x = 0; \sin x \cos 4x = 0; x = \pi n, x = \frac{\pi}{8} + \frac{\pi n}{4};$$

$$\text{г) } y' = \frac{1}{2} \cos \frac{x}{2} - \frac{1}{4}; \cos \frac{x}{2} = \frac{1}{2}; x = \pm \frac{2\pi}{3} + 4\pi n.$$

В задачах 883-888, чтобы определить характер экстремума, необходимо проверить знак производной справа и слева от точки, где она равна нулю. Если слева $y' > 0$, а справа $y' < 0$, то это максимум, если слева $y' < 0$, а справа $y' > 0$, то это минимум.

$$\text{883. а) } y' = 4x - 7 = 0; x = \frac{7}{4} - \min;$$

$$\text{б) } y' = -5 - 2x = 0; x = -2,5 \text{ max};$$

$$\text{в) } y' = 8x - 6 = 0; x = \frac{3}{4} - \min;$$

$$\text{г) } y' = -6x - 12 = 0; x = -2 - \max.$$

$$\text{884. а) } y' = x^2 - 5x + 6 = 0; x = 2, x = 3; x = 2 - \max, x = 3 - \min;$$

$$\text{б) } y' = 3x^2 - 27 = 0; x = \pm 3; x = -3 - \max, x = 3 - \min;$$

$$\text{в) } y' = 3x^2 - 14x - 5 = 0; x = \frac{7+8}{3} = 5; x = -\frac{1}{3}; x = 5 - \min, x = -\frac{1}{3} - \max;$$

$$\text{г) } y' = -6x^2 + 42x = 0; x^2 - 7x = 0; x(x-7) = 0; x = 0, x = 7; x = 7 - \max, x = 0 - \min.$$

$$\text{885. а) } y' = -25x^4 + 9x^2; x^2(9 - 25x^2) = 0; x = 0, x^2 = \frac{9}{25}, x = \pm \frac{3}{5}; x = \frac{3}{5} - \max, x = -\frac{3}{5} - \min;$$

$$\text{б) } y' = 4x^3 - 12x^2 - 16x; 4x(x^2 - 3x - 4) = 0; x = 0, x = 4, x = -1; x = 4, x = -1 - \min, x = 0 - \max;$$

$$\text{в) } y' = 4x^3 - 100x; 4x(x^2 - 25) = 0; x = 0, x = \pm 5; x = -5, x = 5 - \min, x = 0 - \max;$$

$$\text{г) } y' = 10x^4 + 20x^3 - 30x^2; 10x^2(x^2 + 2x - 3) = 0; x = 0, x = -3, x = 1, x = -3 - \max, x = 1 - \min.$$

$$\text{886. а) } y' = 1 - \frac{4}{x^2} = 0; x = \pm 2; x = 2 - \min; x = -2 - \max;$$

$$\text{б) } y' = 1 - \frac{9}{x^2} = 0; x = \pm 3; x = 3 - \min; x = -3 - \max.$$

$$\text{887. а) } y' = 1 - \frac{1}{\sqrt{x-2}} = 0; \sqrt{x-2} = 1; x = 3 - \min;$$

$$\text{б) } y' = \frac{4}{2x-1} - 1 = 0; \sqrt{2x-1} = 4; x = 8,5 - \max.$$

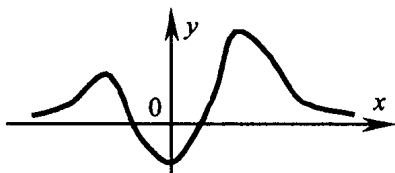
$$\text{888. а) } y' = 1 + 2\sin x; \sin x = -\frac{1}{2}; x = (-1)^{k+1} \frac{\pi}{6} + \pi k; x = -\frac{5\pi}{6} - \max; x = -\frac{\pi}{6} - \min;$$

$$\text{б) } y' = 2\cos x - 1; \cos x = \frac{1}{2}; x = \pm \frac{\pi}{3} + 2\pi n; x = \frac{5\pi}{3} - \min; x = \frac{7\pi}{3} - \max.$$

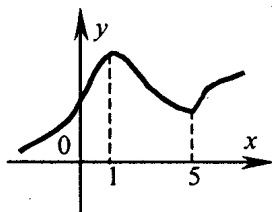
$$\text{889. а) } y' = 3x^2 - 6ax + 27; x^2 - ax + 9 = 0; \frac{D}{4} = a^2 - 9 = 0; a = \pm 3.$$

$$\text{б) } y' = 3x^2 - 6ax + 75; x^2 - 2ax + 25 = 0; \frac{D}{4} = a^2 - 25 = 0; a = \pm 5.$$

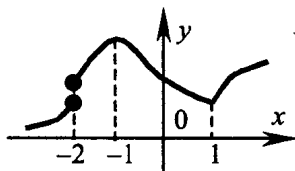
890. а)



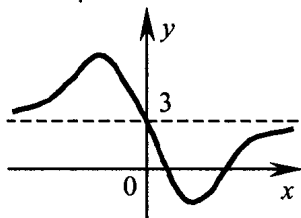
б)



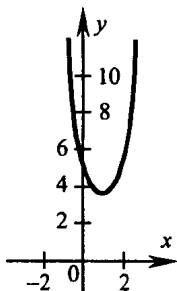
в)



г)



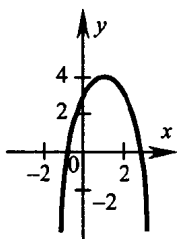
891. а)



$y = 3x^2 - 4x + 5$; $y' = 6x - 4$; $x = \frac{2}{3}$ — min; при $x \geq \frac{2}{3}$ функция возрастает, при

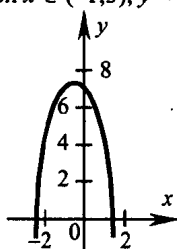
$x \leq \frac{2}{3}$ функция убывает; пересечение с Oy : $(0; 5)$, с Ox : нет; $y > 0$ при $x \in \mathbb{R}$;

б)



$y = 3 + 2x - x^2$; $y' = 2 - 2x$; $x = 1$ — min; при $x \geq 1$ функция убывает при $x \leq 1$ функция возрастает пересечение с Oy : $(0; 3)$, с Ox : $(3; 0)$, $(-1; 0)$; $y > 0$ при $x \in (-1; 3)$; $y < 0$ при $x < -1$, $x > 3$;

в)

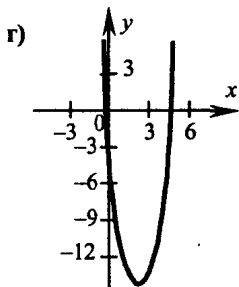


$y = 7 - x - 2x^2$; $y' = -1 - 4x$; $x = -\frac{1}{4}$ — max; при $x \geq -\frac{1}{4}$ функция убывает; при

$x \leq -\frac{1}{4}$ функция возрастает; пересечение с Oy : $(0; 7)$, с Ox :

$$\left(\frac{-1-\sqrt{57}}{4}; 0\right), \left(\frac{-1+\sqrt{57}}{4}; 0\right); y < 0 \text{ при } x \in \left(\frac{-1-\sqrt{57}}{4}; \frac{-1+\sqrt{57}}{4}\right), y > 0 \text{ при}$$

$$x \in \left(-\infty; \frac{-1-\sqrt{57}}{4}\right) \cup \left(\frac{-1+\sqrt{57}}{4}; +\infty\right);$$



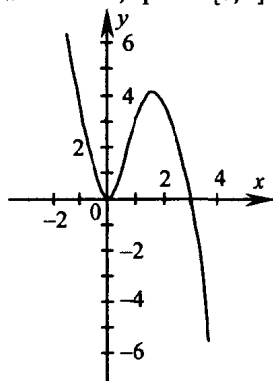
$y = 5x^2 - 15x - 4$; $y' = 10x - 15$; $x = \frac{3}{2}$ — min; при $x \geq \frac{3}{2}$ функция возрастает, при

$x \leq \frac{3}{2}$ функция убывает; пересечение с Oy : $(0; -4)$, с Ox :

$$\left(\frac{15-\sqrt{305}}{10}; 0\right), \left(\frac{15+\sqrt{305}}{10}; 0\right); y > 0 \text{ при}$$

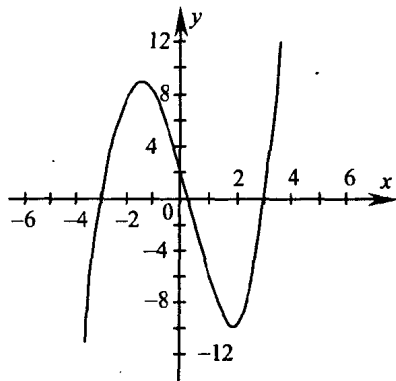
$$x \in \left(-\infty; \frac{15-\sqrt{305}}{10}\right) \cup \left(\frac{15+\sqrt{305}}{10}; +\infty\right), y > 0 \text{ при } x \in \left(\frac{15-\sqrt{305}}{10}; \frac{15+\sqrt{305}}{10}\right).$$

892. а) $y = 3x^2 - x^3$; $y' = 6x - 3x^2$; $x = 0$, $x = 2$; $y(0) = 0$, $y(2) = 12 - 8 = 4$, $x = 0$ — min, $x = 2$ — max; при $x \in [0; 2]$ функция возрастает, при $x \in (-\infty; 0] \cup [2; +\infty)$ убывает.

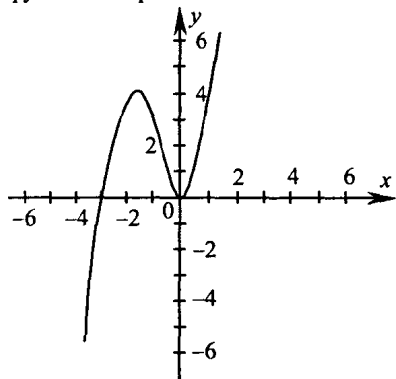


б) $y' = -9x + x^3$; $y' = -9 + 3x^2$; $x^2 = 3$; $x = \pm\sqrt{3}$;

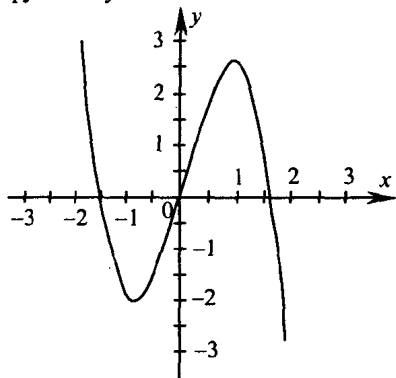
$y(\sqrt{3}) = -9\sqrt{3} + 3\sqrt{3}$; $y(-\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3}$; $x = \sqrt{3} - \min$; $x = -\sqrt{3} - \max$; при $x \in (-\infty; -\sqrt{3}] \cup [\sqrt{3}; +\infty)$ функция возрастает, при $x \in [-\sqrt{3}; \sqrt{3}]$ функция убывает.



в) $y = x^3 + 3x^2$; $y' = 3x^2 + 6x$; $x = 0$, $x = -2$, $y(0) = 0$, $y(-2) = -8 + 12 = 4$; $x = 0 - \min$, $x = -2 - \max$; при $x \in [-2; 0]$ функция убывает, при $x \in (-\infty; -2] \cup [0; +\infty)$ функция возрастает.

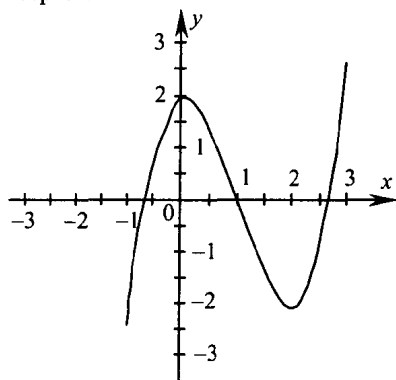


г) $y = 3x - x^3$; $y' = 3 - 3x^2$; $x = \pm 1$, $y(1) = 3 - 1 = 2$, $y(-1) = -3 + 1 = -2$; $x = 1 - \max$, $x = -1 - \min$; при $x \in [-1; 1]$ функция возрастает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция убывает.

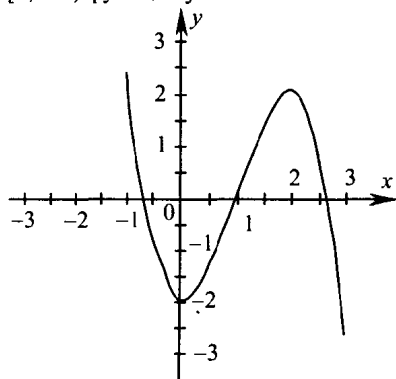


893. а) $y = x^3 + 3x^2$; $y' = 3x^2 + x$; $x = 0$, $x = 2$, $y(0) = 0$, $y(-2) = -8 + 12 = 4$; $x = 0 - \min$,

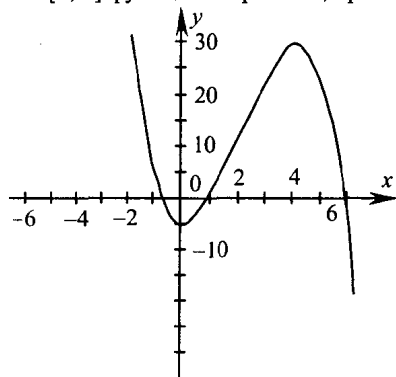
$x = -2$ – max; при $x \in [0; 2]$ функция убывает, при $x \in (-\infty; 0] \cup [2; +\infty)$ функция возрастает.



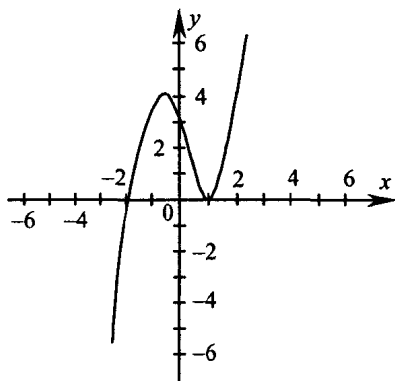
б) $y = -x^3 + 3x - 2$; $y' = -3x^2 + 3$; $x = \pm 1$, $y(1) = -1 + 3 - 2 = 0$, $y(-1) = 1 - 3 - 2 = -4$; $x = 1$ – max, $x = -1$ – min; при $x \in [-1; 1]$ функция возрастает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция убывает.



в) $y = -x^3 + 6x^2 - 5$; $y' = -3x^2 + 12x = -x^2 + 4x$, $x = 0$, $x = 4$, $x = 0$ – min, $x = 4$ – max, при $x \in [0; 4]$ функция возрастает, при $x \in (-\infty; 0] \cup [4; +\infty)$ функция убывает.

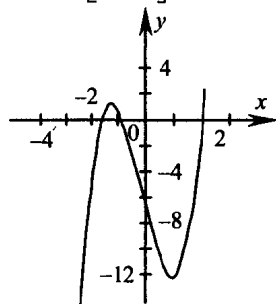


г) $y = x^3 - 3x + 2$; $y' = 3x^2 - 3$; $x = \pm 1$; $x = 1$ – min, $x = -1$ – max; при $x \in [-1; 1]$ функция убывает, при $x \in (-\infty; 0] \cup [4; +\infty)$ функция возрастает.



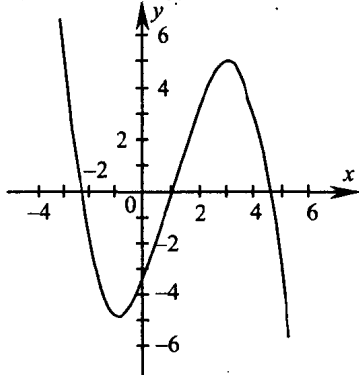
894. а) $y = 2x^3 + x^2 - 8x - 7$; $y' = 6x^2 + 2x - 8$; $x = \frac{-1-7}{6} = -\frac{4}{3}$; $x = 1 - \min$, $x = -\frac{4}{3} - \max$;

при $x \in \left[-\frac{4}{3}; 1\right]$ функция убывает, при $x \in \left(-\infty; -\frac{4}{3}\right] \cup [1; +\infty)$ функция возрастает.



б) $y = -\frac{x^3}{3} + x^2 + 3x - \frac{11}{3}$; $y' = -x^2 + 2x + 3$; $x = 3$, $x = -1$; $x = -1 - \min$, $x = 3 - \max$;

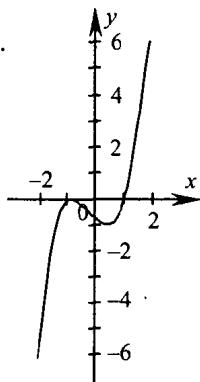
при $x \in [-1; 3]$ функция возрастает, при $x \in (-\infty; -1] \cup [3; +\infty)$ функция убывает.



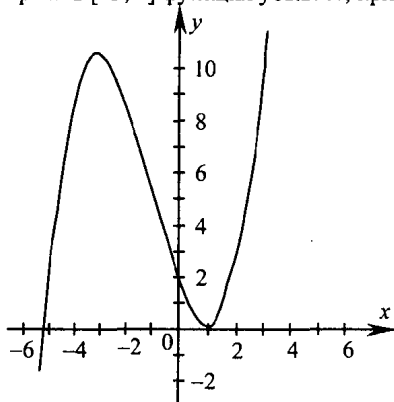
в) $y = x^3 + x^2 - x - 1$; $y' = 3x^2 + 2x - 1$; $x = \frac{-1-2}{3} = -1$, $x = \frac{1}{3}$, $x = \frac{1}{3} - \min$,

$x = -1 - \max$; при $x \in \left[-1; \frac{1}{3}\right]$ функция убывает, при $x \in (-\infty; -1] \cup \left[\frac{1}{3}; +\infty\right)$

функция возрастает.



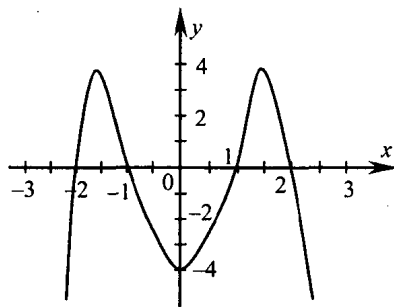
г) $y = \frac{x^3}{3} + x^2 - 3x + \frac{5}{3}$; $y' = x^2 + 2x - 3$, $x = -3$, $x = 1$; $x = 1$ — min, $x = -3$ — max;
при $x \in [-3; 1]$ функция убывает, при $x \in (-\infty; -3] \cup [1; +\infty)$ функция возрастает.



895. а) $y = -x^4 + 5x^2 - 4$; $y' = -4x^3 + 10x$, $2x(-2x^2 + 5) = 0$, $x = 0$, $x = \sqrt{\frac{5}{2}}$, $x = -\sqrt{\frac{5}{2}}$;

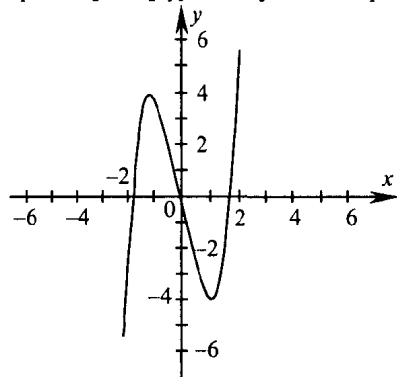
$x = 0$ — min, $x = \pm\sqrt{\frac{5}{2}}$ — max; при $x \in \left(-\infty; -\sqrt{\frac{5}{2}}\right] \cup \left[0; \sqrt{\frac{5}{2}}\right]$ функция возрастает,

при $x \in \left[-\sqrt{\frac{5}{2}}; 0\right] \cup \left[\sqrt{\frac{5}{2}}; +\infty\right)$ функция убывает;



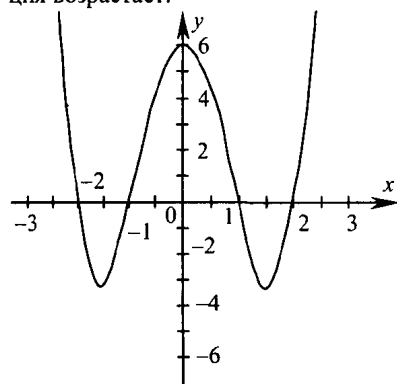
б) $y = x^5 - 5x$; $y' = 5x^4 - 5$; $x^4 - 1 = 0$; $x = \pm 1$; $x = 1 - \min$, $x = -1 - \max$;

при $x \in [-1; 1]$ функция убывает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция возрастает.



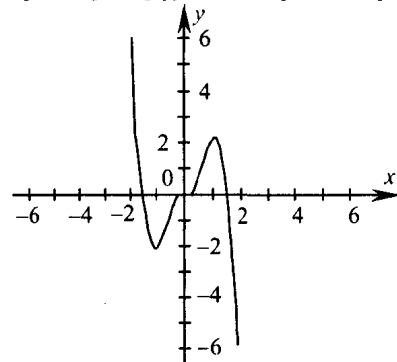
в) $y = 2x^4 - 9x^2 + 7$; $y' = 8x^3 - 18x$; $2x(4x^2 - 9) = 0$; $x = 0 - \max$, $x = \pm \frac{3}{2} - \min$;

при $x \in (-\infty; -\frac{3}{2}] \cup [0; \frac{3}{2}]$ функция убывает, при $x \in [-\frac{3}{2}; 0] \cup [\frac{3}{2}; +\infty)$ функция возрастает.

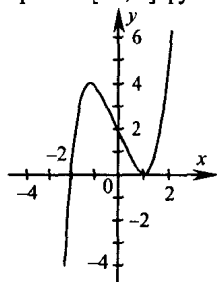


г) $y = 5x^3 - 3x^5$; $y' = 15x^2 - 15x^4$; $15x^2(1 - x^2) = 0$; $x = 0$, $x = \pm 1$; $x = 1 - \max$, $x = -1 - \min$;

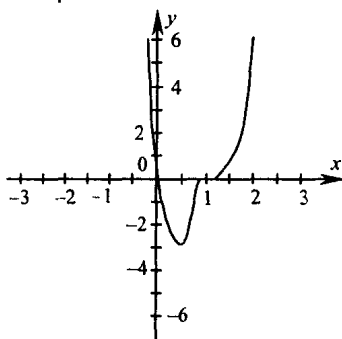
при $x \in [-1; 1]$ функция возрастает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция убывает.



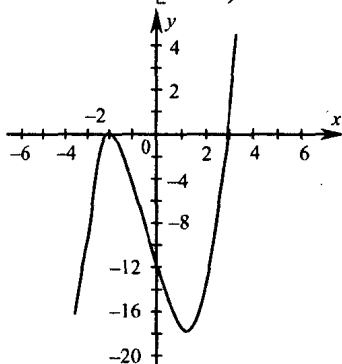
896. а) $y = (x-1)^2(x+2)$; $y' = 2(x-1)(x+2) + (x-1)^2 = (x-1)(2x+4+x-1) = (x-1)(3x+3) = 0$; $(x-1)(3x+1) = 0$, $x = 1$, $x = -1$, $x = -1$ — max, $x = 1$ — min, при $x \in [-1; 1]$ функция убывает, при $x \in (-\infty; -1] \cup [1; +\infty)$ функция возрастает.



- б) $y = \frac{256}{9}x(x-1)^3$; $y' = \frac{256}{9}((x-1)^3 + 3(x-1)^2x) = \frac{256}{9}(x-1)^2(4x-1)$;
 $\frac{256}{9}(x-1)^2(4x-1) = 0$; $x = \frac{1}{4}$ — min, $x = 1$; при $x \geq \frac{1}{4}$ функция возрастает, при $x \leq \frac{1}{4}$ функция убывает.

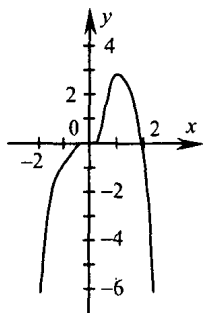


- в) $y = (x+2)^2(x-3)$; $y' = 2(x+2)(x-3) + (x+2)^2 = (x+2)(3x-4)$, $(x+2)(3x-4) = 0$,
 $x = -2$ — max, $x = \frac{4}{3}$ — min, при $x \in \left[-2; \frac{4}{3}\right]$ функция убывает, при $x \in (-\infty; -2] \cup \left[\frac{4}{3}; +\infty\right)$ функция возрастает.



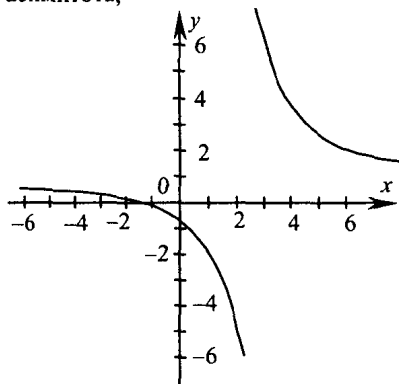
$$г) y = x^3(2-x); y' = (3x^2(2-x) - x^3) = x^2(6-4x), x^2(6-4x) = 0, x = 0, x = \frac{3}{2} - \max;$$

при $x \geq \frac{3}{2}$ функция убывает, при $x \leq \frac{3}{2}$ функция возрастает.



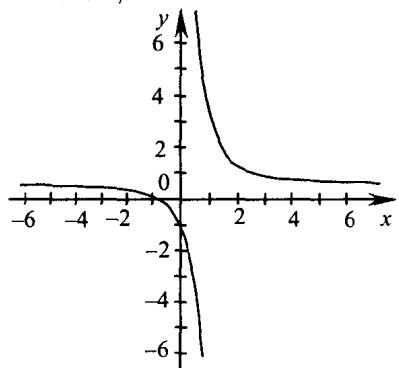
$$897. а) y' = \frac{x-3-x-2}{(x-3)^2} = -\frac{5}{(x-3)^2}; -\frac{5}{(x-3)^2} = 0, \text{ везде убывает, } x \neq 3; x = 3 -$$

асимптота;



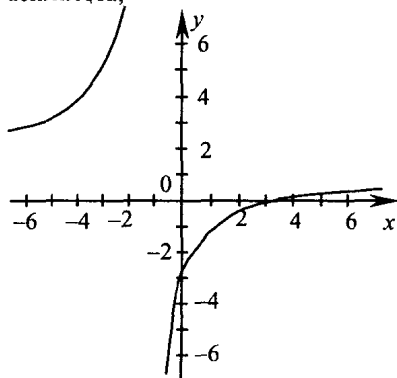
$$б) y' = \frac{6x-2-6x-3}{(3x-1)^2} = -\frac{5}{(3x-1)^2}; -\frac{5}{(3x-1)^2} = 0, \text{ везде убывает, } x \neq \frac{1}{3}; x = \frac{1}{3} -$$

асимптота;



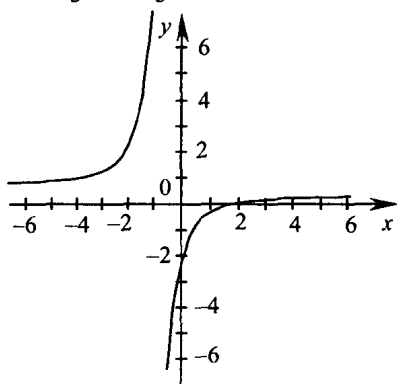
в) $y' = \frac{x+1-x+3}{(x+1)^2} = \frac{4}{(x+1)^2}; \frac{4}{(x+1)^2} = 0$ — везде возрастает, $x \neq -1$; $x = -1$ —

асимптота;

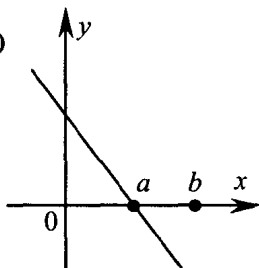


г) $y' = \frac{6x+4-6x+9}{(3x+2)^2} = \frac{13}{(3x+2)^2}; \frac{13}{(3x+2)^2} = 0$ — везде возрастает,

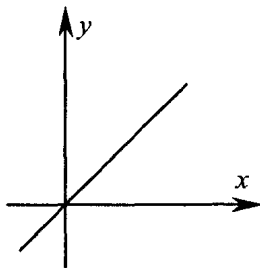
$x \neq -\frac{2}{3}$; $x = -\frac{2}{3}$ — асимптота.

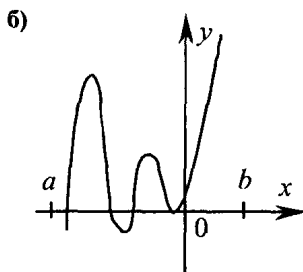
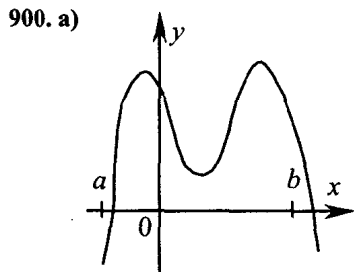
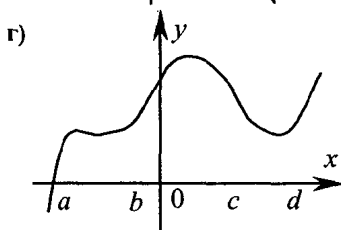
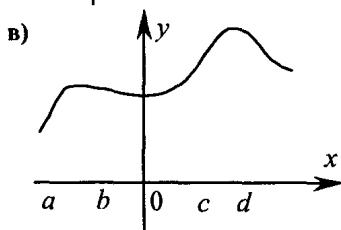
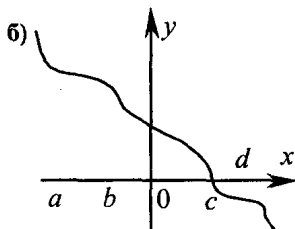
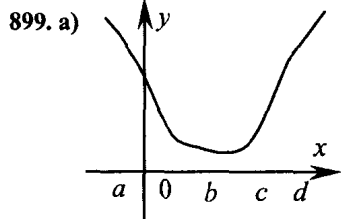
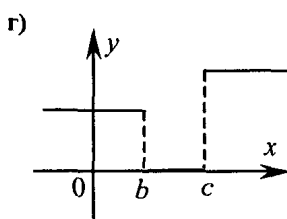
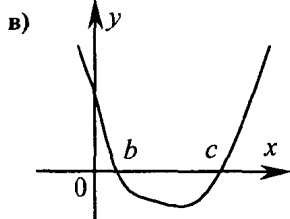


898. а)



б)





901. а) да; б) да; в) нет; г) нет.

902. а) $y' = \cos x - \frac{1}{2}$; $\cos x = \frac{1}{2}$; $x = \pm \frac{\pi}{3} + 2\pi n$; $x = -\frac{\pi}{3} + 2\pi n - \min$; $x = \frac{\pi}{3} + 2\pi n - \max$;

возрастает при $x \in \left[-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right]$, убывает при $x \in \left[\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right]$;

б) $y' = \frac{1}{2} + \sin x$; $x = (-1)^{k+1} \frac{\pi}{6} + \pi k$; $x = -\frac{\pi}{6} + 2\pi n - \max$; $x = \frac{7\pi}{6} + 2\pi n - \min$;

возрастает при $x \in \left[-\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n\right]$, убывает при

$x \in \left[\frac{7\pi}{6} + 2\pi n; \frac{11\pi}{6} + 2\pi n\right]$;

$$\text{в) } y' = \frac{\sqrt{2}}{2} - \sin x; x = (-1)^k \frac{\pi}{4} + \pi k; x = \frac{\pi}{4} + 2\pi l - \max; x = \frac{3\pi}{4} + 2\pi l - \min;$$

$$\text{возрастает при } x \in \left[\frac{3\pi}{4} + 2\pi l; \frac{9\pi}{4} + 2\pi l \right], \text{ убывает при } x \in \left[\frac{\pi}{4} + 2\pi l; \frac{3\pi}{4} + 2\pi l \right];$$

$$\text{г) } y = x - \sin x; y' = 1 - \cos x; x = 2\pi l, \text{ возрастает на } R.$$

$$\text{903. а) } y' = -\frac{1}{2} \sin \frac{x}{2}; \sin \frac{x}{2} = 0; x = 2\pi l; x = 4\pi l - \max, x = 2\pi + 4\pi l - \min; \text{ возрастает}$$

$$\text{при } x \in [-2\pi + 4\pi l; 4\pi l], \text{ убывает при } x \in [4\pi l; 2\pi + 4\pi l],$$

$$\text{б) } y' = -\frac{1}{3} \cos \frac{x}{3}; x = \frac{3\pi}{2} + 6\pi l - \min; x = -\frac{3\pi}{2} + 6\pi l - \max; \text{ убывает при}$$

$$x \in \left[6\pi l - \frac{3\pi}{2}; \frac{3\pi}{2} + 6\pi l \right], \text{ возрастает при } x \in \left[\frac{3\pi}{2} + 6\pi l; \frac{9\pi}{2} + 6\pi l \right].$$

$$\text{904. а) } y' = 1 - 2 \cos 2x; \cos 2x = -\frac{1}{2}; 2x = \pm \frac{\pi}{3} + 2\pi l; x = \pm \frac{\pi}{6} + \pi l; x = \frac{\pi}{6} + \pi l - \min;$$

$$x = \frac{\pi}{6} + \pi l - \max; \text{ убывает при } x \in \left[\pi l - \frac{\pi}{6}; \frac{\pi}{6} + \pi l \right], \text{ возрастает при } x \in \left[\frac{\pi}{6} + \pi l; \frac{5\pi}{6} + \pi l \right];$$

$$\text{б) } y' = 1 - 2 \sin \frac{x}{2}; \sin \frac{x}{2} = \frac{1}{2}; x = (-1)^k \frac{\pi}{3} + 2\pi k; x = \frac{\pi}{3} + 4\pi l - \max; x = \frac{5\pi}{3} + 4\pi l - \min;$$

$$\text{убывает при } x \in \left[\frac{\pi}{3} + 4\pi l; \frac{5\pi}{3} + 4\pi l \right], \text{ возрастает при } x \in \left[-\frac{7\pi}{3} + 4\pi l; \frac{\pi}{3} + 4\pi l \right].$$

$$\text{905. а) I: } x \geq 3; y = x - 5; y' = 1; \text{ возрастает при } x \geq 3;$$

$$\text{II: } x \leq 3; y = 1 - x; y' = -1; \text{ убывает при } x \leq 3;$$

$$\text{Ответ: } x \in (-\infty; 3] - \text{убывает, } x \in [3; +\infty) - \text{возрастает; } x = 3 - \min.$$

$$\text{б) I: } x \geq 1, x < 0; y = -\frac{1}{x} + 1; y' = \frac{1}{x^2} - \text{везде возрастает;}$$

$$\text{II: } x \in (0; 1]; y = -1 + \frac{1}{x}; y' = -\frac{1}{x^2} - \text{везде убывает;}$$

$$\text{Ответ: } x \in (0; 1] - \text{убывает, } x \in (-\infty; 0) \cup [1; +\infty) - \text{возрастает; } x = 1 - \min.$$

$$\text{в) I: } x \geq 2, x \leq -3; y = x^2 + x - 6; y' = 2x + 1; x = -\frac{1}{2}; \text{ возрастает при } x \geq -\frac{1}{2},$$

$$\text{убывает при } x \leq -\frac{1}{2};$$

$$\text{II: } x \in [-3; 2]; y = -x^2 - x + 6; y' = -2x - 1; x = -\frac{1}{2}; \text{ возрастает при } x \leq -\frac{1}{2},$$

$$\text{убывает при } x \geq -\frac{1}{2};$$

$$\text{Ответ: } x \in (-\infty; -3] \cup \left[-\frac{1}{2}; 2 \right] - \text{убывает; } x \in \left[-3; -\frac{1}{2} \right] \cup [2; +\infty) - \text{возрастает;}$$

$$x = -3, x = 2 - \min; x = -\frac{1}{2} - \max.$$

$$\text{г) I: } x \geq 0; y = x^2 - 2x; y' = 2x - 2; x = 1; \text{ возрастает при } x \geq 1, \text{ убывает при } x \leq 1;$$

II: $x \leq 0$; $y = x^2 + 2x$; $y' = 2x + 2$; $x = -1$; возрастает при $x \geq -1$, убывает при $x \leq -1$;

Ответ: $x \in (-\infty; -1] \cup [0; 1]$ – убывает; $x \in [-1; 0] \cup [1; +\infty)$ – возрастает;

$x = \pm 1$ – min; $x = 0$ – max.

906. а) $y = |x^3 - 3x|$; $y = |x(x - \sqrt{3})(x + \sqrt{3})|$;

I: $x \in [-\sqrt{3}; 0] \cup [\sqrt{3}; +\infty)$; $y' = 3x^2 - 3$; $x = \pm 1$; возрастает при $x \in (-\infty; -1] \cup [1; +\infty)$,

убывает при $x \in [-1; 1]$;

II: $x \in (-\infty; -\sqrt{3}] \cup [0; \sqrt{3}]$; $y' = 3 - 3x^2$, $x = \pm 1$; возрастает при $x \in [-1; 1]$, убывает при $x \in (-\infty; -1] \cup [1; +\infty)$;

Ответ: $x \in (-\infty; -\sqrt{3}] \cup [-1; 0] \cup [1; \sqrt{3}]$ – убывает,

$x \in [-\sqrt{3}; -1] \cup [0; 1] \cup [\sqrt{3}; +\infty)$, $x = \pm \sqrt{3}$ – возрастает, $x = 0$ – min, $x = \pm 1$ – max.

б) $y = |x - x^3|$; $y = |x(\sqrt{3} - x)(\sqrt{3} + x)|$;

I: $x \in (-\infty; -\sqrt{3}] \cup [0; \sqrt{3}]$; $y' = 1 - 3x^2$, $x = \pm \frac{1}{\sqrt{3}}$; возрастает при

$x \in \left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$, убывает при $x \in \left(-\infty; -\frac{1}{\sqrt{3}}\right) \cup \left[\frac{1}{\sqrt{3}}; +\infty\right)$;

II: $x \in [-\sqrt{3}; 0] \cup [\sqrt{3}; +\infty)$; $y' = 3x^2 - 1$; $x = \pm \frac{\sqrt{3}}{3}$; возрастает при

$x \in \left(-\infty; -\frac{1}{\sqrt{3}}\right) \cup \left[\frac{1}{\sqrt{3}}; +\infty\right)$, убывает при $x \in \left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$;

Ответ: $x \in (-\infty; -1] \cup \left[-\frac{1}{\sqrt{3}}; 0\right] \cup \left[\frac{1}{\sqrt{3}}; 1\right]$ – убывает,

$x \in \left[-1; -\frac{1}{\sqrt{3}}\right] \cup \left[0; \frac{1}{\sqrt{3}}\right] \cup [1; +\infty)$, $x = \pm 1$, $x = 0$ – min, $x = \pm \frac{1}{\sqrt{3}}$ – max.

907. а) $y' = 5x^4 + 3 > 0 \forall x \in (-\infty; +\infty)$; б) $y' = \frac{2}{x^2} + \frac{3}{x^4} > 0 \forall x \in (-\infty; +\infty)$;

в) $y' = x^6 + 21x^2 + 2$;

г) $y' = 21 + \frac{5}{x^6} > 0 \forall x > 0$.

908. а) $y' = 7 + 2\sin 2x > 0 \forall x \in (-\infty; +\infty)$; б) $y' = \frac{3}{\cos^2 x} > 0 \forall x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$;

в) $y' = \frac{1}{\sin^2 x} > 0 \forall x \in (0; \pi)$;

г) $y' = 10 + 3\cos 3x > 0 \forall x \in (-\infty; +\infty)$.

909. а) $y = 2x^3 + 2x^2 + 11x - 35$, $y' = 6x^2 + 4x + 11$, $D = 16 - 4 \cdot 6 \cdot 11 < 0$, следовательно, $6x^2 + 4x + 11 > 0 \forall x \in (-\infty; +\infty)$;

б) $y = 3x^3 - 6x^2 + 41x - 137$; $y' = 9x^2 - 12x + 41$, $\frac{D}{4} = 36 - 41 \cdot 9 < 0$, следовательно, $9x^2 - 12x + 41 > 0 \forall x \in (-\infty; +\infty)$.

$$910. \text{ а) } y' = \frac{16x + 4 - 16x}{(4x + 1)^2} = \frac{4}{(4x + 1)^2} > 0 \quad \forall x \in \left(-\frac{1}{4}; +\infty\right);$$

$$\text{ б) } y' = \frac{2x - 10 - 2x + 13}{(x - 5)^2} = \frac{3}{(x - 5)^2} > 0 \quad \forall x \in (-\infty; 5).$$

$$911. \text{ а) } y = -x^3 - 5x + 3, y' = -3x^2 - 5 < 0 \quad \forall x \in (-\infty; +\infty);$$

$$\text{ б) } y = -2x^5 - 7x^3 - x + 8, y' = -10x^4 - 21x^2 - 1 < 0 \quad \forall x \in (-\infty; +\infty);$$

$$\text{ в) } y = -x^3 + 3x^2 - 6x + l, y' = -3x^2 + 6x - 6 = -3(x^3 - 2x + 1) = -3(x - 1)^2 < 0 \quad \forall x \in (-\infty; +\infty);$$

$$\text{ г) } y = -4x^3 + 4x^2 - 2x + 9, y' = -12x^2 + 8x - 2, \frac{D}{4} = 16 - 24 < 0 \Rightarrow 12x^2 + 8x - 2 < 0$$

$$\forall x \in (-\infty; +\infty).$$

$$912. \text{ а) } y' = \frac{3x + 6 - 3x - 7}{(x + 2)^2} = -\frac{1}{(x + 2)^2} < 0 \quad \forall x \in (-\infty; +\infty);$$

$$\text{ б) } y' = \frac{-8x - 4 + 8x - 2}{(2x + 1)^2} = -\frac{6}{(2x + 1)^2} < 0 \quad \forall x \in \left(-\infty; -\frac{1}{2}\right).$$

$$913. \text{ а) } y' = -7\sin x - 15\cos 3x - 22 < 0 \quad \forall x \in (-\infty; +\infty), \text{ убывает на } R;$$

$$\text{ б) } y' = -21\sin 7x - 4\cos \frac{x}{2} - 25 < 0 \quad \forall x \in (-\infty; +\infty), \text{ убывает на } R.$$

$$914. \text{ а) } y' = 3x^2 + a, \text{ при } a \geq 0 \text{ возрастает на } R;$$

$$\text{ б) } y' = x^2 - 2ax + 5, \frac{D}{4} = a^2 - 5, \text{ при } a \in [-\sqrt{5}; \sqrt{5}] \text{ возрастает на } R.$$

$$915. \text{ а) } y' = a + \sin x, \text{ при } a > 1 \text{ возрастает на } R;$$

$$\text{ б) } y' = 4\cos 2x - a, \text{ при } a < -4 \text{ возрастает на } R.$$

$$916. \text{ а) } y' = b - 2x - 3x^2, D = 4 - 4 \cdot (-3) \cdot b = 4 + 12b < 0, \text{ при } b < -\frac{1}{3} \text{ убывает на } R;$$

$$\text{ б) } y' = -\frac{1}{\sqrt{x+3}} + b < 0, b < \frac{1}{\sqrt{x+3}}, \text{ при } b < 0 \text{ убывает при } \forall x \in (-3; +\infty);$$

$$\text{ в) } y' = 3x^2 + 2bx + 3. \text{ При } x = 0 \quad y' = 3, \text{ следовательно, ни при каких } b \text{ функция не может убывать на всей области определения.}$$

$$\text{ г) } y' = -2b - \frac{1}{2\sqrt{1-x}} < 0, \text{ при } b \geq 0.$$

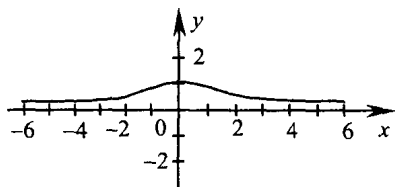
$$917. \text{ а) } y' = 6x^2 - 6x = 6x(x - 1), \text{ возрастает } x \leq 0, x \geq 1 \text{ при } a \leq -1, a \geq 2, \text{ следовательно, } a - 1 \geq 1, a + 1 \leq 0; a \geq 2, a \leq -1.$$

$$\text{ б) } y' = -3x^2 + 3 = -3(x^2 - 1), \text{ возрастает } x \in [-1; 1], \text{ убывает } x \leq -1, x \geq 1 \text{ при}$$

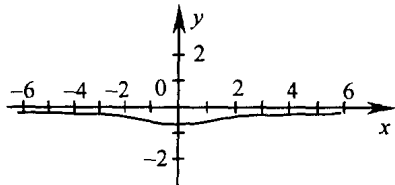
$$a \in \left(-\infty; -\frac{3}{2}\right] \cup [1; +\infty), \text{ следовательно } a > 1, a + \frac{1}{2} < -1; a \geq 2, a \leq -\frac{3}{2} \text{ при}$$

$$a \in \left(-\infty; -\frac{3}{2}\right] \cup [1; +\infty).$$

$$918. \text{ а) } y' = -\frac{1}{(x^2 + 1)^2}, y' < 0 \text{ при } x > 0, y' > 0 \text{ при } x < 0, \text{ асимптота } y = 0, x = 0 - \text{max};$$

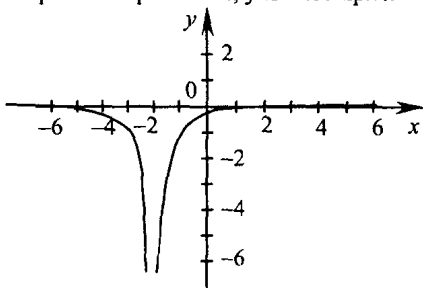


б) $y' = -\frac{2}{(x^2+4)^2}$, $y' < 0$ при $x < 0$, $y' > 0$ при $x > 0$, асимптота $y = 0$, $x = 0$ — min.



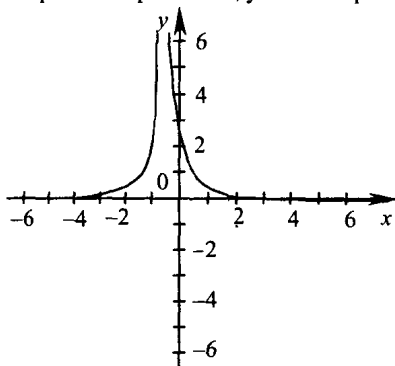
919. а) $y = -\frac{1}{x^2+4x+4} = -\frac{1}{(x+2)^2}$; $y' = \frac{2}{(x+2)^3}$,

возрастает при $x > -2$, убывает при $x < -2$, асимптота $x = -2$;



б) $y = \frac{1}{x^2+4x+1} = \frac{1}{(x+1)^2}$; $y' = -\frac{2}{(x+1)^3}$

возрастает при $x < -1$, убывает при $x > -1$, асимптота $x = -1$.

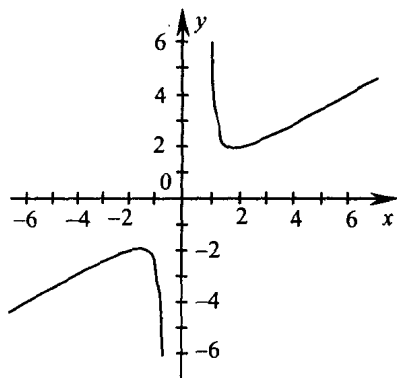


920. а) $y' = \frac{1}{2} - \frac{2}{x^2}$; $y' > 0$: $\frac{1}{2} - \frac{2}{x^2} > 0$, $x^2 > 4$,

$y' > 0$ при $x \in (-\infty; -2) \cup (2; +\infty)$, $y' < 0$ при $x \in (-2; 0) \cup (0; 2)$,

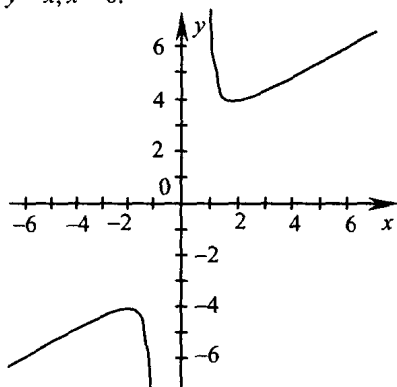
при $x \in (-\infty; -2] \cup [2; +\infty)$ возрастает, при $x \in [-2; 0) \cup (0; 2]$ убывает;

$x = 2$ — min, $x = -2$ — max, асимптоты: $y = \frac{x}{2}$, $x = 0$;



б) $y = \frac{x^2 + 4}{x} = x + \frac{4}{x}$; $y' = 1 - \frac{4}{x^2}$; $x = \pm 2$; $1 - \frac{4}{x^2} > 0$; $\frac{x^2}{4} > 1$; $x^2 > 4$; $|x| > 2$;

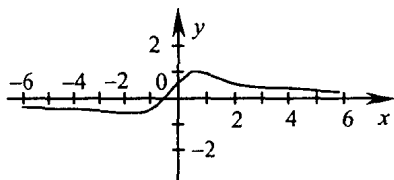
при $|x| \geq 2$ возрастает, при $|x| \leq 2$ убывает; $x = -2$ — min, $x = 2$ — max; асимптоты $y = x$, $x = 0$.



921. а) $y' = \frac{4x^2 + 4x - 4x^2 - 2}{(x^2 + 2)^2} = \frac{2x}{(x^2 + 2)^2}$, $y' > 0$ при $x^2 + x - 4 < 0$;

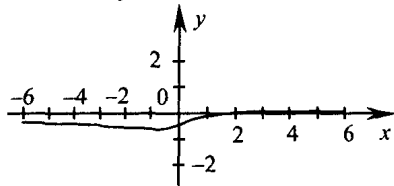
при $x \in \left(\frac{-1 - \sqrt{17}}{2}; \frac{-1 + \sqrt{17}}{2} \right)$ возрастает, при

$x \in \left(-\infty; \frac{-1 - \sqrt{17}}{2} \right) \cup \left(\frac{-1 + \sqrt{17}}{2}; +\infty \right)$ убывает, асимптота $y = 0$;

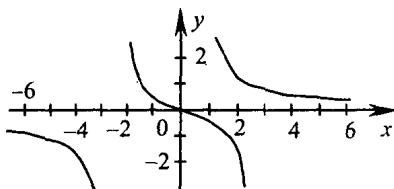


$$6) y' = \frac{x^2 + 5 - 2x^2 + 4x}{(x^2 + 5)^2} = \frac{-x^2 + 4x + 5}{(x^2 + 5)^2}; y' > 0 \text{ при } x^2 - 4x - 5 < 0; x \leq -1; x = 5;$$

при $x \in [-1; 5]$ возрастает, при $x \leq -1, x \geq 5$ убывает; $x = -1$ — min, $x = 5$ — max, асимптота $y = 0$.



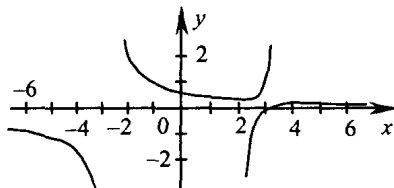
$$922. a) y' = \frac{x^2 - 4 - 2x^2}{(x^2 - 4)^2} = -\frac{x^2 + 4}{(x^2 - 4)^2}, \text{ убывает при всех } x \neq 2, \text{ асимптоты } x = \pm 2, y = 0;$$



$$6) y' = \frac{x^2 - 8 - 2x^2 + 6x}{(x^2 - 8)^2} = \frac{-x^2 + 6x - 8}{(x^2 - 8)^2}, y' > 0 \text{ при } x^2 - 6x + 8 < 0; x = 4, x = 2;$$

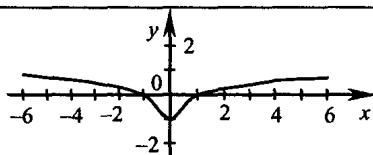
при $x \in [2; 2\sqrt{2}] \cup (2\sqrt{2}; 4]$ возрастает, при $x \leq 2, x \geq 4$ убывает;

$x = 2$ — min, $x = 4$ — max, асимптоты: $x = \pm 2\sqrt{2}, y = 0$.

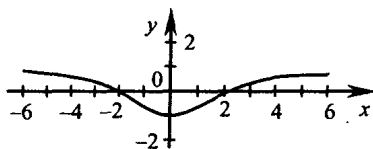


$$923. a) y' = \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2};$$

при $x \geq 0$ возрастает, при $x \leq 0$ убывает, $x = 0$ — min;

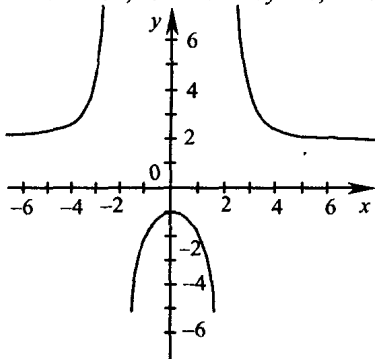


б) $y' = \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}$; при $x \geq 0$ возрастает, при $x \leq 0$ убывает, $x = 0$ —



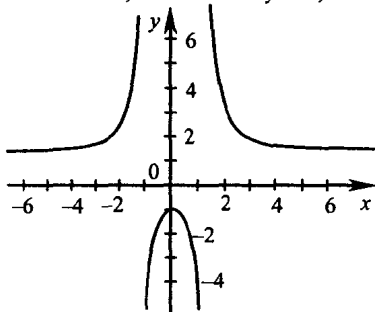
924. а) $y' = \frac{2x^3 - 8x - 2x^3 - 8x}{(x^2 - 4)^2} = \frac{-16x}{(x^2 - 4)^2}$;

при $x \in [0; 2) \cup (2; +\infty)$ убывает, при $x \in (-\infty; -2) \cup (-2; 0]$ возрастает;
 $x = 0$ — макс, асимптоты: $y = 0$, $x = \pm 2$;

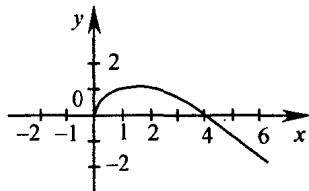


б) $y' = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$;

при $x \in [0; 1) \cup (1; +\infty)$ убывает, при $x \in (-\infty; -1) \cup (-1; 0]$ возрастает;
 $x = 0$ — макс, асимптоты: $y = 0$, $x = \pm 1$.

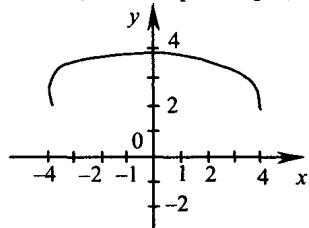


925. а) $y' = \frac{1}{\sqrt{x}} - 1$, при $x \in [1; +\infty)$ убывает, при $x \in [0; 1]$ возрастает, $x = 1$ – max;

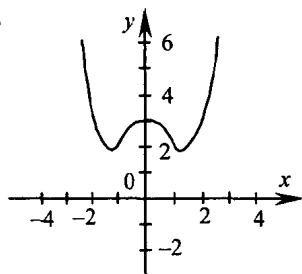


б) $y' = \frac{1}{2\sqrt{x+4}} - \frac{1}{\sqrt{9-3x}} = \frac{\sqrt{9-3x} - 2\sqrt{x+4}}{2\sqrt{x+4}\sqrt{9-3x}}$; $\sqrt{9-3x} > 2\sqrt{x+4}$; $9 - 3x > 4x + 16$;

$7x < -7$, $x < -1$; при $x \in [-4; -1]$ возрастает, при $x \in [-1; 3]$ убывает; $x = -1$ – max.



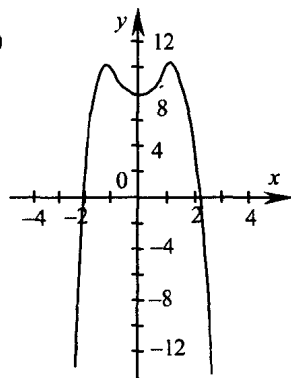
926. а)



б) Количество корней в данном уравнении – это количество пересечения графиков $y = x^4 - 2x^2 + 3$ и $y = a$.

Из рисунка видно, что такой случай имеет место, когда прямая $y = a$ касается графика функции в точке $(0; y(0))$, $y(0) = 3$, следовательно, $a = 3$.

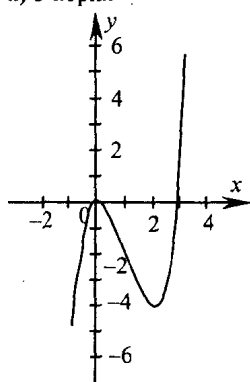
927. а)



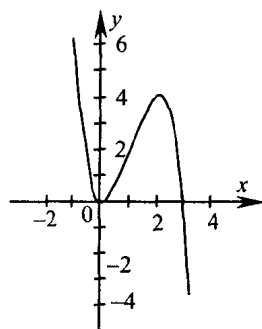
б) Чтобы уравнение не имело корней, необходимо, чтобы прямая лежала выше графика функции $y = -x^4 + 2x^2 + 8$.

Найдем точки максимума: $y' = -4x^3 + 4x = 4x(1 - x^2) = 0$, $x = 0$ – точка минимума, $x = \pm 1$ – точки максимума, $y(1) = y(-1) = -1 + 2 + 8 = 9$. Следовательно, $a > 9$.

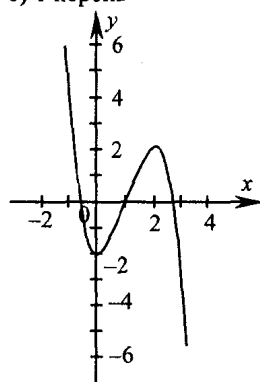
928. а) 3 корня



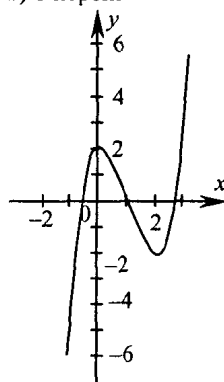
в) 3 корня



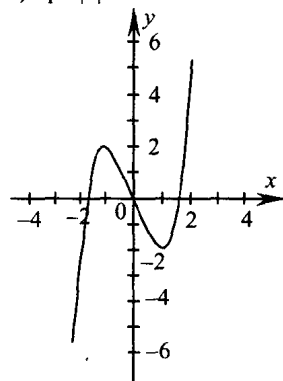
б) 1 корень



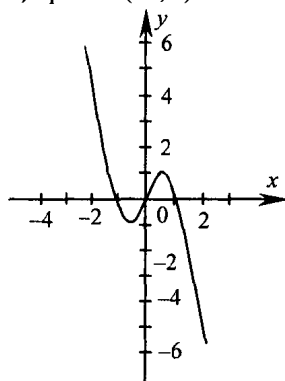
г) 1 корень



929. а) при $|a| > 2$



б) при $a \in (-2; 2)$



930. а) $y_1 = x^3 + 5$, $y_2 = 15 - x$,

$y_1' = 3x^2 + 5$, возрастает на R ; $y_2' = -1$, убывает на R , один корень $x = 2$;

б) $y_1' = 5x^4 + 9x^2 + 7$, возрастает на R , один корень $x = -1$;

в) $2x^5 + 3x^3 + 12x - 17 = 0$; $y' = 10x^4 + 9x^2 + 12$, возрастает на R , один корень $x = -1$;

г) $x^5 + 4x^3 + 8x - 13 = 0$, $y' = 5x^4 + 12x^2 + 8$, возрастает на R , один корень $x = -1$.

931. а) $\sin 5x - 2\cos x - 8x - x^5 + 2 = 0$, $y_1 = \sin 5x - 2\cos x - 8x$, $y_2 = x^5 - 2$, $y_1 = 5\cos 5x + 2\sin x - 8$ — убывает на R ; $y_2' = 5x^4 + 8$ — возрастает на R , только одно решение $x = 0$.

б) $4\cos 3x + 5\sin \frac{x}{2} + 15x = 4 - x^3$, $y_1 = 4\cos 3x + 5\sin \frac{x}{2} + 15x$, $y_2 = 4 - x^3$,

$y_1' = -12\sin 3x + \frac{5}{2}\sin \frac{x}{2} + 15$ — возрастает на R ; $y_2' = 4 - 3x^2$ — убывает на R ,

только одно решение $x = 0$.

932. а) $3\cos \frac{\pi x}{2} + 5\sin \frac{\pi x}{2} + 18x = 46 - x^5 - 22x^3$; $y_1 = 3\cos \frac{\pi x}{2} + 5\sin \frac{\pi x}{2} + 18x$;

$y_2 = 46 - x^5 - 22x^3$, $y_1' = -\frac{3\pi}{2}\sin \frac{\pi x}{2} + \frac{5\pi}{2}\cos \frac{\pi x}{2} + 18$, возрастает на R ;

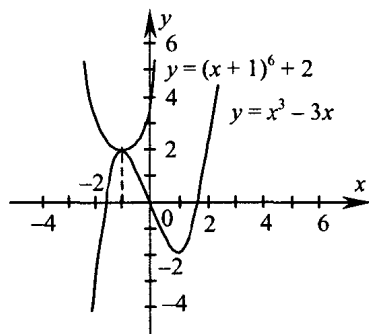
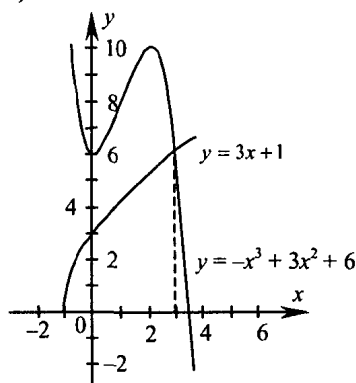
$y_2' = -5x^4 - 66x^2$, убывает на R , один корень $x = 1$;

б) $2\sin \frac{\pi}{2}x - 2\cos \pi x - 8x = x^5 - 50$, $y_1 = 2\sin \frac{\pi}{2}x - 2\cos \pi x - 8x$, $y_2 = x^5 - 50$,

$y_2' = \pi x^4$, $y_1' = \pi \cos \frac{\pi x}{2} + 2\pi \sin \pi x - 8$, убывает на R ; $y_2' = 5x^4$, возрастает на R , один корень $x = 2$.

933. а) $x = 3$

б) $x = -1$.



§36. Применение производной для отыскания наибольших и наименьших значений величин

934. а) $y' = 3 \Rightarrow$ функция растёт на R ; $x_{\min} = -1$, $y(-1) = -9$, $y_{\min} = -9$; $x_{\max} = 4$, $y(4) = 6$, $y_{\max} = 6$.

б) $y' = \frac{8}{x^2} > 0 \Rightarrow$ функция растёт при $x \in \left[\frac{1}{4}; 8\right]$; $x_{\min} = \frac{1}{4}$, $y\left(\frac{1}{4}\right) = -32$, $y_{\min} = -32$;

$x_{\max} = 8$, $y(8) = -1$, $y_{\max} = -1$.

в) $y' = -\frac{1}{2} \Rightarrow$ функция убывает на R ;

$x_{\min} = 6$, $y(6) = 1$, $y_{\min} = 1$; $x_{\max} = -2$, $y(-2) = 5$, $y_{\max} = 5$.

$$\text{г) } y' = -\frac{3}{x^2} < 0 \Rightarrow \text{функция убывает при } [0,3; 2]; x_{\max} = 0,3, y(0,3) = 10,$$

$$y_{\max} = 10; x_{\min} = 2, y(2) = \frac{3}{2}, y_{\min} = \frac{3}{2}.$$

935. а) $y' = 2x - 8, x = 4$ — точка минимума;

$$y(4) = 16 - 32 + 19 = 3; y(-1) = 1 + 8 + 19 = 28; y(5) = 25 - 40 + 19 = 4; y_{\max} = 28, y_{\min} = 3.$$

$$\text{б) } y' = 2x + 4, x = -2, -2 \notin [0; 2]; y(0) = -3, y(2) = 4 + 8 - 3 = 9; y_{\max} = 9, y_{\min} = -3.$$

$$\text{в) } y' = 4x - 8, x = 2; y(7) = 8 - 16 + 6 = -2; y(-1) = 2 + 8 + 6 = 16; y(4) = 32 - 32 + 6 = 6; y_{\max} = 16, y_{\min} = -2.$$

$$\text{г) } y' = -6x + 6, x = 1; y(1) = -3 + 6 - 10 = -7; (-2) = -12 - 12 - 10 = -34;$$

$$y(9) = -243 + 54 - 10 = -199; y_{\max} = -7, y_{\min} = -199.$$

$$936. \text{ а) } y' = 2\cos x; x = \frac{\pi}{2}; y = -\frac{\pi}{2}; y_{\max} = 2; y_{\min} = -2; \quad \text{б) } y_{\max} = 2; y_{\min} = -2;$$

$$\text{в) } y_{\max} = 6; y_{\min} = 0;$$

$$\text{г) } y_{\max} = \frac{1}{2}; y_{\min} = -\frac{1}{2}.$$

$$937. \text{ а) } y_{\max} = -\frac{\sqrt{3}}{3}; y_{\min} = -\sqrt{3};$$

$$\text{б) } y_{\min} = -3\sqrt{3}; y_{\max} = 0;$$

$$\text{в) } y_{\max} = 0; y_{\min} = -\frac{2\sqrt{3}}{3};$$

$$\text{г) } y_{\max} = \frac{1}{2}; y_{\min} = 0.$$

$$938. \text{ а) } y_{\max} = 3; y_{\min} = 0;$$

$$\text{б) } y_{\max} = 2; y_{\min} = 0;$$

$$\text{в) } y_{\max} = -2; y_{\min} = -4;$$

$$\text{г) } y_{\max} = -2; y_{\min} = -3.$$

$$939. \text{ а) } y_{\max} = 192; y_{\min} = 0;$$

$$\text{б) } y_{\max} = -\frac{6}{100\,000} = -\frac{3}{50\,000}; y_{\min} = -192;$$

$$\text{в) } y_{\max} = 0; y_{\min} = -3;$$

$$\text{г) } y_{\max} = 9; y_{\min} = 0.$$

$$940. \text{ а) } y_{\max} = 1; y_{\min} = 0; \text{ б) } y_{\max} = 1; y_{\min} = 0; \text{ в) } y_{\max} = 1; y_{\min} = 0; \text{ г) } y_{\max} = 1; y_{\min} = -1.$$

$$941. y' = 3x^2 - 18x + 24 = 0; x^2 - 6x + 8 = 0; x = 4, x = 2;$$

$$y_{\max} = y(2) = 8 - 36 + 48 - 1 = 19; y_{\min} = y(4) = 64 - 144 + 96 - 1 = 15;$$

$$\text{а) } y(-1) = -1 - 9 - 24 - 1 = -35; y(3) = 17; y_{\min} = y(-1) = -35; y_{\max} = y(2) = 19;$$

$$\text{б) } y(3) = 27 - 81 + 72 - 1 = 17; y(6) = 35; y_{\min} = y(4) = 15; y_{\max} = y(6) = 15;$$

$$\text{в) } y_{\max} = y(2) = 19; y_{\min} = y(-2) = -8 - 36 - 48 - 1 = -93;$$

$$\text{г) } y_{\max} = y(5) = 125 - 225 + 120 - 1 = 19; y_{\min} = y(4) = 15.$$

$$942. y' = 3x^2 + 6x - 45 = 0; x^2 + 2x - 15 = 0; x = -5, x = 3;$$

$$y_{\max} = y(-5) = -125 + 75 + 225 - 2 = 173; y_{\min} = y(3) = 27 + 27 - 135 - 2 = -83;$$

$$\text{а) } y_{\max} = 173; y_{\min} = -2;$$

$$\text{б) } y_{\max} = -43; y_{\min} = -72;$$

$$\text{в) } y_{\max} = 173; y_{\min} = 45;$$

$$\text{г) } y_{\max} = -2; y_{\min} = -72.$$

$$943. y' = 3x^2 - 18x + 15 = 0; x^2 - 6x + 5 = 0; x = 5, x = 1;$$

$$y_{\max} = y(1) = 1 - 9 + 15 - 3 = 4; y_{\min} = y(5) = 125 - 225 + 75 - 3 = -28;$$

$$\text{а) } y_{\max} = 4; y_{\min} = -3;$$

$$\text{б) } y_{\max} = -12; y_{\min} = -28;$$

$$\text{в) } y_{\max} = 4; y_{\min} = -28;$$

$$\text{г) } y_{\max} = 4; y_{\min} = -28.$$

944. $y' = 4x^3 - 24x^2 + 20x = 0$; $4x(x^2 - 6x + 5) = 0$; $x = 0$, $x = 5$, $x = 1$; $y(0) = 1$;
 $y_{\max} = y(1) = 1 - 8 + 10 + 1 = 4$; $y_{\min} = y(5) = 625 - 1000 + 250 + 1 = -124$;
 а) $y_{\max} = 10$; $y_{\min} = -7$; б) $y_{\max} = 4$; $y_{\min} = -124$;
 в) $y_{\max} = 121$; $y_{\min} = -44$; г) $y_{\max} = 148$; $y_{\min} = -124$.

945. $y' = 1 - \frac{4}{(x-1)^2} = 0$; $(x-1)^2 = 4$; $x = -1$, $x = 3$;

$y_{\max} = y(3) = 3 + 2 = 5$, $y_{\min} = y(-1) = -1 + \frac{4}{-2} = -1 - 2 = -3$;

а) $y_{\max} = 6$; $y_{\min} = 5$; б) $y_{\max} = -3$; $y_{\min} = -4$.

946. а) $y' = -\frac{1}{\sin^2 x} + 1 \leq 0$; $y_{\max} = 1 + \frac{\pi}{4}$; $y_{\min} = -1 + \frac{3\pi}{4}$;

б) $y' = 2\cos x - 1$; $x = \pm \frac{\pi}{3} + \pi n$, $x = \frac{\pi}{3}$; $y_{\max} = y\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$; $y_{\min} = -\pi$;

в) $y' = -2\sin x + 1$; $\sin x = \frac{1}{2}$; $x = \frac{\pi}{6}$; $y_{\max} = y\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6}$; $y_{\min} = -\frac{\pi}{2}$;

г) $y' = \frac{1}{\cos^2 x} - 1$; $x = \pi n$, $x = 0$; $y_{\max} = y\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$; $y_{\min} = y(0) = 0$.

947. а) $y' = 3x^2 - 4x = 0$; $x(3x - 4) = 0$; $x = 0$, $x = \frac{4}{3}$;

y_{\max} не существует; $y_{\min} = y\left(\frac{4}{3}\right) = \frac{64}{27} - \frac{32}{9} + 1 = -\frac{5}{27}$;

б) $y' = 1 - \frac{1}{\sqrt{x}}$; $x = 1$; y_{\max} не существует; $y_{\min} = y(1) = 1 - 2 = -1$;

в) $y' = x^4 - 2x = 0$; $x(x^3 - 2) = 0$; $x = 0$, $x = \sqrt[3]{2}$; $y_{\max} = y(0) = 0$; y_{\min} не существует;

г) $y' = \frac{4x^3}{(x^4 + 1)^2} = 0$; y_{\max} не существует; $y_{\min} = y(0) = 0$.

948. а) $y' = 1 - \frac{1}{x^2}$; $x = \pm 1$; $y_{\max} = y(-1) = -1 - 1 = -2$; y_{\min} не существует;

б) $y' = \frac{3x^2 + 9 - 6x^2}{(x^2 + 3)^2} = -3 \cdot \frac{x^2 - 3}{x^2 + 3}$; $x = \pm\sqrt{3}$; $y_{\max} = y(\sqrt{3}) = \frac{3\sqrt{3}}{3+3} = \frac{\sqrt{3}}{2}$; $y_{\min} = y(0) = 0$;

в) $y' = -2 + \frac{1}{2x^2} = 0$; $4x^2 = 1$; $x = \pm \frac{1}{2}$; $y_{\max} = y\left(\frac{1}{2}\right) = -1 - 1 = -2$; y_{\min} не существует;

г) $y' = \frac{1}{\sqrt{2x+6}} - 1 = 0$; $\sqrt{2x+6} = 1$; $x = -\frac{5}{2}$; $y_{\max} = y\left(-\frac{5}{2}\right) = 1 + \frac{5}{2} = 3,5$, y_{\min} не существует.

$$949. \text{ а) } \begin{cases} a+b=24 \\ ab=\max \end{cases}; \begin{cases} a=24-b \\ 24b-b^2=y \end{cases}; y'=24-2b; b=12, y(12)=144; b=12, a=12.$$

$$\text{ б) } ab=484; a=\frac{484}{b}; \frac{484}{b}+b=y; y'=1-\frac{484}{b^2}; b=22, a=22.$$

$$950. \text{ а) } a-b=10; a=10+b; 10b+b^2=y; y'=2b+10; b=-5, a=5.$$

$$\text{ б) } a-b=98; a=98+b; b^2+98b=y; y'=2b+98; b=-49, a=49.$$

$$951. \text{ а) } a(a+36)=y; a^2+36a=y; y'=2a+36; a=-18, b=-18+36=18.$$

$$\text{ б) } a(a-28)=y; a^2-28a=y; y'=2a-28; a=14, b=-14.$$

$$952. \text{ а) } \begin{cases} a+b=3 \\ y=3a+b^3 \end{cases}; \begin{cases} a=3-b \\ y=3a+b^3 \end{cases}; y=9-3b+b^3; y'=3b^2-3; b=\pm 1, \text{ но т.к. по условию } b>0, \text{ то } b=1, a=2.$$

$$\text{ б) } \begin{cases} a+b=5 \\ y=ab^3 \end{cases}; \begin{cases} a=5-b \\ y=ab^3 \end{cases}; e=5b^3-b^4; y'=15b^2-4b^3=b^2(15-4b); b=0, b=\frac{15}{4}$$

$$\text{ но т.к. по условию } b>0, \text{ то } b=\frac{15}{4}; a=\frac{5}{4}.$$

$$953. \text{ а) } \begin{cases} 2a+2b=56 \\ ab=y \end{cases}; \begin{cases} a=28-b \\ 28b-b^2=y \end{cases}; y'=28-2b; 2b=28; b=14, a=14;$$

$$\text{ б) } \begin{cases} a+b=36 \\ ab=y \end{cases}; \begin{cases} a=36-b \\ 36b-b^2=y \end{cases}; y'=36-2b; 2b=36; b=18, a=18.$$

$$954. \text{ а) } \begin{cases} a+b=100 \\ ab=y \end{cases}; \begin{cases} a=100-b \\ 100b-b^2=y \end{cases}; y'=100-2b; 2b=100; b=50, a=50;$$

$$\text{ б) } \begin{cases} a+b=120 \\ ab=y \end{cases}; \begin{cases} a=120-b \\ 120b-b^2=y \end{cases}; y'=120-2b; 2b=120; b=60, a=60.$$

$$955. \text{ а) } \begin{cases} ab=16 \\ 2a+2b=y \end{cases}; \begin{cases} a=\frac{16}{b} \\ 2a+2b=y \end{cases}; y=\frac{32}{b}+2b; y'=2-\frac{32}{b^2}; b=4, a=4;$$

$$\text{ б) } \begin{cases} ab=64 \\ 2a+2b=y \end{cases}; \begin{cases} a=\frac{164}{b} \\ 2a+2b=y \end{cases}; y=\frac{128}{b}+2b; y'=2-\frac{128}{b^2}; b=8, a=8.$$

$$956. \begin{cases} ab=2500 \\ 2a+2b=y \end{cases}; \begin{cases} a=\frac{2500}{b} \\ 2a+2b=y \end{cases}; y=\frac{5000}{b}+2b; y'=2-\frac{5000}{b^2}; b=50, a=50.$$

$$957. KD=DM=x; BH_1=\frac{3\sqrt{2}}{2}; DH_2=\frac{x\sqrt{2}}{2} (H_1 \text{ и } H_2 - \text{ точки пересечения } BD \text{ с } PE \text{ и } KM \text{ соответственно});$$

$$S = \frac{1}{2}(PE + KM) \cdot H_1 H_2 = \frac{1}{2}(x\sqrt{2} + 3\sqrt{2}) \left(8\sqrt{2} - \frac{3\sqrt{2}}{2} - \frac{x\sqrt{2}}{2} \right) = \frac{1}{2}(39 + 10x - x_2);$$

$$S' = \frac{1}{2}(10 - 2x) = 0; x = 5; S = \frac{1}{2}(39 + 50 - 25) = 32.$$

958. а) $y_{\max} = \sqrt{2}; y_{\min} = 0;$

б) $y_{\max} = \sqrt{2}; y_{\min} = 1;$

в) $y_{\max} = \sqrt{2}; y_{\min} = 0;$

г) $y_{\max} = \sqrt{2}; y_{\min} = 0.$

959. а) $y = 2 - 5\sin(x + a); y_{\max} = 7; y_{\min} = -3;$

б) $y = 3\sin x - 4\cos x + l; y = 5\sin(x - \alpha) + l; y_{\max} = 6, y_{\min} = -4.$

960. а) $y' = 4x^3 + 24x^2 + 48x + 32 = 0; x^3 + 6x^2 + 12x + 8 = 0; (x + 2)(x^2 - 2x + 4 + 6x) = 0;$
 $(x + 2)(x^2 + 4x + 4) = (x + 2)^3 = 0; x = -2;$

$y(-2) = 16 - 64 + 96 - 64 + 21 = 133; y(-3) = 81 - 216 + 216 - 96 + 21 = 6;$

$y(0) = 21; y_{\min} = y(-2) = 5, y_{\max} = y(6) = 21.$

б) $y' = 4x^3 - 12x^2 + 12x - 4 = 0; x^3 - 3x^2 + 3x - 1 = 0;$

$(x - 1)^3 = 0; x = 1; y_{\max} = y(4) = 71, y_{\min} = y(1) = 1 - 4 + 6 - 4 - 9 = -10.$

961. а) $y = x^2 - 5|x| + 6, [0; 4];$ на этом промежутке $x \geq 0 \Rightarrow y = x^2 - 5x + 6; y' = 2x - 5;$

$x = \frac{5}{2}; y_{\max} = y(0) = 6; y_{\min} = y\left(\frac{5}{2}\right) = \frac{25}{4} - \frac{25}{2} + 6 = -\frac{25}{4} + 6 = -\frac{1}{4}.$

б) $y = x^2 - 5|x| + 6, [-5; 0];$ на этом промежутке $x \leq 0 \Rightarrow y = x^2 + 5x + 6; y' = 2x + 5;$

$x = -\frac{5}{2}; y_{\max} = y(0) = 6; y_{\min} = y\left(-\frac{5}{2}\right) = \frac{25}{4} - \frac{25}{2} + 6 = -\frac{25}{4} + 6 = -\frac{1}{4}.$

в) $y = x^2 + 8|x| + 7, [1; 5];$ на этом промежутке $x \geq 0 \Rightarrow y = x^2 + 8x + 7; y' = 2x + 8;$
 $x = -4$ — не подходит;

$y_{\max} = y(5) = 25 + 40 + 7 = 72; y_{\min} = y(1) = 16.$

г) $y = x^2 + 8|x| + 7, [-8; -2];$ на этом промежутке $x \leq 0 \Rightarrow y = x^2 - 8x + 7; y' = 2x - 8 -$
 $-x = 4$ — не подходит; $y_{\max} = y(-8) = 64 + 64 + 7 = 135; y_{\min} = y(-2) = 4 + 32 + 7 = 43.$

962. а) $y = x^3 - 3x, (-\infty; 0]; y' = 3x^2 - 3; x = -1; y_{\max} = y(-1) = -1 + 3 = 2; y_{\min}$ — не существует.

б) $y = x^3 - 3x, [0; +\infty); y_{\min} = y(1) = -2; y_{\max}$ — не существует.

963. а) $y' = \frac{x^4 + 3 - 4x^2}{(x^4 + 3)^2} = \frac{3 - 3x^4}{(x^4 + 3)^2}; x = \pm 1; y_{\max} = y(1) = \frac{1}{4}; y_{\min}(0) = 0;$

б) $y_{\min} = y(-1) = -\frac{1}{4}; y_{\max} = y(0) = 0.$

964. а) $y = \sin^2 \frac{x}{2} \sin x = \frac{1}{2} \sin x - \frac{1}{2} \cos x \sin x = \frac{1}{2} \sin x - \frac{1}{4} \sin 2x;$

$y' = \frac{1}{2} \cos x - \frac{1}{2} \cos 2x = 0; \sin \frac{3x}{2} \sin \frac{x}{2} = 0; x = 2\pi n, x = \frac{2\pi n}{3}; x = -\frac{2\pi}{3};$

$y\left(-\frac{2\pi}{3}\right) = -\frac{3}{4} \cdot \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{8} = y_{\min}; y_{\max} = 0.$

$$6) y' = -\cos \frac{x}{2} \cdot \sin \frac{x}{2} \cos x - \cos^2 \frac{x}{2} \sin x = 0; \cos \frac{x}{2} \left(\sin \frac{x}{2} \cos x + \cos \frac{x}{2} \sin x \right) = 0;$$

$$\cos \frac{x}{2} \cdot \sin \frac{3x}{2} = 0; x = \pi + 2\pi n, x = \frac{2\pi k}{3};$$

$$y(0) = 1; y(\pi) = 0; y\left(\frac{2\pi}{3}\right) = \frac{1}{4} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{8}; y_{\min} = -\frac{1}{8}, y_{\max} = 1.$$

$$965. \text{ а) I: } x \geq 1; y = x^2 - 3x + 4; y' = 2x - 3 = 0; x = \frac{3}{2};$$

$$y(1) = 1 - 4 + 5 + (1 - 1) = 2; y\left(\frac{3}{2}\right) = \frac{9}{4} - 6 + 5 + \left(1 - \frac{3}{2}\right) = \frac{7}{4}; y(4) = 8;$$

$$\text{II: } x \leq 1; y = x^2 - 5x + 6; y' = 2x - 5 = 0; x = \frac{5}{2} \text{ — не подходит; } y(0) = 6,$$

$$y_{\min} = \frac{7}{4}; y_{\max} = 8;$$

$$6) \text{ I: } x \geq 1; y = x^3 - 3x - 1; y' = 3x^2 - 3; x = 1; y(1) = -3; y(3) = -17;$$

$$\text{II: } x \leq 1; y = 1 - 3x - x^3; y' = -3 - 3x^2; y(1) = -3 = y_{\max}; y(-1) = 5; y_{\min} = -3; y_{\max} = 17.$$

$$966. \text{ а) } y' = 2 - \frac{8}{\sqrt{16x-4}}; x = \frac{5}{4}; y\left(\frac{5}{4}\right) = \frac{5}{2} - 4 = -\frac{3}{2}; y\left(\frac{1}{4}\right) = \frac{1}{2}; y\left(\frac{17}{4}\right) = \frac{17}{2} - 8 = \frac{1}{2}; y \in \left[-\frac{3}{2}; \frac{1}{2}\right].$$

$$6) y' = \frac{1}{\sqrt{x-1}} - \frac{1}{2}; x = 5; y(5) = 4 - \frac{5}{2} = \frac{3}{2}; y(1) = -\frac{1}{2}; y(10) = 6 - 5 = 1; y \in \left[-\frac{1}{2}; \frac{3}{2}\right].$$

$$967. \text{ а) } y' = \sqrt{x+2} + \frac{x}{2\sqrt{x+2}} = \frac{2x+4+x}{2\sqrt{x+2}} = \frac{3x+4}{2\sqrt{x+2}};$$

$$x = -\frac{4}{3}; y\left(-\frac{4}{3}\right) = -\frac{4}{3}\sqrt{-\frac{4}{3}+2} = -\frac{4\sqrt{6}}{9}; y \in \left[-\frac{4}{9}\sqrt{6}; +\infty\right);$$

$$6) y' = \sqrt{1-2x} - \frac{x}{\sqrt{1-2x}} = \frac{1-2x-x}{\sqrt{1-2x}} = \frac{1-3x}{\sqrt{1-2x}}; x = \frac{1}{3};$$

$$y\left(\frac{1}{3}\right) = \frac{1}{3}\sqrt{1-\frac{2}{3}} = \frac{1}{3\sqrt{3}}; y \in \left(-\infty; \frac{\sqrt{3}}{9}\right].$$

$$968. \text{ а) } y' = \sqrt{x+a} + \frac{x}{2\sqrt{x+a}} = \frac{3x+2a}{2\sqrt{x+a}}; x = -\frac{2a}{3}; y\left(-\frac{2a}{3}\right) = -\frac{2a}{3}\sqrt{\frac{a}{3}} = -6\sqrt{3};$$

$$a\sqrt{a} = 27; a = 27^{\frac{2}{3}} = 9.$$

$$6) y' = -\sqrt{x} + \frac{a-x}{2\sqrt{x}} = \frac{a-x-2x}{2\sqrt{x}} = \frac{a-3x}{2\sqrt{x}}; x = \frac{a}{3}; y\left(\frac{a}{3}\right) = \frac{2a}{3}\sqrt{\frac{a}{3}} = 10\sqrt{5};$$

$$a\sqrt{a} = 15\sqrt{15}; a = 15.$$

$$969. \text{ а) } a_0 = 1; \begin{cases} a_1 + 8d = 1 \\ (a_1 + 3d)(a_1 + 6d)(a_1 + 7d) = y \end{cases}; \begin{cases} a_1 = 1 - 8d \\ (1 - 5d)(1 - 2d)(1 - d) = y \end{cases};$$

$$y = (1 + 10d^2 - 7d)(1 - d) = 1 - d + 10d^2 - 10d^3 - 7d + 7d^2 = 1 - 8d + 17d^2 - 10d^3;$$

$$y' = -8 + 34d - 30d^2 = 0; 15d^2 - 17d + 4 = 0; D = 289 - 240 = 49;$$

$$d_1 = \frac{17+7}{30} = \frac{4}{5}; d_2 = \frac{1}{3}; y\left(\frac{4}{5}\right) = (1-4)\left(1-\frac{8}{5}\right)\left(1-\frac{4}{5}\right) = -3 \cdot \left(-\frac{3}{5}\right) \frac{1}{5} = \frac{9}{25};$$

$$y\left(\frac{1}{3}\right) = \left(1-\frac{5}{3}\right)\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) = -\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = -\frac{4}{27}; d = \frac{4}{5}.$$

$$\text{ б) } \begin{cases} a_1 + d = 6 \\ (a_1 + 2d)a_1(a_1 + 5d) = y \end{cases}; \begin{cases} a_1 = 6 - d \\ (6 + d)(6 - d)(6 + 4d) = y \end{cases}; y = (36 - d^2)(6 + 4d) =$$

$$= 216 + 144d - 6d^2 - 4d^3; y' = 144 - 12d - 12d^2 = 0; d^2 + d - 12 = 0; d_1 = 3; d_2 = -4;$$

$$y(3) = 9 \cdot 3 \cdot 18 = 486; y(-4) = 2 \cdot 10 \cdot (-10) = -200; d = -4.$$

$$970. \text{ а) } y_1 = 2x^2; y_2 = 4x. \text{ Длина отрезка равна } 4x - 2x^2 = f(x), x \in [0; 2]; f'(x) = 4 - 4x;$$

$$x_0 = 1; f(1) = 4 - 2 = 2;$$

$$\text{ б) } y_1 = x^2; y_2 = -2x. \text{ Длина отрезка равна } -2x - x^2 = f(x), x \in [-2; 0]; f'(x) = -2 - 2x;$$

$$x_0 = -1; f(-1) = 2 - 1 = 1.$$

$$971. \text{ а) } \sqrt{(0-x)^2 + (1,5-y)^2} = f(x); f(x) = \sqrt{x^2 + (1,5-x^2)^2} = \sqrt{x^2 + \frac{9}{4} - 3x^2 + x^4} =$$

$$= \sqrt{x^4 - 2x^2 + \frac{9}{4}}; f'(x) = \frac{4x^3 - 4x}{2\sqrt{x^4 - 2x^2 + \frac{9}{4}}} = 2 \cdot \frac{x^3 - x}{\sqrt{x^4 - 2x^2 + \frac{9}{4}}};$$

$$x = 0, x = \pm 1; f'(0) = \frac{3}{2}; f'(1) = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2} = f(-1); (1; 1), (-1; 1);$$

$$\text{ б) } f(x) = \sqrt{(4,5-x)^2 + y^2} = \sqrt{\frac{81}{4} - 9x + x^2 + x} = \sqrt{\frac{81}{4} - 8x + x^2};$$

$$f'(x) = \frac{2x-8}{2\sqrt{\frac{81}{4} - 8x + x^2}}; x = 4; y(4) = 2; (4; 2).$$

972. Пусть h – высота трапеции;

$$S = \frac{1}{2} \left((15 + 2\sqrt{225 - h^2} + 15) h \right) = (15 + \sqrt{225 - h^2}) h;$$

$$S' = 15 + \sqrt{225 - h^2} - \frac{h^2}{\sqrt{225 - h^2}}; 15\sqrt{225 - h^2} + 225 - 2h^2 = 0;$$

$$50\,625 - 225h^2 = 50\,625 - 900h^2 + 4h^2; 4h^2 - 675h^2 = 0; h^2(4h^2 - 675) = 0;$$

$$h^2 = \frac{675}{4}; a = 15 + 2\sqrt{225 - \frac{675}{4}} = 15 + 2 \cdot \frac{15}{2} = 30.$$

973. а) Пусть α – угол между основанием и боковой стороной x – сторона прямоугольника, которая совпадает с высотой, y – его другая стороной.

$$\text{Тогда } \operatorname{tg} \alpha = 5 = \frac{x}{80-y}; 80-y = \frac{x}{5}; y = 80 - \frac{x}{5}; S = 80x - \frac{x^2}{5}; S' = 80 - \frac{2x}{5};$$

$$x = 200, \text{ но } x \in (0; 100] \Rightarrow x = 100, y = 60; S = 6000.$$

$$\text{б) } a = 24, b = 8, h = 12.$$

Пусть α – угол между большим основанием трапеции и ее боковой стороной, x – сторона прямоугольника, которая совпадает с высотой, y – его другая сторона.

$$\operatorname{tg} \alpha = \frac{3}{4} = \frac{x}{24-y}; 24-y = \frac{4x}{3}; y = 24 - \frac{4x}{3}; S = 24x - \frac{4x^2}{3}; S' = 24 - \frac{8x}{3};$$

$$x = 9, y = 12; S = 108.$$

974. а) Пусть x – сторона прямоугольника, лежащая на AB , y – его другая сторона, $\alpha = \angle DCB$.

$$\text{Тогда } \operatorname{tg} \alpha = \frac{7-5}{9-3} = \frac{2}{6} = \frac{1}{3} = \frac{x}{9-y}; x = 3 - \frac{1}{3}y; y = -3x + 9; S = -3x^2 + 9x;$$

$$S' = -6x + 9; x = \frac{3}{2}; y = -\frac{9}{2} + 9 = \frac{9}{2}, \text{ но } AE \cdot AB = 21 \Rightarrow S_{\max} = 21.$$

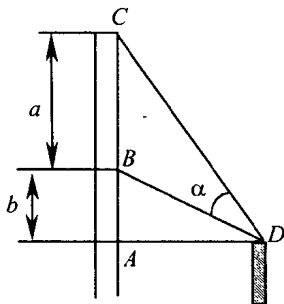
$$\text{б) } a = 7, b = 18, c = 3, m = 1;$$

Пусть x – сторона прямоугольника, лежащая на AB , y – его другая сторона, $\alpha = \angle DCB$.

$$\text{Тогда } \operatorname{tg} \alpha = \frac{7-1}{18-3} = \frac{2}{5} = \frac{x}{18-y}; 18-y = \frac{5}{2}x; y = 18 - \frac{5}{2}x;$$

$$S = 18x - \frac{5}{2}x^2; S' = 18 - 5x; x = \frac{18}{5}; y = 9; S = 32,4; AE \cdot AB = 21 \Rightarrow S_{\max} = 32,4.$$

$$975. AC = a; AB = b; CB = AC - AB = a - b; AD = x; BD = \sqrt{x^2 + b^2}; CD = \sqrt{x^2 + a^2}.$$



$$\text{По теореме косинусов } (a-b)^2 = x^2 + b^2 + x^2 + a^2 - 2\sqrt{(x^2 + b^2)(x^2 + a^2)} \cos \alpha;$$

$$2x^2 + 2ab = 2\sqrt{(x^2 + b^2)(x^2 + a^2)} \cos \alpha; \cos \alpha = \frac{x^2 + ab}{\sqrt{(x^2 + b^2)(x^2 + a^2)}};$$

$$f(x) = \frac{x^2 + ab}{\sqrt{(x^2 + b^2)(x^2 + a^2)}} = \frac{x^2 + ab}{\sqrt{x^4 + x^2(a^2 + b^2) + b^2a^2}};$$

$$f'(x) = \frac{2x\sqrt{x^4 + x^2(a^2 + b^2)} + b^2a^2 - \frac{(x^2 + ab)(4x^3 + 2x(a^2 + b^2))}{2\sqrt{x^4 + x^2(a^2 + b^2)} + b^2a^2}}{(x^2 + b^2)(x^2 + a^2)} = 0;$$

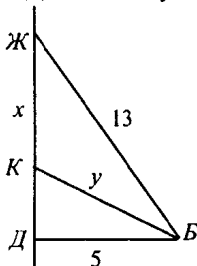
$$2x\sqrt{(x^2 + b^2)(x^2 + a^2)} = x(x^2 + ab) \frac{2x^2 + a^2 + b^2}{\sqrt{(x^2 + b^2)(x^2 + a^2)}};$$

$$2(a^2b^2 + x^2a^2 + x^2b^2 + x^4) = 2x^4 + 2x^2ab + x^2a^2 + x^2b^2 + a^3b + ab^3;$$

$$2ab(ab - x^2) + a^2(x^2 - ab) + b^2(x^2 - ab) = 0; (x^2 - ab)(a - b)^2 = 0; x = \sqrt{ab};$$

$$\alpha_{\max} \text{ при } x = \sqrt{ab}.$$

976. ЖД = 12; x – путь пешехода по дороге, y – путь пешехода по лесу.



$$\text{Суммарное время } t = \frac{x}{5} + \frac{y}{3}; DK = 12 - x;$$

$$y = \sqrt{25 + (12 - x)^2}; t' = \frac{1}{5} + \frac{2x - 24}{2 \cdot 3\sqrt{x^2 - 24x + 169}} = \frac{1}{5} + \frac{x - 12}{3\sqrt{x^2 - 24x + 169}} = 0;$$

$$\frac{3}{5}\sqrt{x^2 - 24x + 169} = 12 - x; 9x^2 - 9 \cdot 24x + 9 \cdot 169 = 25(144 - 24x + x^2);$$

$$16x^2 - 384x + 2079 = 0; x_1 = 8,25; x_2 = 15,75 - \text{не подходит};$$

$$x = \frac{33}{4}; y = \sqrt{25 + 3,75^2} = \frac{25}{4}; t = \frac{x}{5} + \frac{4}{5} = \frac{33}{20} + \frac{25}{12} = \frac{56}{15} \approx 3 \text{ часа } 44 \text{ минуты}.$$

977. Пусть x – длина стороны основания бака, y – его высота.

$$V = x^2y = 32; S = x^2 + 4xy = x^2 + \frac{128}{x}; S'(x) = 2x - \frac{128}{x^2} = 0; x^3 = 64; x = 4, y = 2.$$

Ответ: 4 дм, 4 дм, 2 дм.

978. $V = x^2y = 343$ (x – длина стороны основания бака, y – его высота);

$$S = 2x^2 + 4xy = 2x^2 + \frac{1372}{x}; S' = 4x - \frac{1372}{x^2} = 0; x = 7, y = 7.$$

Ответ: 7 м, 7 м, 7 м.

979. Пусть $2x$ и $3x$ – длины сторон основания короба, y – его высота.

$$V = 6x^2y = 576; S = 12x^2 + 6xy + 4xy = 12x^2 + \frac{960}{x}; S' = 24x - \frac{960}{x^2} = 0;$$

$$x^3 = 40; x_0 = 2\sqrt[3]{5}, y_0 = \frac{576}{6} \cdot \frac{1}{4\sqrt[3]{25}} = 24 \frac{\sqrt[3]{5}}{5}.$$

Ответ: $4\sqrt[3]{5}$ м; $6\sqrt[3]{5}$ м; $\frac{24\sqrt[3]{5}}{5}$ м.

980. $V = \sqrt{(d^2 - x^2)}x$ (x — длина бокового ребра призмы);

$$V' = d^2 - 3x^2 = 0; x = \frac{d}{\sqrt{3}} = \frac{\sqrt{3}}{3}d.$$

981. $a = 2\sqrt{p^2 - h^2}; S_{\text{осн}} = 4(p^2 - h^2); V = \frac{1}{3} \cdot 4(p^2 - h^2)h;$

$$V'(h) = \frac{4}{3}(p^2 - 3h^2) = 0; h = \frac{f}{\sqrt{3}} = \frac{p\sqrt{3}}{3}.$$

982. Пусть x — диаметр цилиндра, h — его высота; $2x + 2h = p;$

$$V = \pi \left(\frac{x}{2}\right)^2 h = \pi \left(\frac{p-2h}{2}\right)^2 h; V' = \left(\frac{\pi}{4}(p^2 - 4ph + 4h^2)h\right) = \frac{\pi}{4}(p^2 - 8ph + 12h^2) = 0;$$

$$12h^2 - 8ph + p^2 = 0; h_{1,2} = \frac{8p \pm 4p}{24}; h_1 = \frac{p}{6}; h_2 = \frac{p}{2};$$

h_2 — не подходит, т.к. $2h_2 = p$, тогда $x = 0$, чего быть не может; значит $h = \frac{p}{6}.$

983. $S = 2\pi Rh + 2\pi R^2$ — площадь боковой поверхности; $V = \pi R^2 h;$

$$h = \frac{V}{\pi R^2}; S = \frac{2V}{R} + 2\pi R^2; S'(R) = -2\frac{V}{R^2} + 4\pi R = 0; R^3 = \frac{V}{2\pi}; R = \sqrt[3]{\frac{V}{2\pi}}.$$

Содержание

Глава 1. ТРИГОНОМЕТРИЧЕСКИЕ ФУНКЦИИ

§ 1. Введение	3
§ 2. Числовая окружность	4
§ 3. Числовая окружность на координатной плоскости	7
§ 4. Синус и косинус	10
§ 5. Тангенс и котангенс	15
§ 6. Тригонометрические функции числового аргумента	16
§ 7. Тригонометрические функции углового аргумента	21
§ 8. Формулы приведения	23
§ 9. Функция $y = \sin x$, ее свойства и график	26
§ 10. Функция $y = \cos x$, ее свойства и график	34
§ 11. Периодичность функции $y = \sin x$, $y = \cos x$	40
§ 12. Как построить график функции $y = mf(x)$, если известен график функции $y = f(x)$	41
§ 13. Как построить график функции $y = f(kx)$, если известен график функции $y = f(x)$	44
§ 14. График гармонического колебания	48
§ 15. Функции $y = \operatorname{tg} x$, $y = \operatorname{ctg} x$, их свойства и графики	48

Глава 2. ТРИГОНОМЕТРИЧЕСКИЕ УРАВНЕНИЯ

§ 16. Первые представления о решении тригонометрических уравнений	54
§ 17. Арккосинус и решение уравнения $\cos t = a$	56
§ 18. Арксинус и решение уравнения $\sin t = a$	59
§ 19. Арктангенс и решение уравнения $\operatorname{tg} x = a$. Арктангенс и решение уравнения $\operatorname{ctg} x = a$	62
§ 20. Тригонометрические уравнения	66

Глава 3. ПРЕОБРАЗОВАНИЕ ТРИГОНОМЕТРИЧЕСКИХ ВЫРАЖЕНИЙ

§ 21. Синус и косинус суммы аргументов	79
§ 22. Синус и косинус разности аргументов	82
§ 23. Тангенс суммы и разности аргументов	87
§ 24. Формулы двойного аргумента	91
§ 25. Формулы понижения степени	100
§ 26. Преобразование сумм тригонометрических функций в произведение	107
§ 27. Преобразование произведений тригонометрических функций в сумму	117
§ 28. Преобразование выражения $A \sin x + B \cos x$ к виду $C \sin(x+t)$	121

Глава 4. ПРОИЗВОДНАЯ

§ 29. Числовые последовательности	129
§ 30. Предел числовой последовательности	136
§ 31. Предел функции	144
§ 32. Определение производной	152
§ 33. Вычисление производных	153
§ 34. Уравнение касательной к графику функции	164
§ 35. Применение производной для исследования функций на монотонность и экстремумы	170
§ 36. Применение производной для отыскания наибольших и наименьших значений величин	195

САМ СЕБЕ РЕПЕТИТОР®

Учебное издание

Белова Анна Александровна

**Алгебра и начала анализа
подробный разбор заданий из учебника
авторов: А.Г. Мордкович, Л.О. Денищева, Т.А. Корешкова и др.**

**+ Задачи повышенной сложности
+ Задачи на построение**

Налоговая льгота — ОКП 005-93-953. (Литература учебная)
Издательство «ВАКО». Изд. лицензия: ИД № 03063 от 18.10.2000.

Подписано к печати с диапозитивов 28.06.2004.
Бумага типографская № 2. Формат 60 × 90/16. Печать офсетная.
Гарнитура Таймс. Усл. печ. листов 13.
Тираж 10 000 экз. Заказ № 4113.

Отпечатано в полном соответствии с качеством
предоставленных диапозитивов на ФГУП ордена «Знак Почета»
Смоленская областная типография им. В. И. Смирнова.
214000, г. Смоленск, проспект им. Ю. Гагарина, 2